

## A LEVEL

*Examiners' report*

# **MATHEMATICS A**

**H240**

For first teaching in 2017

## **H240/01 Summer 2018 series**

Version 1

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## Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

## Paper H240/01 series overview

H240/01 is one of the three examination components for the new revised A Level examination for GCE Mathematics A. It is a two hour paper consisting of 100 marks, which tests Pure Mathematics topics. Pure Mathematics topics are also tested on the first half of Papers 2 and 3, and any Pure Mathematics topic could be tested on any of the three papers.

This was the first live paper for this new A Level and all the candidates had prepared for this examination in one year. The marks were generally very good as many candidates are also Further Mathematics candidates.

To do well on this paper, candidates need to be familiar with all of the Pure content of the specification (section 1) and be able to apply their knowledge of Pure Mathematics to problems set in context. They are expected to make efficient use of their calculator where appropriate; for example, this could include the solution of quadratic equations rather than having to factorise or use the quadratic formula.

When a question requires 'detailed reasoning' candidates are expected to show full and convincing detail in their solution. On all questions candidates should show their method clearly, which allows partial credit to be awarded should a fully correct answer not be achieved. They should justify steps taken in their method, including giving a reason when discarding solutions to an equation that are not valid.

When multiple attempts are made at a question, candidates should clearly identify which attempt they wish to be marked and cross out all other attempts. Not doing so will result in the final attempt being marked, even if this is not best solution. Candidates should ensure that their presentation is of a standard that allows the examiner to discern their intent, and this includes ensuring that handwriting is legible.

If questions are set in context then the candidate's response should also be in context, including units as appropriate. When considering the limitations of a model, or the implications for long term behaviour of a model, then candidates should ensure that their responses are sufficiently detailed, precise and include examples as appropriate. They should also be careful about including additional, incorrect, statements.

## Question 1

- 1 The points  $A$  and  $B$  have coordinates  $(1, 5)$  and  $(4, 17)$  respectively. Find the equation of the straight line which passes through the point  $(2, 8)$  and is perpendicular to  $AB$ . Give your answer in the form  $ax + by = c$ , where  $a$ ,  $b$  and  $c$  are constants. [4]

This question was very well answered, with most candidates gaining full credit. As with all solutions, it is important for candidates to show clear details of the method used; on this question some candidates made an error when finding the gradient. If the correct method was shown then a mark was credited, but if an incorrect value for the gradient was all that was shown then the method mark cannot be credited. This question requested that the final answer be given in a specified form, so candidates need to pay heed to this. Non-integer values for  $a$ ,  $b$  and  $c$  were accepted but best practice would be to use integer coefficients when giving the equation of a line in this form.

## Question 2 (i)

- 2 (i) Use the trapezium rule, with four strips each of width 0.5, to estimate the value of

$$\int_0^2 e^{x^2} dx$$

giving your answer correct to 3 significant figures. [3]

This question was also very well answered, with many fully correct solutions seen. The most successful candidates started by identifying the relevant  $x$  values, used exact  $y$  values rather than decimal approximations and made effective use of brackets.

## Question 2 (ii)

- (ii) Explain how the trapezium rule could be used to obtain a more accurate estimate. [1]

The best explanations referred to using more trapezia, of a narrower width, over the same interval. Many candidates mentioned either using more trapezia or decreasing the width; giving just one reason was condoned in the mark scheme.

## Question 3

- 3 In this question you must show detailed reasoning.

Find the two real roots of the equation  $x^4 - 5 = 4x^2$ . Give the roots in an exact form. [4]

Most candidates were able to provide sufficient detail to be convincing in this 'detailed reasoning' question. It was expected that candidates would show clearly how they solved the disguised quadratic; this was usually by factorising (with or without a substitution) but some used the formula instead. There also had to be a clear reason given as to why  $x^2 = -1$  had no roots. Some candidates referred to the roots being imaginary (content from Further Maths) whereas others explained that square numbers had to be greater than or equal to zero; just stating that  $\sqrt{-1}$  could not be done was not sufficient.

