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AS LEVEL

Examiners' report

FURTHER MATHEMATICS B (MEI)

H635
For first teaching in 2017

Y414/01 Summer 2018 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper Y414/01 series overview

Y414 is one of six available options. Candidates must take Core Pure (Y410) and then choose two of the optional papers to be credited AS Level in Further Mathematics B (MEI). Candidates are expected to know the content of AS Level Mathematics (H630) and Y410.

In this option, candidates apply numerical approaches to four topics from mathematics: solution of equations, differentiation, integration and approximating functions. They are expected to use a spreadsheet to implement the methods and to use error analysis to improve numerical approximations.

To do well on this paper, candidates need to be comfortable using simple spreadsheet formulae and interpreting spreadsheet output. They also need to be able to use their calculators efficiently, particularly when applying iterative methods, and to explain their reasoning clearly and concisely.

Question 1

1 The coordinates of point A are (1.05, 2.71) and the coordinates of point B are (1.07, 3.08). In each case the x and y values have been rounded to two decimal places. The gradient of the line AB is m.

Find the range of values of m, commenting on your answer. **[4]**

Candidates who did well on this question identified the correct intervals in the numerator and denominator and went on to identify the correct bounds. Candidates who did very well recognised that the large range obtained was due to the subtraction of nearly equal numbers in the denominator, and commented accordingly.

Candidates who did less well made slips in arithmetic or with one or more of the intervals. They either omitted the comment or simply noted that there was a large difference between the numbers obtained.

Question 2 (i)

2 The table in Fig. 2 shows some values of $\log_3 x$ which are correct to 6 decimal places.

x	2	2.25	2.5	2.75	3
$\log_3 x$	0.630930	0.738140	0.834044	0.920799	1

Fig. 2

(i) Use Simpson's rule to calculate an approximation to $\int_2^3 \log_3 x \, dx$, giving your answer correct to 6 decimal places. **[3]**

Candidates who did well either calculated M_2 and T_2 and used them to find S_4, or calculated S_4 directly. Candidates who did less well worked with M_1 and T_1 and found S_2 only, or made errors in the formula for S_4 because they made bracket errors or were unable to correctly identify the value *h.*

Question 2 (ii)

(ii) Explain why it is unlikely that the answer to part **(i)** is in fact accurate to 6 decimal places. **[1]**

Candidates who did well commented on the value of h or the fact that the original values are only given to 6 decimal places.

Question 3

We have adjusted the mark scheme to allow for a minor issue with this question.

Question 3 (i)

3 The spreadsheet output in Fig. 3 shows Table 1 and Table 2.

	A	B	C	D	E	F	G
1	x	2	2.1	2.01	2.001	2.0001	2.00001
2	f(x)	0.64	0.625876972	0.638573473	0.639857204	0.639985719	0.639998572
3	Table 1						
4							
5	h	0.1	0.01	0.001	0.0001	0.00001	
6	dy/dx	–0.1412303	–0.142652654	–1.14279594	–0.142810279	–0.14281171	
7	difference	–0.0014224	–0.000143287	–1.4339E-05	–1.43402E-06		
8	ratio	0.1007378	0.100073651	0.10000709			
9	Table 2						

Fig. 3

Table 1 shows values of a function, $y = \mathrm{f}(x)$, for different values of x. Table 2 shows approximations to $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 2$, along with the differences between successive approximations and the ratio of these differences.

The formula in cell C6 is

= (D2 – \$B2)/C5 .

Equivalent formulae are in cells D6, E6 and F6.

(i) Explain why the symbol \$ is used. **[1]**

Candidates who did well in this question either referred to the column being fixed in subsequent calculations or to the cell being fixed in calculations on the same row.

The responses of some candidates suggested that they understood, but their explanation was either imprecise or contained an error such as "as the formula is dragged down".

Question 3 (ii)

(ii) State what method is being used to approximate $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 2$. **[1]**

Question 3 (iii)

(iii) Use extrapolation to find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 2$ as accurately as you can, justifying your answer. **[4]**

Candidates who did well extrapolated from – 0.14281171 with r = 0.1 and showed all the decimal places in their answer before rounding their answer to a suitable precision and making a suitable supporting comment.

Those who did less well extrapolated from a different starting point or used r = 0.5. Some candidates made slips in the calculation.

