Qualification Accredited

Oxford Cambridge and RSA

AS LEVEL

Examiners' report

FURTHER MATHEMATICS B (MEI)

H635
For first teaching in 2017

Y413/01 Summer 2018 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects, which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper Y413/01 series overview

June 2018 was the first examination series for the Linear H635 AS Further Maths B (MEI) optional paper Y413 (Modelling with Algorithms). It was extremely pleasing to note that candidates were well prepared for the examination making good attempts on all questions. Candidates seemed to have sufficient time to answer the questions and nearly all seemed to make good use of the Printed Answer Booklet (that is very few answers to one question appeared in the part for another). Based on the responses seen by examiners this series the following general points should be considered by centres in preparation for candidates for future sittings of this paper.

- Diagrams should be completed in pencil so that candidates have the opportunity to rub out and replace their answer rather than trying to correct in pen on what is likely to be a diagram, which does not have sufficient space for multiple attempts. This was especially true in question 1 (Dijkstra's algorithm) in which it was difficult at times for examiners to know if crossing out was candidates' genuinely trying to correct an error or was the candidates attempt to update the working values.
- In essence Y413 is a methods paper and so it is vital that candidates make their method and application of any corresponding algorithm clear; spotting and writing down the solution (for example in sorting questions) will rarely gain any credit.
- The specification (on page 8) contains a section on the meanings of a number of instructions that will be used in examination questions – one of the key words that could appear (as it did in this paper) is the word 'determine' – the word 'determine' in this mathematical context requires that justification should be given for any results found and not just the writing down or stating of the answer.
- Candidates are reminded to use the number of marks available as the main guide to how detailed their answer should be and not the space given in the Printed Answer Booklet. It is always hoped the PAB should provide sufficient space for the candidates' response, which often includes additional space so candidates can correct any errors. With regards to the number of marks provided a guide to the detail of answer required it is very unlikely in the case of, for example, question 3(iii) that a single worded answer of 'quadratic' would score all the 3 marks available.

Question 1(i)

1 Fig. 1 shows a network. The weights on the arcs are distances.

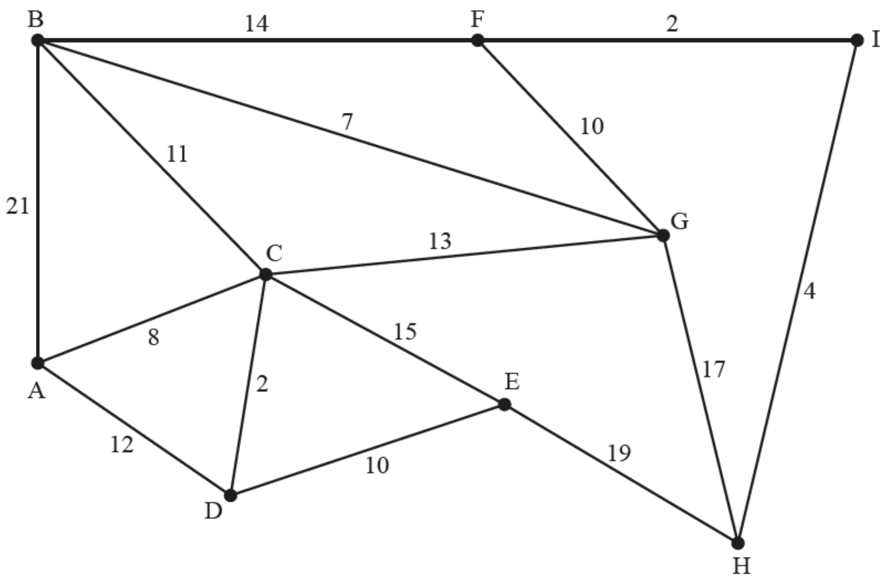

Fig. 1

(i) Apply Dijkstra's algorithm to the copy of the network in the Printed Answer Booklet to find the shortest path from A to H.

State each of the following.

- the shortest path
- its length

[6]

The majority of candidates scored full marks in this first part. Centres should note that it is vital when applying Dijkstra's algorithm that candidates show all the working values at each node (in this case examiners had to see the correct working values of 12 and 10 at D to indicate that the algorithm was initially being applied correctly) and that the order of labelling is correct (a small number of candidates labelled two different nodes with the same label).