Exemplar 1

3	$x^4 - 5 = 4x^2$
	$x^4 - 4x^2 - 5 = 0$ Let $x^2 = y$
	then $y^2 - 4y - 5 = 0$
	$(y-5)(y+1) = 0$
	$y-5=0$ $y+1=0$
	$y=5$ $y=-1$
	so $x^2 = 5$ but this doesn't
	$x = \pm\sqrt{5}$ lead to real
	solutions as $x^2 \neq -1$
	If $x \in \mathbb{R}$
	then $x^2 \in \mathbb{R} : x^2 \geq 0$
	$x = \sqrt{5}$
	$x = -\sqrt{5}$

This response gives a clear justification for why  $x^2 = -1$  does not lead to any real solutions, thus gaining full credit for this solution.

Question 4

4 Prove algebraically that  $n^3 + 3n - 1$  is odd for all positive integers  $n$ . [4]

Candidates were expected to provide an algebraic proof, considering both odd and even  $n$  and justifying their method. Many candidates made it clear which case was being considered, whereas others introduced  $2k$  and  $2k + 1$ , but with no explanation of what these terms represented. The algebraic manipulation was usually correct and candidates then wrote the expressions in a useable form. Some candidates then simply stated that the expression must be odd, but did not justify this statement. A number of candidates provided a worded argument based on the use of odd and even numbers. As long as each step was explained then partial credit was given, but full credit in this question was only available for convincing use of algebra.

## Exemplar 2

4	<p><math>n^3 + 3n - 1</math>, where <math>n</math> is a positive integer.</p> <p>If <math>n</math> is a +ve integer, it can either be an even number or an odd number.</p> <p>Consider case where <math>n</math> is even:</p> <p><math>n = 2p</math></p> <p>Case I: <math>(2p)^3 + 3(2p) - 1</math>  <math>= 8p^3 + 6p - 1</math>  <math>= 2(4p^3 + 3p) - 1</math> (<math>\because 4p^3 + 3p \in \mathbb{Z}</math>)  <math>= 2k - 1</math> which is an odd number  <math>\therefore</math> when <math>n</math> is even, <math>n^3 + 3n - 1</math> is odd.</p> <p>Case II: Consider when <math>n</math> is odd  <math>n = 2q + 1</math></p> <p>Case I: <math>(2q + 1)^3 + 3(2q + 1) - 1</math>  <math>= (2q + 1)^2(2q + 1) + 6q + 3 - 1</math>  <math>= (4q^2 + 4q + 1)(2q + 1) + 6q + 3 - 1</math>  <math>= 8q^3 + 4q^2 + 8q^2 + 4q + 2q + 1 + 6q + 3 - 1</math>  <math>= 8q^3 + 12q^2 + 12q + 3</math>  <math>= 8q^3 + 12q^2 + 12q + 2 + 1</math>  <math>= 2(4q^3 + 6q^2 + 6q + 1) + 1</math>  <math>= 2j + 1</math> (<math>\because 4q^3 + 6q^2 + 6q + 1 \in \mathbb{Z}</math>)          which is odd <math>\therefore</math> when <math>n</math> is odd, <math>n^3 + 3n - 1</math> is also odd.</p> <p>Therefore in both cases, <math>n^3 + 3n - 1</math> is odd  <math>\therefore n^3 + 3n - 1</math> is odd for all positive integers <math>n</math>.</p>
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This candidate states that they are using  $2p$  to represent an even number; simply using  $2p$  with no justification would not have gained the first mark. Having substituted and rearranged they then clearly explain that their expression is of the form  $2k - 1$ , which represents an odd number. An alternative explanation would be to say that for all  $p$ ,  $2(4p^3 + 3p)$  is even as it has a factor of 2 hence  $2(4p^3 + 3p) - 1$  is odd. Simply concluding 'odd' but with no justification would not have gained the second mark. This candidate's proof of why the expression is odd when  $n$  is odd is equally convincing, so this response gained full credit.