Question 3 (iv)

(iv) Calculate an approximation to the value of f(2.05). **[2]**

Candidates who did well used the approximation f(2 + 0.05) ≈ f(2) + 0.05×f'(2) together with their answer to part (iii).

Those who did less well used linear interpolation.

Question 4 (i)

4 The equation $e^x - x^2 - 2x = 0$ has a root α, where $0 < \alpha < 1$ and a root β, where $2 < \beta < 3$.

(i) Show how to obtain the iterative formula

$$x_{r+1} = \frac{e^{x_r} - x_r^2}{2}.$$

[2]

Candidates who did well in this question started with the original equation and added when the rearrangement had been successfully obtained.

Candidates who did less well omitted the subscripts altogether or did not include all of them.

Question 4 (ii)

Fig. 4 shows part of the curve $y = \dfrac{e^x - x^2}{2}$ and part of the straight line $y = x$.

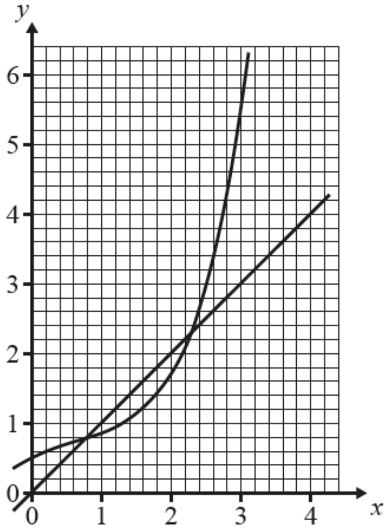


Fig. 4

(ii) Explain why the iteration in part **(i)** will

- successfully find α if a suitable starting value is chosen,
- fail to find β however close the starting value is to the root. **[2]**

Candidates who did well referred to g'(α) and g'(β) and the conditions for convergence and divergence in their explanations.

Candidates who did less well made vague comments such as the gradient is less than 1.

Question 4 (iii)

(iii) Use the iteration in part **(i)** to find α correct to 6 significant figures. **[2]**

Candidates who did well in this question gave sufficient detail of their working to convince the examiner that the iteration had been used and were able to successfully deduce when 6 d.p. accuracy had been achieved.

Candidates who did less well simply stated the answer (which could be obtained by use of a solver function).

Question 4 (iv)

The relaxed iteration

$$x_{r+1} = (1-\lambda)x_r + \lambda\left(\frac{e^{x_r} - x_r^2}{2}\right)$$

is used to find β.

(iv) Use $x_0 = 2$ with $\lambda = -0.8$ to find the value of β correct to 6 decimal places. **[3]**

Candidates who did well in this question gave sufficient detail of their working to convince the examiner that the relaxed iteration had been used. They showed at least three correct iterates in their working. They were able to successfully deduce when 6 d.p. accuracy had been achieved.

Question 4 (v)

(v) Determine what happens when the relaxed iteration is used with $\lambda = 0.8$ and a starting value of 2. **[2]**

Candidates who did well understood that the request to "determine what happens" necessitated some justification of any stated conclusion. They started with $x_0 = 2$ and generated a sufficient number of iterates for a convincing explanation.

Question 5 (i)

5 The value of shares in Sunfield plc was £2.21 per unit on the first day it came under new management. One week later the value of one unit of shares was £4.00 and after three weeks the value of one unit was £7.34. The information is summarised in Fig. 5.1, where x is the value in pounds per unit, and t is the time in weeks since coming under new management.

