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A LEVEL

Examiners' report

MATHEMATICS A

H240
For first teaching in 2017

H240/02 Summer 2018 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper H240/02 series overview

This is the first example of the A-level Pure Mathematics and Statistics paper produced for the new specification. It differs slightly in style from papers in previous years in the following ways.

- There is a greater emphasis on proof.
- There is a greater emphasis on interpretation, rather than calculation.
- It reflects the fact that candidates are expected to have used the Pre-release Large Data Set as a tool for studying statistical concepts and techniques.
- It assumes the use of more powerful calculators than in the past. This affects the paper in two ways.
    1. There are fewer marks for certain types of skill, such as calculating binomial probabilities and solving quadratic equations.
    2. Some questions contain the instruction **In this question you must show detailed working**. In these questions, candidates are required to demonstrate their understanding of the relevant concepts by showing their working, rather than by presenting an answer gained simply by pressing a few buttons. Consequently, in these questions, marks will not be credited unless correct working is seen.
- The paper is divided into separate Pure Maths and Statistics sections.

Most candidates found this paper very accessible, with some candidates gaining very high marks. However, some of the final section B questions appeared rushed or not attempted, suggesting that perhaps some candidates appeared to run out of time. In each section, the questions are placed approximately in order of increasing difficulty. Consequently, some candidates would be well advised to attempt the first few (easier) Pure Maths questions and the first few (easier) Statistics questions, before attempting the more challenging questions in each section.

Many candidates used trial and improvement methods, for example in questions 5(iii), 13 (i) and 13(iii)(a). Some marks are available for these methods, but not necessarily full marks. In practice, many of these candidates scored no marks for their attempts. With some exceptions, such methods are generally to be discouraged.

For answers to all the questions, centres are referred to the published mark scheme.

Section A overview

Candidates handled all the traditional questions well. They also dealt well with the interpretation required in question 3. Question 5 covered "proof" rather more fully than in the past, and was less well answered, although a significant number of candidates did score well on this question.

Question 1(i)

1 **(i)** Express $2x^2 - 12x + 23$ in the form $a(x+b)^2 + c$. **[4]**

Most candidates answered this correctly. A few found *a* and *b* correctly but made an error in finding *c*. This most frequently came from an incorrect first step such as $2(x – 3)^2 + 11.5 – 9$ or $2(x – 3)^2 + 23 – 9$.

Question 1(ii)

(ii) Use your result to show that the equation $2x^2 - 12x + 23 = 0$ has no real roots. **[1]**

Most candidates answered this correctly. Some of those who discussed the turning point lost a mark because they merely stated that it is a minimum, rather than showing that this is so.

Question 1(iii)

(iii) Given that the equation $2x^2 - 12x + k = 0$ has repeated roots, find the value of the constant k. **[2]**

This question was very well answered.

Question 2(i)

2 The points A and B have position vectors $\begin{pmatrix}1\\-2\\5\end{pmatrix}$ and $\begin{pmatrix}-3\\-1\\2\end{pmatrix}$ respectively.

(i) Find the exact length of AB. **[2]**

This question was very well answered. A few candidates made sign errors.

Question 2(ii)

(ii) Find the position vector of the midpoint of AB. **[1]**

A surprisingly large number of candidates simply halved their answer to part (i). Perhaps they did not understand the concept of a "position vector".

Question 2(iii)

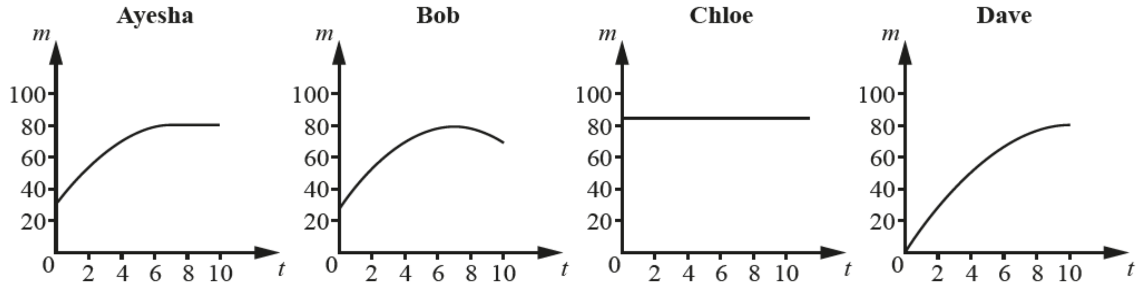
The points P and Q have position vectors $\begin{pmatrix}1\\2\\0\end{pmatrix}$ and $\begin{pmatrix}5\\1\\3\end{pmatrix}$ respectively.