AfL

The Printed Answer Booklet provided the standard key for the completion of the boxes at each node together with a specific instruction to candidates that they should not cross out their temporary labels. A number of candidates still did so. It is vital that examiners can read all the values at each node (so that they can check that the algorithm has been applied correctly) and it is strongly recommended that teachers emphasis that no temporary labels are crossed out in classroom practice.

Question 1(ii)

A problem requires a spanning tree that must include the arcs AD and GH.

(ii) The arcs AD and GH are chosen first. Apply Kruskal's algorithm to the ordered list of arcs in the Printed Answer Booklet to complete the tree. You should draw your tree and give its total length. **[4]**

While most candidates answered this part extremely well many did not follow the specific instruction in the question, which was to apply Kruskal's algorithm to the ordered list of arcs in the Printed Answer Booklet. So while some candidates had a correct tree and correct total length they only scored 2 of the 4 marks available as they did not indicate on the list of arcs how they had applied the algorithm. The simplest way to indicate the successful application of the algorithm is to tick the arcs that are to be included in the tree and to put a cross next to (or a single horizontal line through) any arc that is not to be included. A number of candidates did not include the arcs AD and GH in their total and so gave a total length of 36 (rather than the correct 65).

Question 1(iii)

(iii) How could you modify the network in Fig. 1 so that Prim's algorithm would find a spanning tree that includes arcs AD and GH? **[1]**

Candidates struggled with this part with many not realising that one of the most efficient ways of ensuring that arcs AD and GH would be included in a spanning tree when using Prim's algorithm would be to set the weight of these two arcs to zero (or any value such that AD < 8 and GH < 7). A number of candidates incorrectly stated that these two arcs should be included first (however, Prim's algorithm requires that the tree is connected at all times as it grows). Some candidates tried to argue from the point of view of removing arcs from the network and while this approach is appropriate it has to be, the correct arcs that are removed (eg AC and HI). While many candidates correctly stated that removing AC would ensure the inclusion of AD, unfortunately many incorrectly stated that removing FG would ensure that arc GH would be included.

Question 2(i)

2 Fig. 2 represents a system of one-way roads through which traffic flows from a source, S, to a sink, T. The weights on the arcs show the capacities of the roads in cars per minute.

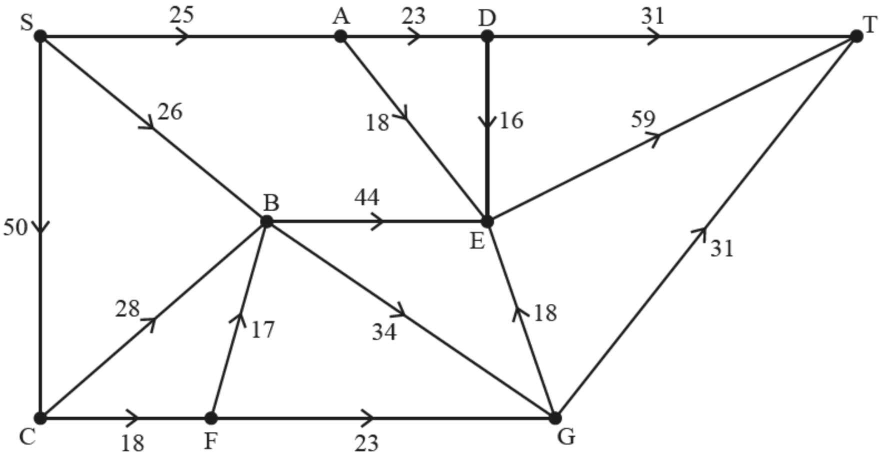

Fig. 2

(i) Explain why the maximum possible flow along FG must be less than 23 cars per minute. **[1]**

Nearly all candidates gave the correct answer that the only flow into FG came from CF and as the maximum capacity of CF was 18 it was therefore not possible to have a maximum flow of 23 cars per minute along FG. It should be noted that in questions like this a specific answer relating to the given network and values given in the question is required and not a general statement about why a flow may not be possible. A number of candidates thought that the number on arc FG implied that the flow (and not the capacity) was 23 and so they gave an argument based on the fact that it could not be less than 23 as it had to be 23.