t	0	1	3
x	2.21	4.00	7.34

Fig. 5.1

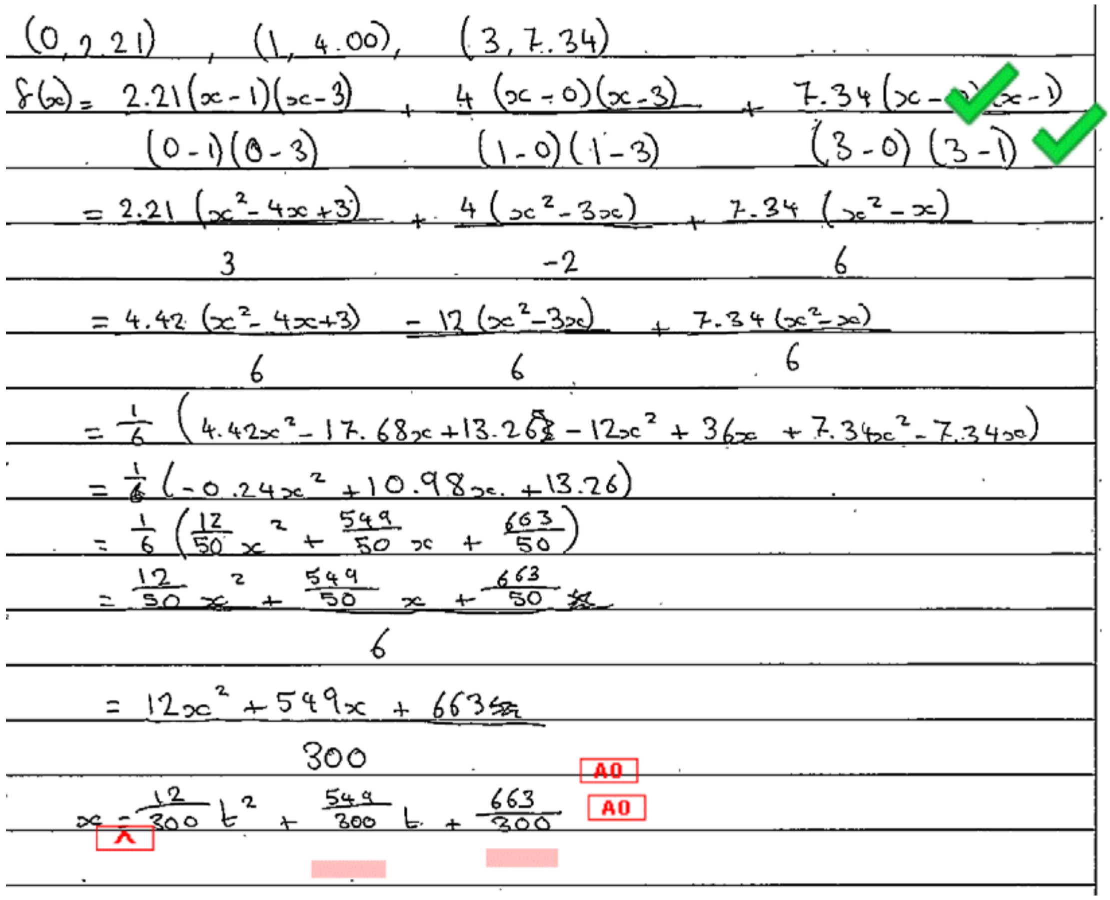
The Public Relations Director uses this information to propose a quadratic model connecting the value of a unit of shares, in pounds, and the time, in weeks, since coming under new management.

(i) Use Lagrange's interpolation formula to find a quadratic model for these data, giving your answer in the form $x = at^2 + bt + c$, where, a, b and c are constants to be found. **[4]**

Candidates who did well substituted in the given formula correctly and wrote the coefficients in their simplest form.

Candidates who did less well worked in terms of x instead of t, or made slips in the arithmetic, or left the coefficients unsimplified.

Exemplar 1


(0, 2.21), (1, 4.00), (3, 7.34)

$f(x) = \frac{2.21(x-1)(x-3)}{(0-1)(0-3)} + \frac{4(x-0)(x-3)}{(1-0)(1-3)} + \frac{7.34(x-0)(x-1)}{(3-0)(3-1)}$

$= \frac{2.21(x^2-4x+3)}{3} + \frac{4(x^2-3x)}{-2} + \frac{7.34(x^2-x)}{6}$

$= \frac{4.42(x^2-4x+3)}{6} - \frac{12(x^2-3x)}{6} + \frac{7.34(x^2-x)}{6}$

$= \frac{1}{6}(4.42x^2 - 17.68x + 13.26 - 12x^2 + 36x + 7.34x^2 - 7.34x)$

$= \frac{1}{6}(-0.24x^2 + 10.98x + 13.26)$

$= \frac{1}{6}\left(\frac{12}{50}x^2 + \frac{549}{50}x + \frac{663}{50}\right)$

$= \frac{\frac{12}{50}x^2 + \frac{549}{50}x + \frac{663}{50}}{6}$

$= \frac{12x^2 + 549x + 663}{300}$

$x = \frac{12}{300}t^2 + \frac{549}{300}t + \frac{663}{300}$

In this response, the formula was applied correctly but there was a sign error in the first term of the answer and the constant term was left unsimplified, resulting in the loss of the last two A marks.

