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Examiners' report

FURTHER MATHEMATICS A

H235
For first teaching in 2017

Y533/01 Summer 2018 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper Y533/01 series overview

This is one of the optional modules in the new specification for AS Further Maths, alongside the compulsory Pure Core, and the other optional modules in Statistics, Discrete Mathematics and Additional Pure Mathematics. This component complements the Mechanics content of the AS Mathematics course and requires candidates to solve practical problems using a variety of techniques including energy, power, momentum, connected particles, pendulums and dimensional analysis. To do well on this paper, candidates needed to be able to work systematically with the standard principles of Mechanics. They often needed make sense of more complex situations, for example, correctly combining several forces and/or types of energy in a variety of situations, such as gravitational potential energy, kinetic energy and friction along a slope.

Candidates who did well on this paper generally did the following:

- Recognised the different types of energy present in a problem and combined them systematically
- Recognised the different forces present and were aware of the correct way to resolve them
- Recognised the different directions that forces were acting, or when energy was being reduced or increased
- Made good use of diagrams to help them understand the problem
- Were able to use Newton's Second Law (F = ma) confidently, with all the forces present on the left-hand side of the equation
- Were able to apply their knowledge to complex and unfamiliar problems that combined several areas, such as Q2 and Q6, recognising which information was important at each stage
- Were efficient and effective in applying their knowledge, always choosing the simplest possible route to the solution

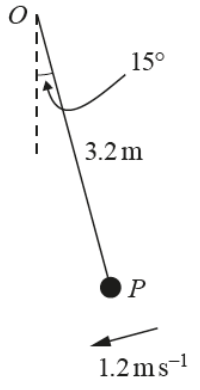
Candidates who did less well on this paper generally did the following:

- Were unfamiliar with the laws of indices required in the dimensional analysis (Q3)
- Did not recognise all the forces and/or energy types present in a situation
- Did not correctly account for the direction in which forces were acting. Examples include incorrect signs; resolving in the wrong direction
- Did not correctly account for all aspects of energy change. Examples include adding gravitational potential energy to kinetic energy rather than subtracting it; not realising that initial kinetic energy had to be taken into account
- Confused different quantities, such as initial and final velocity, or used the wrong values
- Used very laborious and inefficient methods, leading to errors in both method and application

There was no evidence that any time constraints had led to a candidate underperforming. Scripts where there was no response to the final question often had earlier sections of the paper which had not been tackled.

Question 1 (i)

1



A particle P of mass m kg is attached to one end of a light inextensible string of length 3.2 m. The other end of the string is attached to a fixed point O. The particle is held at rest, with the string taut and making an angle of 15° with the vertical. It is then projected with velocity 1.2 m s^{-1} in a direction perpendicular to OP and with a downwards component so that it begins to move in a vertical circle (see diagram). In the ensuing motion the string remains taut and the angle it makes with the downwards vertical through O is denoted by θ°.

(i) Find the speed of P at the point on its path vertically below O. **[4]**

This question required candidates to relate loss of potential energy to gain of kinetic energy in a pendulum in order to calculate the final velocity.

Almost all candidates attempted this question, with many being successful and gaining full marks. Most candidates recognised the need to add the initial kinetic energy to the potential energy change. Some worked with an energy balance equation, equating KE + PE before and after the motion. Whilst valid, candidates using this approach were generally less successful due to the extra complexity.

Some candidates did not factor in the initial kinetic energy, whilst others thought that the height change was 3.2cos15° rather than 3.2(1 – cos15°). A small number mistakenly thought that this was a conical pendulum and were therefore unable to gain any marks.

Question 1 (ii)

(ii) Find the value of θ at the point where P first comes to instantaneous rest. **[2]**

This question required candidates to use their prior result from Q1 (i) to find the angle at which the pendulum would come to rest. Essentially, this is the converse of the first part.