### Question 5 (i)

5 The equation of a circle is  $x^2 + y^2 + 6x - 2y - 10 = 0$ .

(i) Find the centre and radius of the circle.

[3]

Solutions to this question were nearly always correct, with most candidates choosing to write the equation in factorised form. There were a few sign errors when stating the centre of the circle, and also a few errors when subtracting the constant term when completing the square each time.

### Question 5 (ii)

(ii) Find the coordinates of any points where the line  $y = 2x - 3$  meets the circle  $x^2 + y^2 + 6x - 2y - 10 = 0$ .

[4]

All candidates attempted to solve the equations simultaneously, either using the expanded equation of the circle or the factorised equation. As this question did not specify 'detailed reasoning', it was expected that candidates would solve the ensuing quadratic on their calculator but instead most still showed the factorisation.

### Question 5 (iii)

(iii) State what can be deduced from the answer to part (ii) about the line  $y = 2x - 3$  and the circle  $x^2 + y^2 + 6x - 2y - 10 = 0$ .

[1]

Part (ii) shows one point of intersection so it was expected that candidates would put this information into context and conclude that the line was a tangent to the circle. If an error had happened in part (ii) resulting in other than one point of intersection then candidates could still get this mark for a correct deduction from their answer.

### Question 6 (i)

6 The cubic polynomial  $f(x)$  is defined by  $f(x) = 2x^3 - 7x^2 + 2x + 3$ .

(i) Given that  $(x - 3)$  is a factor of  $f(x)$ , express  $f(x)$  in a fully factorised form.

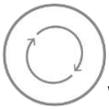
[3]

Most candidates correctly found the quadratic quotient, with the most common methods being algebraic long division and the grid method, which tended to be equally successful. They could then factorise the quadratic quotient correctly. The question asks for  $f(x)$  to be given in fully factorised form, so candidates were expected to give their final answer as the product of three factors. A few candidates found the roots from their calculator and then attempted to work backwards but no correct solutions were seen as the factor was given as  $(x + 0.5)$  rather than the correct factorised form  $(2x + 1)$  or  $2(x + 0.5)$ .

## Question 6 (ii)

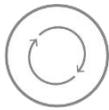
- (ii) Sketch the graph of  $y = f(x)$ , indicating the coordinates of any points of intersection with the axes. [2]

Most candidates were able to provide a sketch of a positive cubic, including the correct behaviour at the extremities. Some candidates marked the points on the axes first resulting in a distorted graph as they then tried to fit the cubic through the points. Candidates were asked for the coordinates of any points of intersection, and most did so although a few omitted the coordinate on the y-axis.



Whilst this question asked for the coordinates, it is good practice to always provide them on a sketch graph even if not explicitly requested.

## Key



Guidance to offer for future teaching and learning practice

## Question 6 (iii)

- (iii) Solve the inequality  $f(x) < 0$ , giving your answer in set notation. [2]

Set notation is new to this specification and some candidates were clearly familiar with the topic whereas others were not able to use the notation correctly, achieving only partial credit.

## Exemplar 3

6(iii)	$x < -\frac{1}{2}, 1 < x < 3$
	$\{x: x < -\frac{1}{2}\} \cup \{x: 1 < x < 3\}$

This response gives a fully correct solution, using correct set notation, so gains full credit.

## Exemplar 4

6(iii)	$(2x+1)(x-1)(x-3) < 0$
	$f(x) \in (-\infty, -0.5) \cup (1, 3)$

This response demonstrates an alternative, but equally, valid use of set notation and also gains full credit.

Exemplar 5

6(iii)	$f(x) < 0 \quad 2x^3 - 7x^2 + 2x + 3 < 0$
	<del>Handwritten scribbles</del>
	$x < -\frac{1}{2}; \quad x < -\frac{1}{2} \quad 1 < x < 3$
	$x < -\frac{1}{2} \cup 1 < x < 3$

*Note: The handwritten work includes green checkmarks for the individual inequalities and a red 'X' for the union notation. Marking boxes 'M1' and 'A0' are present.*

This response gains one mark for identifying at least one correct set of values using inequalities. The use of  $\cup$  shows some familiarity with set notation, but the structure is not fully correct so they do not get the second mark.