Question 5 (ii)

(ii) A shareholder notes that the value of a unit was £5.73 after two weeks. Determine whether the model is consistent with this information. **[2]**

Question 5 (iii)

(iii) According to the model, how will the value of a unit change in the long run? **[1]**

Candidates who did well considered the behaviour of their quadratic for large values of *t* and commented accordingly.

Question 5 (iv)

After seven weeks the management analysed the data and presented the findings in a difference table. The results are shown in Fig. 5.2.

t	x	Δ	Δ^2	Δ^3
0	2.21			
		1.79		
1	4.00		–0.06	
		1.73		–0.06
2	5.73		–0.12	
		1.61		–0.07
3	7.34		–0.19	
		1.42		–0.08
4	8.76		–0.27	
		1.15		–0.07
5	9.91		–0.34	
		0.81		–0.06
6	10.72		–0.40	
		0.41		
7	11.13			

Fig. 5.2

A shareholder proposes a cubic model for the data.

(iv) Explain whether the information in the difference table supports this proposal. **[1]**

Candidates who did well commented appropriately on the third differences.

Question 5 (v)

(v) Use Newton's forward difference interpolation formula to show that the shareholder's model is

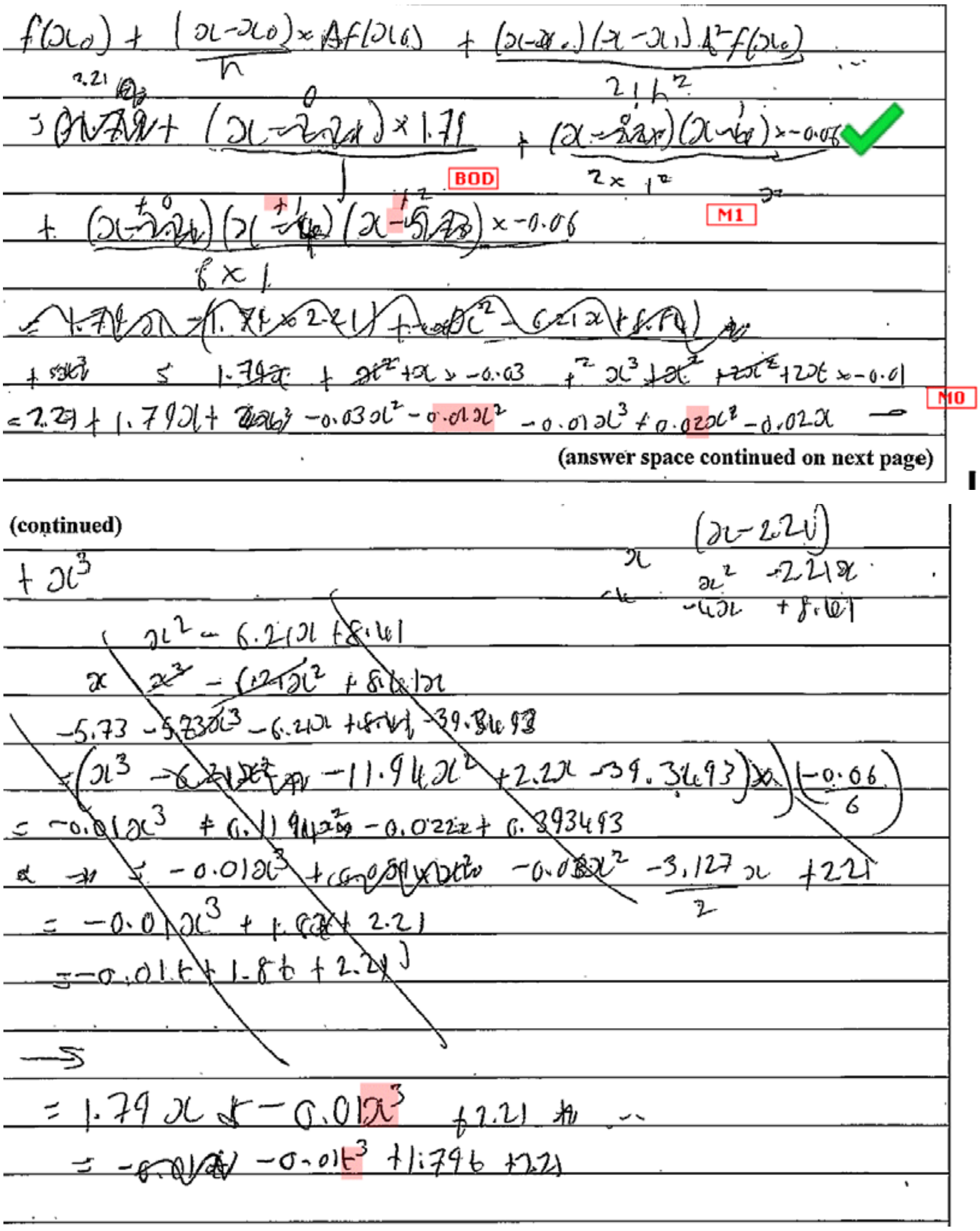
$$x = -0.01t^3 + 1.8t + 2.21.$$

[4]

Candidates who did well showed full details of substitution in the correct formula and clear intermediate steps to generate the given formula. They worked in *t* throughout.

Candidates who did less well worked in *x* instead of *t*. Sometimes algebraic errors were made, but the correct formula still appeared.

Exemplar 2



This candidate's work is quite difficult to read. The first M mark has been credited BOD one slip only – see the highlighted parts. The second M mark has been withheld because there is more than one error in the intermediate step. Note that the candidate works in *x* until the very last line, before abruptly changing to *t* without any explanation. Even if the work had been correct, the two A marks would have been withheld unless there had been a brief explanation given at this point.

Question 5 (vi)

(vi) Identify a limitation of the shareholder's model. **[1]**