Whilst most candidates attempted this question, it proved to be much more challenging, with most responses either gaining full marks or no marks. A small proportion of candidates gained partial credit through showing a valid method but then made calculation errors. The majority gaining full marks used the same systematic approach as in Q1 (i), equating their kinetic energy from part (i) with the potential energy gain expressed in terms of θ. Common errors included thinking that the pendulum would come to rest at the top of the vertical circle; including the original kinetic energy; and using 3.2cosθ instead of 3.2(1 – cosθ) for the height gain. Some candidates attempted to find the height gain from the original KE and add it to the initial height but generally this proved too complicated to follow through.

Question 2 (i)

2 A particle P of mass 3.5 kg is moving down a line of greatest slope of a rough inclined plane. At the instant that its speed is $2.1\,\mathrm{m\,s^{-1}}$ P is at a point A on the plane. At that instant an impulse of magnitude 33.6 N s, directed up the line of greatest slope, acts on P.

(i) Show that as a result of the impulse P starts moving up the plane with a speed of $7.5\,\mathrm{m\,s^{-1}}$. **[2]**

This question required the use of the impulse-momentum equation *I* = *mv* – *mu* and was attempted by all candidates. The key issue was that there was a change of direction after the collision that had to be accounted for.

Most candidates recognised that the slope was not relevant to this part of the question, except for interpreting the direction of the final velocity as being "up the slope". The majority of candidates gained full marks on this question, although there were plenty of instances of candidates working backwards from the given result. Many candidates did this question in a piecemeal fashion, but the most successful candidates started with the correct equation and used a diagram to determine the correct values to substitute into the equation prior to solving.

A significant minority of candidates lost one or both marks as they did not correctly account for the change in direction, having a final velocity and/or impulse with the same sign as the original velocity. Some did not apply the equation correctly, reversing the roles of *u* and *v* in the equation and setting *v* = 2.1. Others corrected their work by effectively using -*I* = *mu* – *mv* instead.

Question 2 (ii)

While still moving up the plane, P has speed $1.5\,\mathrm{m\,s^{-1}}$ at a point B where $AB = 4.2$ m. The plane is inclined at an angle of 20° to the horizontal. The frictional force exerted by the plane on P is modelled as constant.

(ii) Calculate the work done against friction as P moves from A to B. **[4]**

This item required candidates to compare the energy at the start and end of the motion and recognise that the difference was due to the effect of friction and proved very challenging for many.

The most successful candidates recognised and combined the key components – initial and final kinetic energy and gravitational potential energy – in the correct way in order to find the work done against friction. Less successful candidates often missed out one or more elements, equating the work done to the change in kinetic energy for example, and scoring only 1 or 2 marks. Others incorrectly resolved the force of gravity or the frictional force in the horizontal direction, using *mg* x *d*cos*θ* or *Fr* x *d*cos*θ* instead, or added the work done by gravity to the change in KE. A significant number of candidates used the velocities from part (i) instead of the new values in part (ii).

A substantial number of candidates recognised that they could use their knowledge of constant acceleration to find the acceleration and hence the friction and often gained at least some credit as a result.

Question 2 (iii)

(iii) Hence find the magnitude of the frictional force acting on P. **[2]**

Once an answer to part (ii) had been found, most candidates were then able to gain marks in part (iii) provided they had used a valid method in part (ii).

Question 2 (iv)

P first comes to instantaneous rest at point *C* on the plane.

(iv) Calculate *AC*. **[3]**

This question drew on the same methods as in part (iii), but candidates needed to use their value for the friction or acceleration and recognise that the final velocity, and therefore final KE, was 0 in order to find the distance travelled.

Successful candidates set up their energy equations well, considering all relevant quantities, but less successful candidates often considered only the effect of friction or the effect of gravity but not both, or again used u = 2.1.

Some candidates recognised that this could be solved using constant acceleration and tended to be successful.

Question 3 (i)

3 A particle moves in a straight line with constant acceleration. Its initial and final velocities are u and v respectively and at time t its displacement from its starting position is s. An equation connecting these quantities is $s = k(u^\alpha + v^\beta)t^\gamma$, where k is a dimensionless constant.