(iii) Show that $ABPQ$ is a parallelogram. **[3]**

Only a minority of candidates answered this question in the most efficient way, using the vector forms of one pair of sides. Many found the vector forms of <u>both</u> pairs of sides. Then they commented either that both pairs consisted of two parallel lines, or they found the lengths of all four sides and commented that both pairs consisted of lines of equal length. Some candidates found the vector form for two opposite sides and then stated that because these two sides are parallel, *ABPQ* is a parallelogram. A few candidates made sign errors while finding vectors. Some did not use correct vector notation. Some candidates (quite reasonably) replaced the column vector notation by the **i**, **j**, **k** notation. Some candidates discussed the "gradients" of opposite sides.

Question 3(i)

3 Ayesha, Bob, Chloe and Dave are discussing the relationship between the time, t hours, they might spend revising for an examination, and the mark, m, they would expect to gain. Each of them draws a graph to model this relationship for himself or herself.



(i) Assuming Ayesha's model is correct, how long would you recommend that she spends revising? **[1]**

Almost all candidates answered this correctly.

Question 3(ii)

(ii) State one feature of Dave's model that is likely to be unrealistic. **[1]**

Most candidates gave a good answer. A few candidates gave an answer appropriate for part (iii) rather than (ii).

Question 3(iii)

(iii) Suggest a reason for the shape of Bob's graph as compared with Ayesha's graph. **[1]**

Most candidates gave a good answer. A few merely described the shape rather than suggesting a reason.

Question 3(iv)

(iv) What does Chloe's model suggest about her attitude to revision? **[1]**

Most candidates gave a good answer.

Question 4

4 Prove that $\sin^2(\theta + 45)° - \cos^2(\theta + 45)° \equiv \sin 2\theta°$. **[4]**

A large variety of correct methods were seen, some shorter than others. Some candidates made mistakes in quoting formulae, despite the formulae being given on the question paper.

Question 5

This question reflects various aspects of the new specification, paragraph 1.01: **Proof**.

Question 5(i)

5 Charlie claims to have proved the following statement.

"The sum of a square number and a prime number cannot be a square number."

(i) Give an example to show that Charlie's statement is not true. **[1]**

Almost all candidates answered this correctly

Question 5(ii)

Charlie's attempt at a proof is below.

Assume that the statement is not true.
$\Rightarrow$ There exist integers n and m and a prime p such that $n^2 + p = m^2$.
$\Rightarrow p = m^2 - n^2$
$\Rightarrow p = (m - n)(m + n)$
$\Rightarrow p$ is the product of two integers.
$\Rightarrow p$ is not prime, which is a contradiction.
$\Rightarrow$ Charlie's statement is true.

(ii) Explain the error that Charlie has made. **[1]**

Many candidates identified the error correctly although a large variety of incorrect suggestions were seen, such as "He has proved something that is false is correct, but that doesn't prove his point correct.", "He assumes that the statement is not true.", "*p* could be 0", "$m + n$ could be zero.", "He hasn't chosen m and n to be integers.", "p is not the product of two integers, $p = m^2 – n^2$.", "He has not stated that m and n are square numbers.".

Question 5(iii)

(iii) Given that 853 is a prime number, find the square number S such that $S + 853$ is also a square number. **[4]**

Some candidates recognised that the starting point was $m - n = 1$. Most of these proceeded to obtain the correct answer (although a few squared 427 instead of 426). Many candidates, however, did not appreciate the link with part (ii) and attempted trial and improvement, without success.

Question 6(i)

6 **In this question you must show detailed reasoning.**

A curve has equation $y = \frac{\ln x}{x}$.

(i) Find the x-coordinate of the point where the curve crosses the x axis. **[2]**

This question was well answered.