Question 6 (iv)

- (iv) The graph of  $y = f(x)$  is transformed by a stretch parallel to the  $x$ -axis, scale factor  $\frac{1}{2}$ . Find the equation of the transformed graph. [2]

Most candidates were able to replace  $x$  with  $2x$  to obtain a correct equation although a few gave it as an expression, omitting the 'y =' at the start. The best solutions made effective use of brackets to show  $x$  being replaced with  $2x$ , and then expanded them. Other candidates simply wrote down an equation; if this was wrong then there was no evidence to justify that they had been attempting the correct method, but with an error on substitution, as opposed to attempting to double each term.

Question 7 (i)

- 7 Chris runs half marathons, and is following a training programme to improve his times. His time for his first half marathon is 150 minutes. His time for his second half marathon is 147 minutes. Chris believes that his times can be modelled by a geometric progression.
- (i) Chris sets himself a target of completing a half marathon in less than 120 minutes. Show that this model predicts that Chris will achieve his target on his thirteenth half marathon. [4]

Having been told that the training programme was being modelled by a geometric progression, candidates were able to identify the first term and common ratio and use these in the  $n$ th term formula. Some candidates demonstrated that the time on the thirteenth half marathon was indeed less than 120 minutes, but to answer the question fully they also had to show that the time on the twelfth half marathon was more than 120 minutes. Other candidates set up an inequality and solved this for  $n$ , but the signs were not always correct throughout. As this is a modelling question the answer was expected to be in context namely referring to the thirteenth half marathon, although just 'marathon' or 'run' were also condoned.

Question 7 (ii)

- (ii) After twelve months Chris has spent a total of 2974 minutes, to the nearest minute, running half marathons. Use this model to find how many half marathons he has run. [3]

Candidates could correctly equate the expression for the sum of  $n$  terms to 2974, and most were able to solve this equation. Most candidates showed a clear mathematical justification even though there was no specific command in the question requiring this level of detail for full credit. Once again, the final answer was expected to be given in context.

## Question 7 (iii)

- (iii) Give two reasons why this model may not be appropriate when predicting the time for a half marathon. [2]

Candidates were required to consider the limitations of the model, focusing both on the variations between each run and the long term implications. For the first limitation, some candidates were able to give a clear explanation along with an example, often mentioning injury or weather conditions, whereas others explanations were too vague to be given credit. Candidates would be well advised in such questions to give specific examples to support their answer. For the second mark there had to be some consideration that Chris would eventually reach a point where no further improvements would be possible. Once again, there were some eloquently phrased explanations, sometimes giving a specific example. Candidates should ensure that their handwriting is legible, and their phrasing clear, in order to convey their intent to an examiner.

## Exemplar 6

7(iii) Will depend on external factors like weather, injury, tiredness etc. ✓  
 Using this model, will continue to improve until gets times that aren't realistic (very very small) ✓

This response gains one mark for identifying that external factors could affect Chris' performance and gives three examples. Any one of these factors would have been sufficient to gain the mark, but a specific example was required. The second mark is credited for identifying that eventually the model would predict very small times, which is not realistic.

## Exemplar 7

7(iii) Chris' times could have fluctuated for some half marathons and assuming that Chris gets faster, when he could have been slower (Time  $\rightarrow 0$ ). BO BO

This response identifies that times could fluctuate but does not give a specific example so the first mark is not credited. The candidate states that the model assumes that Chris gets faster but does not comment on how realistic this is so the second mark is not credited either. In fact the final part of the sentence suggests that this is actually supporting the idea of fluctuating times, but it is still not specific.