Question 2(ii)(*A*)

(ii) (*A*) The cut α partitions the vertices into the sets $\{S, C\}$, $\{A, B, D, E, F, G, T\}$. Calculate the capacity of cut α. **[1]**

This part was answered extremely well by the majority of candidates with nearly all giving the correct answer of 97.

Question 2(ii)(*B*)

(*B*) The cut β partitions the vertices into the sets {S, A, C, F, G}, {B, D, E, T}. Calculate the capacity of cut β. **[1]**

Understandably this part was not answered as well as (ii)(a) with many candidates not including all the arcs that went in the direction from the source set of nodes to the sink set (which were AD(23), AE(18), SB(26), CB(28), FB(17), GE(18) and GT(31) giving the correct answer of 161). Some candidates incorrectly included arc BG. As this arc was directed from the sink to source, it should not have been included in the calculation of the capacity of that particular cut.

Question 2(iii)

(iii) Given that one of α or β is a minimum cut, write down the maximum possible flow from S to T. **[1]**

Many candidates correctly applied the maximum flow-minimum cut theorem in this part and realised that the least value from parts (ii)(a) and (b) would be the maximum possible flow through the network.

Question 2(iv)

On a particular morning

- road GE is closed for repairs,
- roads FB, AD and DE are full to capacity.

(iv) Use the diagram in the Printed Answer Booklet to show that the maximum possible flow from part **(iii)** is still achievable when GE is not used and FB, AD and DE are full to capacity. **[2]**

The most common misunderstanding in this part was by some candidates who only showed the flow in one particular path from S to T rather than the flow through the entire network (subject to the conditions given in the question). There are two main points that candidates must consider when tackling this type of question. They are the feasibility condition (which says that the flow along each arc must not exceed the capacity of that arc) and the conservation condition (which says that apart from the source (S) and sink (T) nodes the total flow into a node must equal the total flow out of that node).

Misconception An arc that has no flow passing through it, such as GE in this part, should be labelled with a 0 and not simply left blank.

Question 3(i)

3 A list of n numbers is sorted by making passes through an algorithm as follows.

The first pass consists of the following.

> Compare the first number and second number. If necessary, swap them so that the first number is less than or equal to the second number.

The second pass consists of the following.

> Compare the second number and third number. If necessary, swap them so that the second number is less than or equal to the third number. If a swap occurred compare the new second number and the first number. If necessary, swap them so that the first number is less than or equal to the second number.

…

The $(n-1)$th (final) pass consists of the following.

> Compare the $(n-1)$th number and nth number. If necessary, swap them so that the $(n-1)$th number is less than or equal to the nth number. If a swap occurred compare the new $(n-1)$th number and the $(n-2)$th number. If necessary, swap them so that the $(n-2)$th number is less than or equal to the $(n-1)$th number. Continue in this way until no swap occurs between consecutive numbers or the first number has been swapped with the second number.

(i) Sort the following list of six numbers using the above algorithm

12 34 15 23 10 25

You should only show the result at the end of each pass. **[2]**

This part was answered extremely well with many candidates correctly stating the five passes required to sort the given list of numbers. Although the first pass did not change the order of the list, it still had to be stated to indicate the correct application of the algorithm. A small minority of candidates produced more than five passes in their attempt to sort the list. In these cases examiners assumed that candidates were showing the list after each comparison and swap even though the question specifically told candidates to only show the result at the end of each pass.

Question 3(ii)

The algorithm is now applied to a list of n numbers.

(ii) Find, when $n = 6$,

- the minimum possible number of comparisons needed,
- the maximum possible number of comparisons needed.

[2]

The correct answers of 5 for the minimum (in which all numbers are in the correct order at the start) and 15 for the maximum (in which the list is in reverse order at the start) were the most common answers with very few candidates making errors here.

Question 3(iii)

The number of comparisons is used as a measure of the complexity of the above algorithm.