Candidates who did well considered the behaviour of the cubic for large values of *t* and commented accordingly, or they made a sensible comment about extrapolation.

A significant number of candidates made no response to this question.

Question 6 (i)

6 **(i)** Show that the equation $0.1x^3 - 2x + 3 = 0$ has a root α, where $3 < \alpha < 4$. **[1]**

Candidates who did well found and stated the values of f(3) and f(4). They made a suitable supporting comment.

Candidates who did less well stated that there would be a sign change if f(3) and f(4) were to be evaluated, or made slips with the substitution.

Question 6 (ii)

The method of false position is used to find α.
The spreadsheet output in Fig. 6.1 shows some of the results.

	A	B	C	D	E	F	G
1	r	x_r	$f(x_r)$	x_{r+1}	$f(x_{r+1})$		
2	0	3	–0.3	4	1.4	3.176471	–0.14789
3	1	3.176471	–0.14789	4	1.4	3.255155	–0.06114
4	2	3.255155	–0.06114	4	1.4	3.286321	–0.02345
5	3	3.286321	–0.02345	4	1.4	3.298076	–0.00873
6	4	3.298076	–0.00873	4	1.4	3.302428	–0.00322
7	5	3.302428	–0.00322	4	1.4	3.304028	–0.00118
8	6	3.304028	–0.00118	4	1.4	3.304614	–0.00043
9	7	3.304614	–0.00043	4	1.4	3.304829	–0.00016
10	8	3.304829	–0.00016	4	1.4	3.304908	–5.8E-05
11	9	3.304908	–5.8E-05	4	1.4	3.304937	–2.1E-05

Fig. 6.1

The spreadsheet formula in cell F2 is

= (B2*E2–D2*C2)/(E2–C2) .

(ii) What is being calculated in this cell? **[1]**

Question 6 (iii)

(iii) Write down a suitable spreadsheet formula for cell G2. **[1]**

Candidates who did well gave a complete formula with no errors.

Candidates who did less well made syntax errors or omitted "=".

Question 6 (iv)

(iv) Explain why the values in column G are necessary. **[1]**

Candidates who did well homed in on the need for a sign check in the method of false position.

Candidates who did less well opted for a general comment such as "calculating the next estimate".

Question 6 (v)

The spreadsheet formula in cell B3 is

$$\boxed{= \text{IF(G2}<\text{0,F2,B2)}}\,.$$

(v) Explain the purpose of this formula. **[1]**

Candidates who did well referred to cells B3, F2 and B2 in their explanation.