### Question 8 (i)

- 8 (i) Find the first three terms in the expansion of  $(4-x)^{-\frac{1}{2}}$  in ascending powers of  $x$ . [4]

This question was well answered with many fully correct solutions seen. The most successful solutions made effective use of brackets to ensure that the relevant indices were applied to the entire term including the coefficient of  $x$ . Showing clear working allowed partial credit to be credited for attempting a valid method should the final answer not be correct.

### Question 8 (ii)

- (ii) The expansion of  $\frac{a+bx}{\sqrt{4-x}}$  is  $16-x \dots$ . Find the values of the constants  $a$  and  $b$ . [3]

This part of the question was also very well answered. Most candidates appreciated the connection with the previous part of the question, and equated coefficients to find the values of  $a$  and  $b$ . An alternative method was to cross-multiply and then square both sides before equating coefficients, and this was usually equally successful.

### Question 9 (i)

- 9 The function  $f$  is defined for all real values of  $x$  as  $f(x) = c + 8x - x^2$ , where  $c$  is a constant.

- (i) Given that the range of  $f$  is  $f(x) \leq 19$ , find the value of  $c$ . [3]

Many fully correct solutions to this question were seen, with candidates employing a variety of different methods. The most common approaches were to write  $f(x)$  in completed square form and equate the maximum value to 19, or to use differentiation to identify the maximum point. Some candidates attempted rearranged to obtain  $f(x) - 19 \leq 0$  and attempted to use the discriminant, but only the most able identified that the condition for repeated roots should then be used.

### Question 9 (ii)

- (ii) Given instead that  $ff(2) = 8$ , find the possible values of  $c$ . [4]

All candidates understood the meaning of  $ff(2)$  and were able to attempt the correct process for the composition of functions. The more successful method was to first find  $f(2)$ , simplify this to  $c + 12$  and then attempt  $f(c + 12)$ . Candidates who attempted to find  $ff(x)$  before substituting  $x = 2$  were more likely to make mistakes when simplifying their algebraic expression. It was expected that candidates would use their calculators to solve the quadratic equation, but the vast majority instead showed full detail of the method used.

**Question 10 (i)**

10 A curve has parametric equations  $x = t + \frac{2}{t}$  and  $y = t - \frac{2}{t}$ , for  $t \neq 0$ .

- (i) Find  $\frac{dy}{dx}$  in terms of  $t$ , giving your answer in its simplest form. [4]

Most candidates gained at least 3 marks for obtaining a correct expression for the derivative, but the subsequent simplification proved to be more challenging. Candidates who worked with fractions tended to be more successful than those who worked with negative indices. The most common error was 'cancelling' just a single term in the numerator and denominator.

**Question 10 (ii)**

- (ii) Explain why the curve has no stationary points. [2]

Whilst most candidates were able to attempt the correct method, many did not give sufficient detail of their reasoning. Rather than just equating the derivative to 0 examiners expected to see some justification for this, such as a statement that the gradient is 0 at a stationary point. Candidates were then expected to explain why the equation  $t^2 + 2 = 0$  had no real roots before concluding that this meant there were no stationary points.

**Question 10 (iii)**

- (iii) By considering  $x + y$ , or otherwise, find a cartesian equation of the curve, giving your answer in a form not involving fractions or brackets. [4]

Many candidates were able to use the hint given in the question to produce  $x + y = 2t$  but a number were then unsure how to make any further progress. Others appreciated that they could now use this equation to eliminate  $t$  from one of the given parametric equations, but errors when rearranging to the requested form were common, demonstrating a lack of confidence when dealing with algebraic fractions. A minority of candidates ignored the hint given in the question, and attempted to use an alternative method such as rearranging to  $t^2 - xt + 2 = 0$ , solving for  $t$  and substituting into the other equation. Whilst some progress was usually made, these attempts were rarely fully correct. The most elegant solution, provided by a few candidates, was to consider both  $x + y$  and  $x - y$  to produce two equations from which  $2t$  could then be easily eliminated.

### Question 11 (i)

11 In a science experiment a substance is decaying exponentially. Its mass,  $M$  grams, at time  $t$  minutes is given by  $M = 300e^{-0.05t}$ .