(iii) Determine, in the worst case, the complexity of the algorithm in terms of n. **[3]**

As this part specifically asked for candidates to 'determine' the complexity of the algorithm in the worst case an answer of 'quadratic' or O(n^2) with no working scored very few marks. It was hoped that candidates would realise the link between parts (ii) and (iii) and therefore spot the fact that in the (generalised) worst case the total number of comparisons (equivalent to the maximum possible number of comparisons) in the first (n – 1) passes would be equal to 1 + 2 + 3 + … + (n – 1). This could then be summed to $\frac{1}{2}n(n-1)$ and therefore the correct deduction of the complexity being O(n^2) could be made.

Question 4(i)(*A*)

4 The table below shows the unit cost, in hundreds of pounds, of transporting goods from each of three suppliers, A, B and C to each of four depots, W, X, Y and Z. The margins of the table show the stock at each supplier and the demand at each depot.

	Demand	10	17	12	11
Stock		W	X	Y	Z
20	A	18	11	14	10
13	B	19	25	15	20
17	C	24	15	20	18

The following LP formulation can be used to find the minimum total cost of delivering all the required stock.

Minimise $18AW + 11AX + 14AY + 10AZ + 19BW + 25BX + 15BY + 20BZ + 24CW + 15CX + 20CY + 18CZ$

subject to

$$AW + AX + AY + AZ = 20$$
$$BW + BX + BY + BZ = 13$$
$$CW + CX + CY + CZ = 17$$
$$AW + BW + CW = 10$$
$$AX + BX + CX = 17$$
$$AY + BY + CY = 12$$
$$AZ + BZ + CZ = 11$$

(i) Explain the purpose of each of the following lines from the LP formulation.

(*A*) Minimise $18AW + 11AX + \ldots + 18CZ$ **[2]**

As this part had 2 marks candidates were required to make two points regarding this particular line from the LP formulation. The first point was that the summation was related to the total cost of transporting all the required stock and the second was the requirement that the total cost (of transporting all the required stock) needed to be as small as possible and so hence the need to minimise.

Question 4(i)(*B*)

(*B*) $AW + AX + AY + AZ = 20$ **[1]**

Examiners noted that while it was clear that candidates had an understanding of why this constraint was needed many candidates did not give an answer in context and simply mentioned the total stock. A contextual answer, which specifically mentioned the fact that this was the constraint that fixed the total stock that could be shipped from supplier A to each of the four depots to 20, was in fact required.

Question 4(ii)(*A*)

The LP is run in an Online LP Solver and the following output is obtained.

OBJECTIVE FUNCTION VALUE

1) 726.0000

VARIABLE	VALUE	REDUCED COST
AW	9.000000	0.000000
AX	0.000000	0.000000
AY	0.000000	0.000000
AZ	11.000000	0.000000
BW	1.000000	0.000000
BX	0.000000	13.000000
BY	12.000000	0.000000
BZ	0.000000	9.000000
CW	0.000000	2.000000
CX	17.000000	0.000000
CY	0.000000	2.000000
CZ	0.000000	4.000000

The REDUCED COST value indicates by how much more the unit cost needs to be reduced for that particular transportation link to become part of the optimal solution. For example, the unit cost of transporting from B to Z would need to be reduced by at least £900 if BZ was to become part of the solution.

(ii) (*A*) Interpret the output to give a solution to the transportation problem. **[1]**

Many candidates did not read the demand for this part carefully and therefore did not realise that all was required was an answer based on the levels of stock that would be shipped from each supplier to the corresponding depots e.g. A to W is 9 (or 900), A to Z is 11 (or 1100), etc.

Question 4(ii)(*B*)

(*B*) State the minimum total cost of delivering all the required stock. **[1]**

The two most common incorrect answers seen by examiners in this part were by those candidates who either summed the five values in the VALUE column or gave an answer of 726. Examiners had to see the answer of 72 600 to indicate that candidates had understood the fact that the unit costs given in the question were in hundreds of pounds and therefore the solution given in the Online LP Solver had to be multiplied by 100.