Question 6 (vi)

(vi) Write down the value of α to an accuracy that appears justified. **[1]**

Question 6 (vii)

Further analysis is carried out. This is shown in the spreadsheet output in Fig. 6.2.

	J	K	L	M
1	x_r	$x_{r+1} - x_r$		
2	3			
3	3.176471	0.176471		
4	3.255155	0.078684	0.445876	2.526632
5	3.286321	0.031166	0.396089	5.033923
6	3.298076	0.011756	0.377196	12.10282
7	3.302428	0.004352	0.370184	31.48985
8	3.304028	0.0016	0.367603	84.47262
9	3.304614	0.000587	0.366657	229.2012
10	3.304829	0.000215	0.36631	624.5195
11	3.304908	7.87E-05	0.366183	1704.301
12				

Fig. 6.2

The spreadsheet formula in cell L4 is

= K4/K3

and the spreadsheet formula in cell M4 is

= K4/(K3^2) .

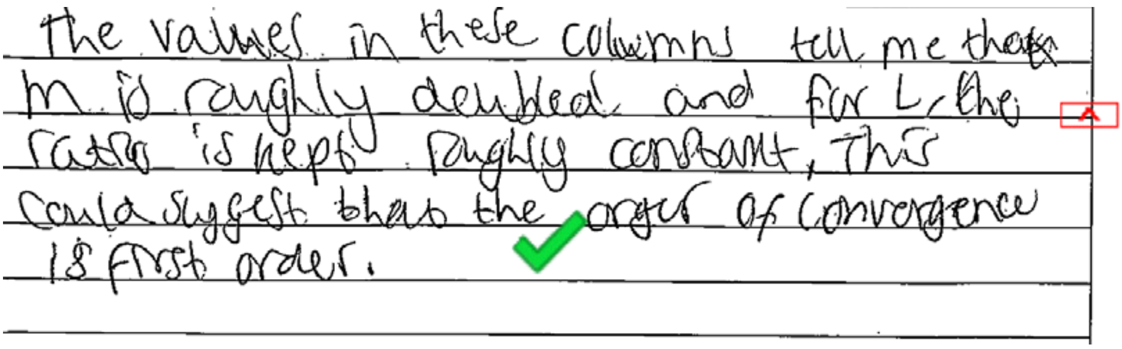
Equivalent formulae are in cells L5 to L11 and M5 to M11.

(vii) Explain what the values in columns L and M tell you about the order of convergence of the sequence of approximations to α found using the method of false position in this case. **[2]**

Candidates who did well commented separately on the values in column L and the values in column M. Their explanations referred to 1st order convergence and 2nd order convergence.

Candidates who did less well gave incomplete explanations.

Exemplar 3


The values in these columns tell me that M is roughly doubled and for L, the ratio is kept roughly constant, This could suggest that the order of convergence is first order.

In this response, both columns of values are referred to. Two interpretations of the values in column L were allowed – that the convergence may be 1st order, or slightly faster than 1st order. The first mark was credited for this. However, the second mark was not credited. To earn this mark candidates needed to comment that the convergence is clearly not 2nd order.

Question 7 (i)

7 Fig. 7.1 shows part of the curve $y = 2^x - x^2 + 3$.

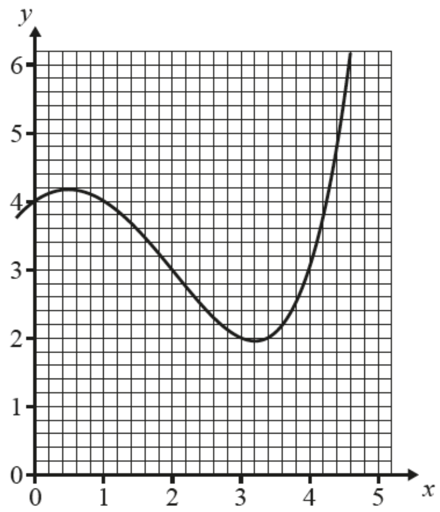


Fig. 7.1

(i) State, with a reason, whether using the trapezium rule to approximate $\int_3^4 (2^x - x^2 + 3)\,dx$ will give an under-estimate or an over-estimate. **[1]**

Candidates who did well gave clear, detailed explanations why an over-estimate is generated related to the tops of the trapezia above the curve.