Question 6(ii)

(ii) The points A and B lie on the curve and have x coordinates 2 and 4. Show that the line AB is parallel to the x-axis. **[2]**

Many candidates just stated or implied that $\frac{\ln 2}{2} - \frac{\ln 4}{4} = 0$, either without proof, or by using their calculator and decimals. These candidates scored no marks, because of the "**detailed reasoning**" instruction.

Question 6(iii)

(iii) Find the coordinates of the turning point on the curve. **[4]**

This question was well answered. A few candidates did not find the *y*-coordinate. Some made mistakes in the differentiation.

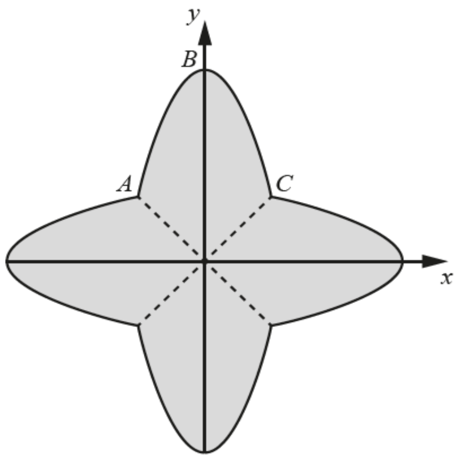
Question 6(iv)

(iv) Determine whether this turning point is a maximum or a minimum. **[5]**

Most candidates attempted a correct method. Some made mistakes in the differentiation. Others made numerical errors when substituting $x = \mathrm{e}$ into $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. Some considered the gradient on either side of the turning point, generally correctly. In both this part and part (iii), candidates who used "e" throughout, rather than its approximate decimal value, produced neater and more efficient solutions.

Question 7

7 The diagram shows a part ABC of the curve $y = 3 - 2x^2$, together with its reflections in the lines $y = x$, $y = -x$ and $y = 0$.



Find the area of the shaded region. **[7]**

A large variety of correct methods were seen. Some were unnecessarily long. Examples of correct, although long, methods were these:

1. Find the inverse function in order to find the equation of the reflection of the given curve in $y = x$. Then solve this with the given function in order to find the point of intersection, C.

2. Rearrange the given function to make x the subject and then find $\int x\,\mathrm{d}y$.

Some candidates found the points where the given curve cuts the x-axis and hence integrated with incorrect limits. Many made mistakes in trying either to add or subtract all or part of the area of the middle square.

Perhaps the neatest method was $4 \times \int_{-1}^{1} (3 - 2x^2 - 1)\,\mathrm{d}x + 4$.

Section B overview

Most candidates seemed well prepared for the content in this section. Answers to the two hypotheses questions (9 and 10(iii)) were reasonably good. Answers to question 11 suggested that most candidates were familiar with the Pre-release material and were accustomed to making interpretative inferences from diagrams and sets of data extracted from this material.

Questions 8(i)(a), 8(i)(b), 8(i)(c), 10 and 13(i) involve normal distributions. Some candidates used the standard normal variable, *z*, in order to answer these questions. Perhaps this reflects habits ingrained in centres over many years of teaching the previous syllabus. In fact, the new, more powerful calculators (required for this syllabus) provide more efficient methods without needing to resort to the use of *z*. However, it should be noted that question 8(ii) <u>does</u> require an understanding of *z*.

In question 9, some candidates used the normal approximation to the binomial distribution. This is an acceptable method, although it lies just outside the specification (see paragraph 2.04h). In this topic area, candidates only need to be familiar with the content in paragraph 2.04d. For an illustration of the level of detail required, see the mark scheme for question 13(i).

Question 8(i)(a)

8 **(i)** The variable X has the distribution N(20, 9).

(a) Find $P(X > 25)$. **[1]**

Most candidates answered this question correctly. A few used a standard deviation of 9.