- (i) Find the time taken for the mass to decrease to half of its original value. [3]

This question was very well answered with candidates identifying the initial mass, and then setting up and solving a relevant equation. Most candidates worked exactly throughout to provide a sufficiently accurate final answer.

### Question 11 (ii)

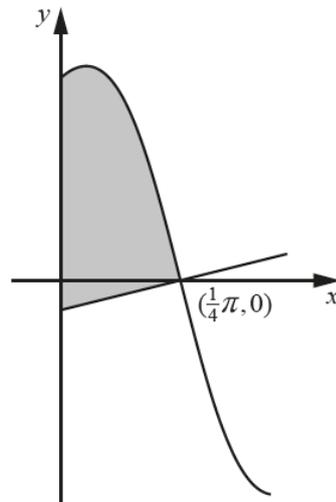
A second substance is also decaying exponentially. Initially its mass was 400 grams and, after 10 minutes, its mass was 320 grams.

- (ii) Find the time at which both substances are decaying at the same rate. [8]

The vast majority of candidates were able to make some progress on this question, and a number of fully correct solutions were seen. Candidates appreciated the need to find an expression for the mass of the second substance, and were able to make a good attempt at finding the two parameters. As the substance was decaying, some candidates used an initial structure of  $M = Ae^{-kt}$ , but sign errors were relatively common when substituting back for  $k$ . A few candidates simply equated the two expressions for the mass, but most realised that it was the derivatives that should be equated and made a reasonable attempt to do so. Solving the ensuing equation was found to be challenging. Some attempted to rearrange first whereas others introduced logarithms straightaway. Sign errors were common, especially in solutions where candidates were working exactly as the coefficient of  $0.1\ln 0.8$  is not obviously negative. Some candidates spoilt an otherwise correct solution by not working to a sufficient degree of accuracy throughout their solution resulting in an incorrect final answer.

## Question 12

12 In this question you must show detailed reasoning.



The diagram shows the curve  $y = \frac{4 \cos 2x}{3 - \sin 2x}$ , for  $x \geq 0$ , and the normal to the curve at the point  $(\frac{1}{4}\pi, 0)$ . Show that the exact area of the shaded region enclosed by the curve, the normal to the curve and the  $y$ -axis is  $\ln \frac{9}{4} + \frac{1}{128}\pi^2$ . [10]

This was a question requiring detailed reasoning, so candidates were expected to show sufficient evidence of method throughout which not seen on all scripts. Candidates were generally successful when using the quotient rule to find the gradient of the curve. They were then expected to show evidence of finding the gradient at the given point; whilst some showed clear detail, others just stated the gradient with no justification. There were a variety of methods seen when finding the area of the triangle; the most common was to find the  $y$  intercept from the equation of the normal whereas others used integration. Some candidates identified that the  $\frac{1}{128}\pi^2$  in the given answer must be the area of the triangle and attempted to fudge their answer to obtain this, in some cases deleting correct work and replacing it with a solution that was now worth less credit. Most candidates were able to correctly integrate the equation of the curve, some by inspection and others by using a substitution of their choosing. The limits were usually used correctly, but not all candidates provided sufficient evidence of use of logarithms before the appearance of  $\ln \frac{9}{4}$ .

Exemplar 8

12

$$y = \frac{4 \cos 2x}{3 - \sin 2x} \quad y' = \frac{(3 - \sin 2x)(-8 \sin 2x) - (4 \cos 2x)(-2 \cos 2x)}{(3 - \sin 2x)^2}$$

$$= \frac{8 \sin^2 2x - 24 \sin 2x + 8 \cos^2 2x}{(3 - \sin 2x)^2} = \frac{8 - 24 \sin 2x}{(3 - \sin 2x)^2}$$