Question 4(iii)(*A*)

(iii) State the effect this change has on

(*A*) the objective function in the original LP formulation **[1]**

Apart from those candidates who got the idea of 'constraint' and 'objective' mixed up in parts (iii)(a) and (b) nearly all candidates who attempted this part gave the correct answer that the only change would be to the coefficient of BX from 25 to 15.

Question 4(iii)(*B*)

(*B*) the constraints in the original LP formulation. **[1]**

The responses seen here were similar to the previous part in that nearly all candidates stated correctly that there would be no change in the constraints (as the costs of transportation are solely covered in the objective function).

Question 4(iv)(*A*)

The modified LP problem (where the unit cost of transporting goods from B to X was reduced to £1500) is run in the same Online LP Solver and the following output is obtained.

OBJECTIVE FUNCTION VALUE

1) 726.0000

VARIABLE	VALUE	REDUCED COST
AW	9.000000	0.000000
AX	0.000000	0.000000
AY	0.000000	0.000000
AZ	11.000000	0.000000
BW	1.000000	0.000000
BX	0.000000	3.000000
BY	12.000000	0.000000
BZ	0.000000	9.000000
CW	0.000000	2.000000
CX	17.000000	0.000000
CY	0.000000	2.000000
CZ	0.000000	4.000000

In an attempt to cut the minimum total cost of delivering the required stock, supplier C decides to reduce the unit cost of supplying one of the four depots by £150. You can assume that the unit cost of transporting goods from B to X is still only £1500 and that no other supplier is going to cut their unit costs.

(iv) (*A*) Determine which of the four depots should have its transportation costs cut by supplier C to reduce the minimum total cost of delivery. Give a reason for your answer. **[2]**

It was clear to most candidates that supplier C should reduce the cost of transportation to depot X (and so scoring one of the 2 marks available) but very few candidates could give an adequate reason for why this was the case. The key here was in the REDUCED COST column (from the LP solver) in that the three other depots (W, Y and Z) all had a reduced cost greater than 1.5. The fact that these three values were all greater than 1.5 indicated that a reduction of only £150 in the transportation costs from any of these three depots would not lead to a reduction in the total cost of transportation.

Question 4iv(*B*)

(*B*) What is the new minimum total cost of delivering all the stock when supplier C makes this change? **[1]**

This part was often left blank and examiners noted that the correct answer of 70 050 (which came from considering 72600 – (17 x 150)) was rarely seen.

Question 5(i)

5 Fig. 5 shows an activity network for a project. Each activity is represented by an arc. The early event times and late event times are shown at each vertex.

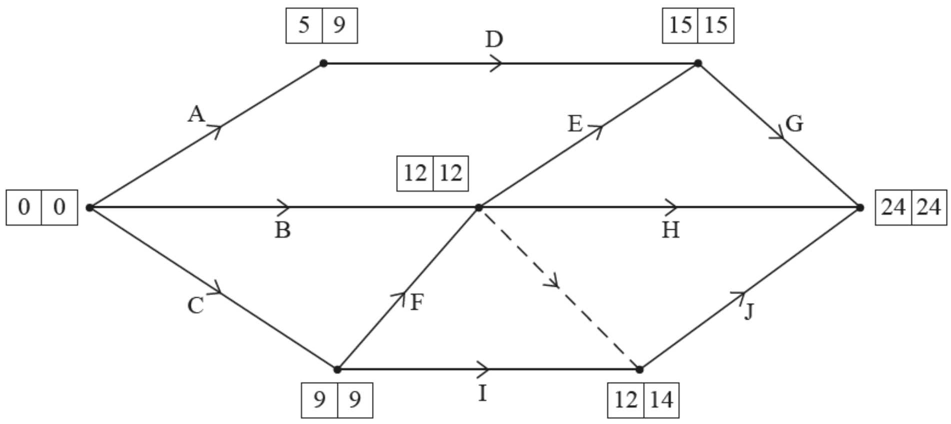

Fig. 5

(i) State the minimum completion time for the project. **[1]**

It is given that

- the project contains two critical paths that both include activity C,
- the total float for activity I is 3.

This was probably the best-answered part of the whole paper with nearly all candidates giving the correct answer of 24.