Question 8(i)(b)

(b) Given that $P(X > a) = 0.2$, find a. **[1]**

Some candidates found $\phi^{-1}(0.2) = 17.5$. A few used a standard deviation of 9

Question 8(i)(c)

(c) Find b such that $P(20 - b < X < 20 + b) = 0.5$. **[3]**

Many candidates could not make the first step, which is to move from the given probability of 0.5 to a probability of either 0.25 or 0.75

Question 8(ii)

(ii) The variable Y has the distribution $N(\mu, \frac{\mu^2}{9})$. Find $P(Y > 1.5\mu)$. **[3]**

Many candidates chose a value for μ, without justification. These could score two out of the three marks. Many candidates used a standard deviation of $\frac{\mu^2}{9}$ and so found that μ did not cancel out. Some could not handle the simple algebra involved in substituting $X = 1.5\mu$ into $\frac{X-\mu}{\frac{\mu}{3}}$.

Question 9

9 Briony suspects that a particular 6-sided dice is biased in favour of 2. She plans to throw the dice 35 times and note the number of times that it shows a 2. She will then carry out a test at the 4% significance level. Find the rejection region for the test. **[7]**

This question was well answered by many candidates. A common error was the omission of a definition of p in the hypotheses. To gain full marks candidates had to use an exact method, using the binomial distribution. This involved finding $P(X \geq 10)$ and $P(X \geq 9)$ or a similar method. Many candidates stated that they were finding, for example, $P(X < 10)$ but their result was actually the value for $P(X \leq 10)$. (Perhaps this is due to insufficient familiarity with the use of the binomial function on the calculator). Some candidates found only one of $P(X \geq 10)$ and $P(X \geq 9)$. These lost a mark. Some gave completely correct probabilities, but gave an incorrect final answer of $X \geq 10$ instead of $X \geq 11$.

Some candidates misread the question and attempted actually to carry out a test.

Some candidates used the normal approximation to the binomial distribution. (These were able to gain a maximum of six marks out of the seven). Many of these candidates arrived correctly at $a = 9.69$, but then gave an answer of $X \geq 10$.

Question 10(i)

10 A certain forest contains only trees of a particular species. Dipak wished to take a random sample of 5 trees from the forest. He numbered the trees from 1 to 784. Then, using his calculator, he generated the random digits 14 781 049. Using these digits, Dipak formed 5 three-digit numbers. He took the first, second and third digits, followed by the second, third and fourth digits and so on. In this way he obtained the following list of numbers for his sample.

147 478 781 104 49

(i) Explain why Dipak omitted the number 810 from his list. **[1]**

This question was very well answered.

Question 10(ii)

(ii) Explain why Dipak's sample is not random. **[1]**

Only a minority of candidates saw the point. Some suggested reasons such as "Only 3-digit numbers can be selected", "1- or 2-digit numbers are less likely than 3-digit", "It's systematic, not random", "He picked the numbers using a pattern", "He only used these digits so, for example, tree number 324 could not be chosen", "The calculator does not give random numbers", "He chose which numbers to use", Many gave that old favourite fall-back answer (not appropriate here): "Every tree does not have an equal chance of being selected".

Question 10(iii)

The mean height of all trees of this species is known to be 4.2 m. Dipak wishes to test whether the mean height of trees in the forest is less than 4.2 m. He now uses a correct method to choose a random sample of 50 trees and finds that their mean height is 4.0 m. It is given that the standard deviation of trees in the forest is 0.8 m.

(iii) Carry out the test at the 2% significance level. **[7]**

This question was well answered by many candidates. Perhaps the most common error was the omission of a definition of μ in the hypotheses. Some candidates made errors in the standard deviation of $\overline{X}$, such as $\frac{0.8}{50}$. A few candidates gave a "definite" conclusion, such as "The mean height is not less than 4.2". These candidates lost the final mark. A few others omitted to give a conclusion in context, just stating, "There is insufficient evidence to reject H_0".

Some candidates used the "critical value" method, comparing the c.v. with the relevant value of *z*. This method is, of course, acceptable, but with the advent of more powerful calculators, the method which finds the relevant probability (here P($\overline{X}$ < 4.0) and compares this with the significance level is quicker and simpler.

Question 11(i)(a)

11 Christa used Pearson's product-moment correlation coefficient, r, to compare the use of public transport with the use of private vehicles for travel to work in the UK.

(i) Using the pre-release data set for all 348 UK Local Authorities, she considered the following four variables.