(i) Use dimensional analysis to find the values of α, β and γ. **[6]**

This question required candidates to use dimensional analysis, a new topic at this level. The question was based around a well-known constant acceleration equation, but required candidates to carry out a complete analysis as if they were not familiar with the equation.

Almost all candidates were able to use the correct terms L, T and LT^{-1} for displacement, time and velocity and most recognised that α = β for consistency. Very few candidates realised that only one of *u* and *v* should be used in the method, since you cannot "add" dimensions. This was not penalised on this occasion.

Successful candidates were able to formulate a satisfactory equation. They then used their analysis to compare the indices of L and T in order to determine α, β and γ. Less successful candidates often produced an incorrect analysis, which meant that further marks that depended on it were rarely gained. Some candidates followed up their initial correct analysis with incorrect applications of the rules of indices, with things such as $L^\alpha + L^\beta = L^{\alpha+\beta}$ or $T^{-\alpha}T^\gamma = T^{-\alpha\gamma}$ being seen in working, leading to incorrect answers.

In this specific question, candidates were able to go straight from the analysis to the solution without further working due to the simplicity of the indices. However, in future examinations, the values may be such that working will be necessary in order to find the correct values.

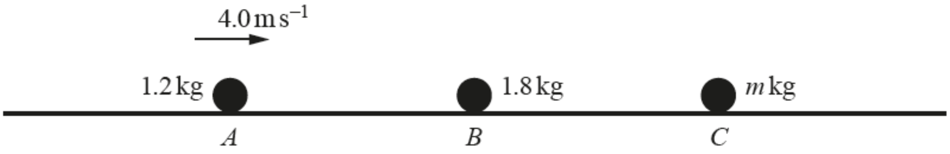
Question 3 (ii)

(ii) By considering the case where the acceleration is zero, determine the value of k. **[2]**

This question required candidates to prove that *k* = ½ from first principles, using the fact that *u* = *v* and that *s* = *ut* under constant acceleration. A minority of candidates were able to make progress with this item, which was generally accessible only to candidates who had scored full marks in part (i). Some tried to use their incorrect values from part (i) whilst a small number tried to make use of their prior knowledge of the relevant constant acceleration equation, which did not gain any credit.

Question 4 (i)

4



Three particles A, B and C are free to move in the same straight line on a large smooth horizontal surface. Their masses are 1.2 kg, 1.8 kg and m kg respectively (see diagram). The coefficient of restitution in collisions between any two of them is $\frac{3}{4}$. Initially, B and C are at rest and A is moving with a velocity of 4.0 m s^{-1} towards B.

(i) Show that immediately after the collision between A and B the speed of B is 2.8 m s^{-1}. **[4]**

Almost all candidates were familiar with using the law of restitution together with conservation of momentum, with few candidates scoring less than full marks in this part.

Question 4 (ii)

(ii) Find the velocity of A immediately after this collision. **[1]**

Candidates successful in part (i) generally answered this part correctly.

Question 4 (iii)

B subsequently collides with C.

(iii) Find, in terms of m, the velocity of B after its collision with C. **[4]**

Part (iii) used the same method as part (i), but on a secondary collision with a particle of unknown mass. Most candidates handled this well, with the most successful proceeding exactly as they had done in part (i). However, some candidates made errors due to the extra complexity of dealing with an unknown mass and some made unforced errors in the restitution equation.

A common issue was that many candidates used inefficient methods to solve the simultaneous equations in part (i) and more so in part (iii), which also led to errors in manipulation. Some candidates used their equations to find V_C instead of V_B as instructed, or rearranged for m, and therefore lost the last 2 marks.