m at  $\frac{1}{4}\pi$

$$= \frac{8 - 24 \sin \frac{\pi}{2}}{(3 - \sin \frac{\pi}{2})^2} = -4$$

∴ m of normal =  $\frac{1}{4}$

$$y = \frac{1}{4}(x - \frac{1}{4}\pi)$$

$$= \frac{1}{4}x - \frac{1}{16}\pi$$

∴ intersects y-axis at  $-\frac{1}{16}\pi$

$$\int_0^{\frac{\pi}{4}} \frac{4 \cos 2x}{3 - \sin 2x} dx = 2 \int_0^{\frac{\pi}{4}} \frac{2 \cos 2x}{3 - \sin 2x} dx \quad \left\{ \frac{f'(x)}{f(x)} = \ln f(x) \right.$$

$$= [2 \ln(3 - \sin 2x)]_0^{\frac{\pi}{4}}$$

$$= -2 \ln(2) + 2 \ln 3 = 2 \ln \frac{3}{2}$$

$$= \ln \left(\frac{3}{2}\right)^2$$

$$= \ln \frac{9}{4}$$

Area between normal and line  $y=0$  (x-axis)

$$= \frac{1}{2} \times \frac{1}{16}\pi \times \frac{1}{4}\pi = \frac{\pi^2}{128}$$

∴ Total Area =  $\frac{\pi^2}{128} + \ln \frac{9}{4}$

$$= \ln \frac{9}{4} + \frac{\pi^2}{128}$$

This response gained full credit for a correct solution with sufficient justification seen throughout. The derivative is correct, and there is clear evidence of substitution to find the gradient at  $x = \frac{1}{4}\pi$ . Had the gradient of  $-4$  just been stated, but with no evidence, then this would have been penalised. The equation of the normal is used to find the intercept on the y-axis, and this is used to find the area of the triangle. The integration is also correct and limits are used correctly. There is then clear evidence of at least one log law being used to obtain the correct area under the curve. The two areas are then summed to justify the given answer.

Question 13 (i) (a)

13 A scientist is attempting to model the number of insects,  $N$ , present in a colony at time  $t$  weeks. When  $t = 0$  there are 400 insects and when  $t = 1$  there are 440 insects.

(i) A scientist assumes that the rate of increase of the number of insects is inversely proportional to the number of insects present at time  $t$ .

(a) Write down a differential equation to model this situation.

[1]

The majority of candidates could state a correct differential equation, including a coefficient of proportionality. The most common error was to have the equation as a function of  $t$  not of  $N$ .

### Question 13 (i) (b)

(b) Solve this differential equation to find  $N$  in terms of  $t$ .

[4]

Candidates who had stated a correct equation in part (a) were usually able to rearrange and integrate correctly. They were then able to use the given boundary conditions to find a particular solution to the differential equation. Having got this far, a few candidates did not gain the final mark due to making errors when rearranging the equation to the requested form, such as square rooting term by term.

### Question 13 (ii)

(ii) In a revised model it is assumed that  $\frac{dN}{dt} = \frac{N^2}{3988e^{0.2t}}$ . Solve this differential equation to find  $N$  in terms of  $t$ . [6]

Candidates identified the need to separate the variables before integrating and many made a good attempt to do so. The majority of candidates chose to leave the 3988 where it was rather than incorporating it with the term in  $N$ . This caused problems for some when rewriting the right-hand side in a form that could be integrated, with  $3988e^{-0.2t}$  being the most common error. When finding a value for the constant of integration most candidates opted to use the first condition given. Once again, not all candidates were able to use the correct method to find  $N$  in terms of  $t$ ; the most common error was to simply invert each of the three terms in their equation.

### Question 13 (iii)

(iii) Compare the long-term behaviour of the two models.

[2]

When considering the first model, candidates simply needed to identify that it predicted that the population would continue to increase and a number of candidates were able to do so. Some spoilt an otherwise correct statement by including additional, incorrect detail such as increasing at a faster rate. Only the most able candidates were able to both identify that the second model would predict that the population will tend towards a limit and also identify the value of the limit. There was no credit for commenting on how realistic each of the two models was, but some candidates did consider this.

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