Number of employees using public transport	x
Number of employees using private vehicles	y
Proportion of employees using public transport	a
Proportion of employees using private vehicles	b

(a) Explain, in context, why you would expect strong, positive correlation between x and y. **[1]**

Most candidates answered correctly, showing a good understanding of the difference between this part and part (i)(b). A few, however, wrote that as the number of employees using public transport increases, the number using private will decrease.

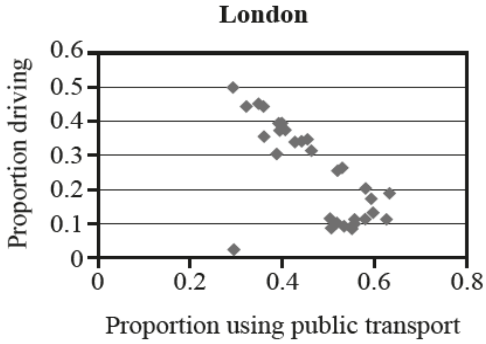
Question 11(i)(b)

(b) Explain, in context, what kind of correlation you would expect between a and b. **[2]**

Many candidates understood the point, although some of these worded their answers badly, referring to the "numbers" (rather than "proportions") using the two types of transport, without making it clear that they were discussing each individual LA rather than all LAs together.

Question 11(ii)(a)

(ii) Christa also considered the data for the 33 London boroughs alone and she generated the following scatter diagram.



One London Borough is represented by an outlier in the diagram.

(a) Suggest what effect this outlier is likely to have on the value of r for the 32 London Boroughs. **[1]**

Some candidates stated that *r* would decrease, or that the value of *r* would decrease, both of which are incorrect. It is possible that what they meant was that the size of *r* would decrease, which is correct, but unfortunately these candidates could not be credited the mark. Some candidates ensured that there was no ambiguity by saying that *r* would "become less negative" or "move closer to 0" or "decrease in magnitude". Some candidates gave inadequate answers such as "It will weaken the correlation" or "It will weaken the value of *r*". There were a few irrelevant answers such as "The outlier will skew the distribution."

Question 11(ii)(b)

(b) Suggest what effect this outlier is likely to have on the value of r for the whole country. **[1]**

Many good answers were seen.

Question 11(ii)(c)

(c) What can you deduce about the area of the London Borough represented by the outlier? Explain your answer. **[1]**

Most candidates recognised the key factor - that a tiny proportion drive to work. But some candidates mistakenly suggested that this is because there is a great deal of public transport available. Others merely stated that few people drive to work. This was not considered an adequate answer to the question. To gain the mark answers had to fall into one of two categories:

1. A sensible suggestion for a possible reason why in this particular area few people drive.

2. A statement that it is likely that a large proportion walk or cycle to work, or that jobs are generally close to home.

Question 12(i)

12 The discrete random variable X takes values 1, 2, 3, 4 and 5, and its probability distribution is defined as follows.

$$\mathrm{P}(X = x) = \begin{cases} a & x = 1, \\ \frac{1}{2}\mathrm{P}(X = x - 1) & x = 2,3,4,5, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(i) Show that $a = \frac{16}{31}$. **[2]**

This question was well answered on the whole, although a few candidates used the probabilities in the table, just finding $1 - (\frac{8}{31} + \frac{4}{31} + \frac{2}{31} + \frac{1}{31})$.

Question 12(ii)

The discrete probability distribution for X is given in the table.

x	1	2	3	4	5
$\mathrm{P}(X = x)$	$\frac{16}{31}$	$\frac{8}{31}$	$\frac{4}{31}$	$\frac{2}{31}$	$\frac{1}{31}$

(ii) Find the probability that X is odd. **[1]**

This question was well answered.

Question 12(iii)

Two independent values of X are chosen, and their sum S is found.

(iii) Find the probability that S is odd. **[2]**

Some candidates did not see that their answer to part (ii) could be used, and started from scratch using the probabilities in the table. Many of these omitted at least one possible pair, and others included all possible pairs, but omitted to double their answer. A few candidates ignored the probabilities (and their answer to part (ii)) and assumed the each value of X is equally likely.

Question 12(iv)

(iv) Find the probability that S is greater than 8, given that S is odd. **[3]**

Most candidates recognised the need to find P(S = 9), but some omitted to include both 4, 5 and 5, 4 . Many then correctly divided by their answer to part (iv).