Question 4 (iv)

(iv) Given that the direction of motion of B is reversed by the collision with C, find the range of possible values of m. **[2]**

Most candidates attempted this question, which made use of their result in part (iii), to solve an inequality and hence find *m*. Successful candidates generally started with $V_B < 0$ and proceeded from there to the solution. Some candidates did not handle the inequality correctly, thinking that cancelling the denominator would affect the other side of the equation or be the basis of a second inequality. Others used equations instead of inequalities and scored 0. Some of those who had previously found V_C then went on to find V_B in order to solve the problem. As a result, they did not get the credit they deserved in part (iii).

Exemplar 1

4(iv)

$$\frac{5.04 - 2.1m}{1.8 + m} < 0$$

$$5.04 - 2.1m < 1.8 + m$$

$$5.04 - 1.8 < m + 2.1m$$

$$5.04 - 1.8 < \cancel{m(1+2.1)}\ 3.1m$$

$$\frac{5.04 - 1.8}{3.1} < m$$

$$\therefore m > 1.045 \ (3dp)$$

In this exemplar, we see that the candidate gains M1 for their $V_B < 0$, but then moves the denominator to the right-hand side of the equation. Some candidates used their denominator as the basis of a second inequality.

Question 5 (i)

5 The engine of a car of mass 1200 kg produces a maximum power of 40 kW.

In an initial model of the motion of the car the total resistance to motion is assumed to be constant.

(i) Given that the greatest steady speed of the car on a straight horizontal road is $42\,\text{m s}^{-1}$, find the magnitude of the resistance force. **[2]**

Almost all candidates were able to find the driving force to successfully answer this part.

Question 5 (ii)

The car is attached to a trailer of mass 200 kg by a light rigid horizontal tow bar. The greatest steady speed of the car and trailer on the road is now $30\,\mathrm{m\,s^{-1}}$. The resistance to motion of the trailer may also be assumed constant.

(ii) Find the magnitude of the resistance force on the trailer. **[2]**

This part required candidates to use their answer from part (i) in a connected particles problem to find the resistance of the trailer.

The overall majority of candidates did this successfully using Newton's First Law on the system as a whole, taking into account the resistances of both the car and the trailer and the changed driving force. The most common mistake seen by candidates was to simply equate the driving force with the resistance of the trailer to give 1333N, gaining 0 marks.

Question 5 (iii) (a)

The car and trailer again travel along the road. At one instant their speed is $15\,\mathrm{m\,s^{-1}}$ and their acceleration is $0.57\,\mathrm{m\,s^{-2}}$.

(iii) **(a)** Find the power of the engine of the car at this instant. **[4]**

This question required candidates to build on their solutions to parts (i) and (ii), using the resistances as in part (ii), but combining them with the use of Newton's Second Law and $P = Fv$ to find the power output.

Candidates who used Newton's Second Law systematically, listing all three forces on one side and mass × acceleration on the other side, were almost always successful in this question, other than the occasional numerical error. Some combined their resistances to give an overall value (usually 1333), which was allowed as long as this was consistent with their values in parts (i) and (ii).

Just under half of candidates did not score any marks on this question. Most commonly, neither of the resistances were taken into account, giving a result of 11970W. Some did not combine the masses of the car and the trailer, whilst a few candidates tried to use the kinetic energy of the car to find the power output.

Question 5 (iii) (b)

(b) Find the magnitude of the tension in the tow bar at this instant. **[2]**

In this question, candidates were required to apply Newton's Second Law to one of the connected particles in order to find the tension. Of those candidates that attempted the question, almost all selected the trailer as the point of focus rather than the car. As with part (iii) (a), many did not account for the resistance and instead simply used the equation $T = ma$. Those who accounted for the resistance mostly gained full marks.

Question 5 (iv) (a)

In a refined model of the motion of the car and trailer the resistance to the motion of each is assumed to be zero until they reach a speed of $10\,\mathrm{m\,s^{-1}}$. When the speed is $10\,\mathrm{m\,s^{-1}}$ or above the same constant resistance forces as in the first model are assumed to apply to each.

The car and trailer start at rest on the road and accelerate, using maximum power.