Question 5(ii)(*A*)

(ii) (*A*) Complete, as far as possible, the table in the Printed Answer Booklet showing the duration of each activity. Note that you will not be able to determine the duration for one of the activities. **[5]**

It was pleasing to note that many candidates completed the table correctly with the durations of the missing activities. The most common incorrect responses included:

- Giving all activities a duration even though the question specifically mentioned that it would not be possible to determine the duration for one activity (which happened to be activity B).
- Giving activity I a duration of 3 rather than 2 (this could have come from confusing total float with duration).
- Not realising that the duration of activity J could be determined from a backward pass through the network. With late event times at the beginning and end of J being 14 and 24 (and due to J's position in the network), this implied that the duration of J had to be 10.

Question 5(ii)(*B*)

(*B*) Explain why one of the activity durations could not be determined in part **(ii)** (*A*). **[1]**

Candidates found it extremely difficult to articulate why it was not possible to determine the duration of activity B – many answers in essence seemed to say that it just was not possible without giving any concrete reasons based on the either network or the event times. The key point that seemed to be missing from candidates' responses was the fact that as C was a critical activity this implied that F was a critical activity too (as activity I could not be critical as it had a total float and therefore the two critical paths must both include F). Therefore the early event time at the end of B is derived from the length of the path(s) though C and F only and as B is not dependent on the completion of any other activity all we can infer is that the duration of B should be less than 12.

Question 5(iii)

At the beginning of the project it becomes apparent that the duration of activity C can be reduced to 4 but this only reduces the minimum completion time of the project by 2.

(iii) Determine, showing your reasoning, the missing activity duration from part **(ii)** (*A*). **[2]**

While many candidates did indeed state the duration of activity B as 10 very few did as requested which was to 'determine, showing your reasoning'. The key point that candidates now had to consider was that the new information regarding the duration of activity C now implied that activity C was no longer critical. Therefore, C was no longer part of a critical path and hence activity B would now become critical therefore making it possible to determine its duration.

Question 6(i)

6 The following LP problem is to be solved.

Maximise $P = 3x + y + 2z$

subject to

$$x + 2y \leqslant 30$$
$$2x + y + z \geqslant 14$$
$$2x + z \leqslant 20$$
$$x, y, z \geqslant 0$$

(i)

- Complete the initial tableau in the Printed Answer Booklet so that the two-stage simplex method may be used to solve this problem.
- Show how the constraints have been made into equations using slack variables, surplus variables and artificial variables.
- Show how the rows for the two objective functions are formed. **[7]**

It is vital in questions like this that the candidate reads the entire question carefully before beginning as a number of candidates only completed the initial tableau and did not show the constraints (as equations) or show how the two objective functions were formed. It was clear that candidates were much more confident in adding slack variables to the two 'less than or equal to' constraints and re-writing the *P* objective function than adding the surplus and artificial variables to the 'greater than or equal to' constraint and forming the second (*Q*) objective function.

AfL

When candidates are asked to derive equations or constraints that need to include variables that have not been defined in the question that it is the responsibility of the candidate to define the variables they use. In this case, for example, many candidates did not make it explicitly clear from the letters they used which was the artificial and which was the surplus variable. Candidates should also be reminded that when filling in an initial tableau that all cells must be completed (especially zeros).

Question 6(ii)

After one iteration of the two-stage simplex method the tableau in Fig. 6.1 is produced.

Q	P	x	y	z	s_1	s_2	s_3	a_1	RHS
1	0	0	0	0	0	0	0	–1	0
0	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	0	$-\frac{3}{2}$	0	$\frac{3}{2}$	21
0	0	0	$\frac{3}{2}$	$-\frac{1}{2}$	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	23
0	0	1	$\frac{1}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	7
0	0	0	–1	0	0	1	1	–1	6

Fig. 6.1

(ii) Explain how the tableau in Fig. 6.1 shows that the first stage has been completed. **[2]**

Again, in this part the number of marks on offer should have indicated the fact that there were two points that needed to be made when discussing whether the first stage had been completed. The first was that the solution was feasible at this stage as $Q = 0$ and the second was that the solution was optimal since there were no non-artificial positive numbers in the Q row. A number of candidates misunderstood the demand here and gave a response based on the pivot column or which variables were now basic.