Question 12(v)

Sheila sometimes needs several attempts to start her car in the morning. She models the number of attempts she needs by the discrete random variable Y defined as follows.

$$P(Y = y+1) = \tfrac{1}{2}P(Y = y) \quad \text{for all positive integers } y.$$

(v) Find $P(Y = 1)$. **[2]**

Some candidates recognised the need for an infinite series, but most could not cope with the fact that the first term is unknown. Many candidates thought that Y cannot be 0, hence $P(Y = 0) = 0$ and hence $P(Y = 1) = 0.5 \times 0 = 0$

Question 12(vi)

(vi) Give a reason why one of the variables, X or Y, might be more appropriate as a model for the number of attempts that Sheila needs to start her car. **[1]**

A choice of either *X* or *Y* with a reasonable justification was acceptable. Some candidates felt that it was unrealistic for Sheila to go on trying after five attempts, so *X* is the better model. Others said that she might well need more than five attempts so *Y* is the better model. One ingenious answer was that *X* is better, because it gives a higher chance of the car starting first time! Unfortunately, this answer did not deal with the question as to which model is more <u>appropriate</u>. A common incorrect response was that *Y* is a good model because according to *Y* the probability that the car starts decreases, rather than increases, with each attempt. Others stated that *Y* is <u>not</u> a good model, quoting exactly the same reason. Some answers did not include a choice of either *X* or *Y*. Another answer was that model *Y* implies that the car never starts. Many answers seemed to imply that using model *X*, the probabilities do not decrease.

Question 13(i)

13 In this question you must show detailed reasoning.

The probability that Paul's train to work is late on any day is 0.15, independently of other days.

(i) The number of days on which Paul's train to work is late during a 450-day period is denoted by the random variable Y. Find a value of a such that $P(Y > a) \approx \frac{1}{6}$. **[3]**

Because this question required "detailed reasoning", correct answers did not necessarily score full marks. Thus, for example, some candidates gave the following working:

X ~ B(450. 0.15); $P(X < a) = \frac{5}{6}$; X = 75. These scored only one mark.

Trial and improvement methods only scored marks if they were very clearly explained, with the distribution fully described <u>and</u> with at least two values close to 75 being tried, with the relevant probabilities actually seen.

The better method was to use the normal approximation to the binomial and the fact that approximately $\frac{2}{3}$ of values lie within the range $\mu - \sigma < X < \mu + \sigma$.

Question 13(ii)

In the expansion of $(0.15 + 0.85)^{50}$, the terms involving 0.15^r and 0.15^{r+1} are denoted by T_r and T_{r+1} respectively.

(ii) Show that $\dfrac{T_r}{T_{r+1}} = \dfrac{17(r+1)}{3(50-r)}$. **[3]**

Some candidates appeared not to understand the definition of T_r and T_{r+1}. Many candidates omitted the relevant powers of 0.85 in their expressions for T_r and T_{r+1}. Some wrote the correct expressions for both T_r and T_{r+1}, but were unable to find any common factors to cancel. Many attempted to expand the given binomial expression, but did not include a general term. Some candidates "fudged" the answers giving, for example, $T_r = 0.85(r + 1)$ and $T_{r+1} = 0.15(50 - r)$ etc.

Question 13(iii)(a)

(iii) The number of days on which Paul's train to work is late during a 50-day period is modelled by the random variable X.

(a) Find the values of r for which $\mathrm{P}(X = r) \leqslant \mathrm{P}(X = r + 1)$. **[4]**

Many candidates did not see the connection with part (ii). These went back to the binomial distribution but very few succeeded. Some candidates carried out a correct method but stopped after obtaining $r \leq 6.65$. Others found $r \leq 6.65$ but then gave the answer $r = 6$. A trial and improvement method could score a maximum of 2 marks in this question.

Question 13(iii)(b)

(b) Hence find the most likely number of days on which the train will be late during a 50-day period. **[2]**

Almost no candidates gave a correct solution based on their answer to part (iii)(a). Some used trial and improvement, but did not consider enough values of X. (At least $X = 6$ and 7 were required). Some rounded their figure of 6.65 from part (iii)(a) to 7. This did not score any marks.

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