(iv) Without carrying out any further calculations,

(a) explain whether the time taken to attain a speed of $20\,\mathrm{m\,s^{-1}}$ would be predicted to be lower, the same or higher using the refined model compared with the original model, **[2]**

This question, together with part (iv) (b) tested the conceptual understanding of candidates. Almost all intuitively realised that less resistance would result in less time taken to reach 20m/s. In order to gain the second mark, candidates had to clearly link the reduction in friction to an increase in acceleration (during the first phase) and hence to a reduction in time taken. Many candidates managed to do this and some went into further detail, also pointing out that there would be an increased resultant force. Less successful candidates only mentioned that there was no friction without explaining the effect of this, or that the acceleration was increased without saying why. Some produced very vague statements that did not gain any credit.

Exemplar 2

5(iv)(a) It would be B1 lower as until it reaches $10\,\mathrm{ms^{-1}}$, there are no restrictive forces acting on it. Only after that is there restriction but it is already travelling quick so it won't take too long to reach $20\,\mathrm{ms^{-1}}$ compared to the original model, which is restricted from the start. B0

We presume that "restrictive" means "resistive" here. There is no clear attempt to relate to acceleration here.

In this response, the candidate produces a long explanation saying that there is less resistance at the start so this part will take less time and therefore it "is already travelling quick" and so will take less time overall. Unfortunately, this statement does not adequately explain that it speeds up faster in the initial phase due to increased acceleration as a result of removing the resistance force.

Question 5 (iv) (b)

(b) explain whether the greatest steady speed of the system would be predicted to be lower, the same or higher using the refined model compared with the original model. **[2]**

In this question, candidates were required to identify clearly or at least imply that the terminal velocity is dependent only on the final resistance, which is unchanged (assuming that the power is unchanged). The best responses gave very full explanations, including that resistance = driving force and speed = power ÷ driving force. Less successful candidates did not adequately convey this, or focussed their explanation on things that were not directly relevant.

Exemplar 3

5(iv)(b) The greatest steady speed would remain the same B1 as the max ~~amount of~~ power produced by the cars engine remains the same. The power depends on the speed and the force. B0

Turn over

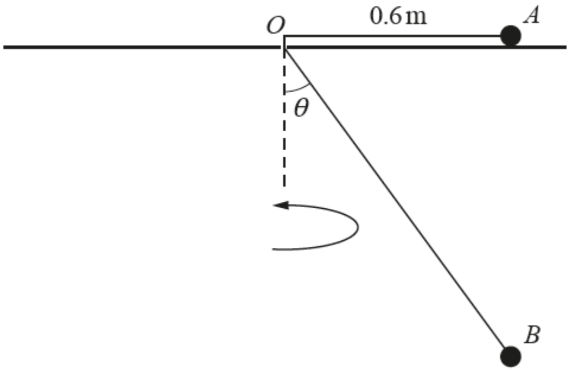
* 0009522661109 *

This answer does not mention that the resistances are the same

In this response, the candidate mentions that the power is constant and "the power depends on the speed and the force". Unfortunately, they have not explained what "the force" is, nor that at terminal velocity the resistance force (which determines the force of the engine and hence the speed of the car) is unchanged in the new model.

Question 6 (i)

6 Two particles A and B are connected by a light inextensible string. Particle A has mass 1.2 kg and moves on a smooth horizontal table in a circular path of radius 0.6 m and centre O. The string passes through a small smooth hole at O. Particle B moves in a horizontal circle in such a way that it is always vertically below A. The angle that the portion of the string below the table makes with the downwards vertical through O is θ, where $\cos\theta = \frac{4}{5}$ (see diagram).



(i) Find the time taken for the particles to perform a complete revolution. **[7]**

This question was about two connected particles moving in horizontal circles with one forming a conical pendulum. Candidates needed to focus on particle B, resolving forces vertically and horizontally using Newton's Law's and the formula for radial acceleration.

This item was challenging for most candidates, who often just wrote any formulas that they thought might be relevant. More successful candidates were able to resolve the tension in the string at B correctly to form suitable equations that could then be solved to find ω and t.