Question 6(iii)

(iii) Using the tableau in Fig. 6.1 as the starting point for the second stage

- reduce the tableau so that the second stage can be started,
- carry out the first iteration of the second stage of the two-stage simplex method, using an entry in the s_2 column as the pivot element. **[3]**

Candidates overall were very proficient in carrying out the simplex algorithm and many identified the correct pivot value (the 1 in the bottom row) and used this to carry out the first iteration as requested. The most common errors were sign errors on certain values or difficulty in dealing with fractions. There were a significant minority were pivoting on an incorrect value with some candidates using a negative value as a pivot.

Question 6(iv)

(iv) State which variables are now basic after this first iteration of the second stage of the two-stage simplex method. **[1]**

Many candidates left this part blank but of those who did attempt it, many correctly stated the three variables that were basic after the first iteration. In this series examiners condoned the inclusion of *P* as a basic variable but centres are reminded that *P* is not a variable in this sense and its inclusion in future series may possibly be penalised.

Question 6(v)

The final tableau is shown in Fig. 6.2.

P	x	y	z	s_1	s_2	s_3	RHS
1	$\frac{3}{2}$	0	0	$\frac{1}{2}$	0	2	55
0	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	15
0	2	0	1	0	0	1	20
0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	1	1	21

Fig. 6.2

(v) Give the maximum value of P, and the corresponding values of x, y and z. **[2]**

Nearly all candidates who attempted this part correctly stated the maximum value of *P* as 55 – many candidates however stated *x* incorrectly as 21 rather than 0.

Question 6(vi)

(vi) The original LP problem is modified by replacing the constraint $2x+y+z \geqslant 14$ with $2x+y+z \geqslant k$. Determine the maximum possible value of k such that the solutions to both the modified and original LP problems would be the same. **[2]**

Of the two possible ways of attempting this last part, the vast majority of candidates used the results from part (v) and substituted these values into $2x + y + z$ to obtain the answer of $k = 35$. Another way of tackling this part was to realise that the surplus variable had a value of 21 which indicated that the corresponding constraint was being underused by 21 and so the solution to the LP problem would be no different if 14 was added to the 21 to give $k = 35$. Once again, as the word 'determine' was given in the question the correct answer with no working only scored 1of the 2 marks available.

Supporting you

For further details of this qualification please visit the subject webpage.

Review of results

If any of your students' results are not as expected, you may wish to consider one of our review of results services. For full information about the options available visit the OCR website. If university places are at stake you may wish to consider priority service 2 reviews of marking which have an earlier deadline to ensure your reviews are processed in time for university applications.

Active Results offers a unique perspective on results data and greater opportunities to understand students' performance.

It allows you to:

- Review reports on the **performance of individual candidates**, cohorts of students and whole centres
- **Analyse results** at question and/or topic level
- **Compare your centre** with OCR national averages or similar OCR centres.
- Identify areas of the curriculum where students excel or struggle and help **pinpoint strengths and weaknesses** of students and teaching departments.

http://www.ocr.org.uk/administration/support-and-tools/active-results/

Attend one of our popular CPD courses to hear exam feedback directly from a senior assessor or drop in to an online Q&A session.

https://www.cpdhub.ocr.org.uk

The small print

We'd like to know your view on the resources we produce. By clicking on the 'Like' or 'Dislike' button you can help us to ensure that our resources work for you. When the email template pops up please add additional comments if you wish and then just click 'Send'. Thank you.

Whether you already offer OCR qualifications, are new to OCR, or are considering switching from your current provider/awarding organisation, you can request more information by completing the Expression of Interest form which can be found here: www.ocr.org.uk/expression-of-interest

Looking for a resource?

There is now a quick and easy search tool to help find **free** resources for your qualification:

www.ocr.org.uk/i-want-to/find-resources/

www.ocr.org.uk

OCR Customer Contact Centre

General qualifications

Telephone 01223 553998
Facsimile 01223 552627
Email general.qualifications@ocr.org.uk