Candidates who did less well gave incomplete explanations such as "convex" or "concave".

Question 7 (ii)

The spreadsheet output in Fig. 7.2 was generated in order to evaluate $\int_3^4 (2^x - x^2 + 3)\,dx$.

The values in columns N and O are estimates of the integral using the midpoint rule and the trapezium rule respectively.

	M	N	O
1	*n*	*Mn*	*Tn*
2	1	2.063708499	2.5
3	2	2.171499782	2.281854249
4	4	2.199007398	2.226677016
5	8	2.205919726	2.212842207
6	16	2.207650029	2.209380966
7	32	2.208082743	2.208515498

Fig. 7.2

(ii) Using only values from column N and/or column O, give a suitable spreadsheet formula for cell O3. **[2]**

Candidates who did well gave a complete formula with no errors.

Candidates who did less well made syntax errors or omitted "=".

Question 7 (iii)

(iii) Use the entries in cells N7 and O7 to write down the value of $\int_3^4 (2^x - x^2 + 3)\,dx$ as accurately as you can, explaining your reasoning. **[2]**

Candidates who did well compared the values in cells N7 and O7 and gave the appropriate value.

Candidates who did less well did further calculations and gave answers to a precision which could not be justified.

Question 7 (iv)

Further analysis shows that the ratio of differences of the midpoint rule approximations and the trapezium rule approximations converge rapidly to the value expected from theory.

(iv) Explain whether it is reasonable to assume that the ratio of differences of a sequence of approximations generated using Simpson's rule would also converge to the value predicted by theory. **[1]**

Candidates who did well referred to the convergence of the trapezium rule and midpoint rule ratios in their answers.

Question 7 (v) (A)

(v) (*A*) Use the information in Fig. 7.2 to obtain two Simpson's Rule estimates of $\int_3^4 (2^x - x^2 + 3)\,dx$, S_{32} and S_{64}. **[3]**

Candidates who did well completed the calculations efficiently and gave their answers correct to 8 or 9 decimal places.

Candidates who did less well confused S_{32} with S_{64}.

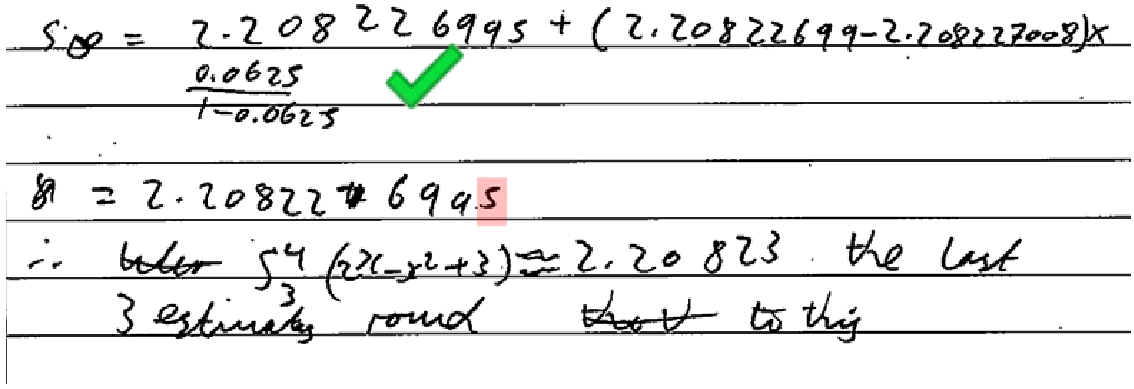
Question 7 (v) (B)

(*B*) Use extrapolation to find the value of $\int_3^4 (2^x - x^2 + 3)\,dx$ as accurately as you can, justifying the precision quoted. **[3]**

Candidates who did well extrapolated from S_{64} using $r = 0.0625$. They then rounded their answer to an appropriate precision and justified their answer.

Candidates who did less well did not justify the precision quoted in their final answer or made slips in the arithmetic.

Exemplar 4



This candidate made a slip in the calculation which cost the first A mark. For the second A mark, candidates needed to quote the highest level of precision they considered they could reasonably justify. A minimum of 7 decimal places was expected here.

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