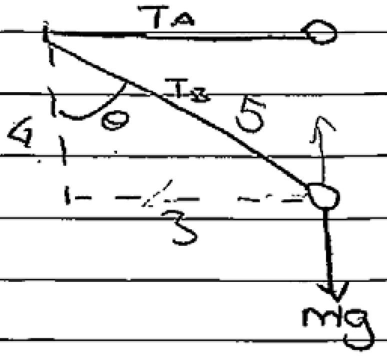
Less successful candidates were often distracted by particle A and tried for example to relate the tension at A to the tension at B in some way(which was the intention of part (ii)). Other common problems included resolving in the wrong direction with candidates frequently stating $T = mg\cos\theta$ rather than $T\cos\theta = mg$, and assuming that they needed to find T in order to find ω.

Many candidates did not recognise the use of a 3, 4, 5 triangle, which gives simple exact values of $\sin\theta$ and $\tan\theta$, opting instead to use $\theta = 36.9°$.

Practice of finding the corresponding $\sin\theta$ and $\tan\theta$ expressions from a given $\cos\theta = a/b$ using a sketch and Pythagoras' Theorem (and the equivalents from a given $\sin\theta$ or $\tan\theta$ value), will help avoid complex decimal coefficients in algebraic expressions and ensure access to exact answers when required.

Exemplar 4

6(i)



$mg = T_B\cos\theta$ M1

$mg = \frac{4T_B}{5}$

$T_B\cos\theta = T_A = mg$

$\boxed{T_A = 1.2\frac{\omega^2}{0.6}}$ $\quad T_A = mg$

$\boxed{T_B\sin\theta = m\frac{\omega^2}{0.6}}$ $\quad T_B = \frac{5mg}{4}$

$\frac{3T_B}{5} = m\frac{\omega^2}{0.6}$

$\cancel{\frac{3T_B \times 0.6}{5\times\omega^2}} = m$ $\quad 3T_B = \frac{15mg}{4}$ $\quad T_B\sin\theta = \frac{15mg}{20}$

B1

$\frac{15mg}{5\times 4} = \frac{15mg}{20} = \frac{m\omega^2}{0.6}$

$mg = 1.2\frac{\omega^2}{0.6}$

$\frac{3T_B}{5}$ $\quad T_B = \frac{5mg}{4}$

$\frac{15mg}{20} = T_B\sin\theta$

$\frac{15mg}{20} = \frac{m\omega^2}{0.6}$

$\frac{15g}{20} = \frac{\omega^2}{0.6}$ $\quad \sqrt{\frac{15g\times 0.6}{20}} = \omega = 2.1$

2.1 rads^{-1}

$\frac{2\pi}{2.1} = 4.18$ M1

$\therefore$ 4.18 s

In this exemplar, the candidate has attempted a variety of approaches, some correct and some incorrect. For example, at one point they have equated the weight of particle B with the tension at A. We have given them credit wherever possible.

For resolving T vertically, i.e. $T\cos\theta = mg$, they have gained M1. Unfortunately, they have used $a = \omega^2/r$ instead of $r\omega^2$ or v^2/r when resolving horizontally, and therefore do not have two suitable equations to solve. Notice however, that they have also written T as $mg/\cos\theta$ and multiplied by $\sin\theta$ to get $\frac{3}{4}mg$ ($= mg\tan\theta$), seen as 15/20. We have taken this to imply that $\tan\theta = \frac{3}{4}$ and therefore given B1 here. Later on, they have produced a value for ω, albeit from wrong equations, but gain M1 for correctly using it to get a value of t.

Question 6 (ii)

(ii) Find the mass of B. **[3]**

This question required candidates to equate the tension at B with the tension at A in order to determine the mass of particle B. It also required rearranging one of their equations from the start of part (i) and using their value of ω or v. Most candidates who had successfully completed part (i) went on to complete part (ii) although some made errors, such as assuming that the centripetal force at A was the same as at B.

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