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Oxford Cambridge and RSA

AS/A LEVEL GCE

Examiners' report

MATHEMATICS (MEI)

3895-3898, 7895-7898

4754/01A Summer 2018 series

Version 1

Contents

Introduction

Our examiners’ reports are produced to offer constructive feedback on candidates’ performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates’ performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper 4754/01A series overview

Applications of Advanced Mathematics (core 4) 4754 is the fourth mandatory component of 7895 A Level Mathematics (MEI). This component is made up of this examination paper and a separate comprehension task. This is the final assessment series for this specification, although there is a resit opportunity in summer 2019.

On the whole, candidates found Paper A this year slightly more demanding than last year although the standard of work in the majority of cases was very high. This paper was accessible to all candidates but there were sufficient questions for the more able candidates to show their skills.

Candidates made similar errors as in previous years and these included:

- Sign and basic algebraic errors (Questions 3, 4, 6(i), 6(ii), 8(ii) and 8(iii))
- Failure to include a constant of integration (Question 8(ii))
- Inappropriate accuracy, for example in Question 5 (i) and (ii), candidates either gave insufficient accuracy (answers to 2 significant figures) or they gave too much accuracy (answers to 4 or more significant figures). Candidates are reminded to give answers to 1 decimal place for questions involving trigonometry (Questions 6(ii) and 7(i))
- Failure to give exact answers when required (Question 2 and 4(ii))
- Failure to give sufficient detail when verifying given results (Questions 6(i) and 8(ii))

Quite a number of candidates did not attempt some parts but there did not appear to be a shortage of time.

Centres are again reminded that as Papers A and B are marked separately any supplementary sheets used must be attached to the appropriate paper. Furthermore, centres are requested that Papers A and B are not attached to each other and are sent separately for marking.

Section A overview

Section A focuses upon routine calculations and structured problem solving questions. This section is worth 36 marks. Candidates made a good attempt at all the questions in Section A, with very few parts of questions with no response provided.

Question 1

1 Express $\sin\theta - 2.4\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Hence write down the maximum value of the function $f(\theta) = 1 - \sin\theta + 2.4\cos\theta$, where $0 \leqslant \theta \leqslant 2\pi$. **[5]**

The majority of candidates correctly calculated the values of *R* and α although some lost the first method mark by not including *R* in the expanded trigonometric statements $R\cos\alpha = 1$ and $R\sin\alpha = 2.4$. Some candidates did not give α in radians and a small minority stated *R* as 6.76 rather than the correct 2.6. While candidates found the first four marks in this part relatively straightforward many could not write down the maximum value of $f(\theta)$ even though the question gave the hint of 'hence'; many either gave the maximum as 2.6 (the value of *R*) or as 3.4 rather than realising that $f(\theta) = 1 - 2.6\sin(\theta - 1.17...) \Rightarrow f_{max} = 1 - 2.6(-1) = 3.6$.

Question 2

2 The finite region bounded by the curve $y = \ln x$, the *x*-axis, the *y*-axis and the line $y = 1$ is rotated through 360° about the *y*-axis. Find the exact volume of the solid of revolution generated. **[4]**

In this question the vast majority of candidates considered both the correct integral (with correct limits) for the volume of revolution generated by rotating the given curve about the *y*-axis and went on to integrate correctly. A number of candidates, however, misread the question and instead tried to calculate the volume of revolution generated by rotating the curve about the *x*-axis. While nearly all candidates who correctly rotated about the *y*-axis stated that $V = \pi\int_0^1 \left(e^y\right)^2 dy$ a number then wrote $\int_0^1 e^{y^2} dy = \left[\frac{1}{3}e^{y^3}\right]_0^1$ or, for those that did have the correct integral, some stated that $\int_0^1 e^{2y} dy = \left[2e^{2y}\right]_0^1$. Of those that did integrate correctly a number forgot the π in their final answer or did not give an exact answer. Finally, a number of candidates, who had the correct answer of $\frac{1}{2}\pi\left(e^2 - 1\right)$ then, for some inexplicable reason, went on to halve (or even double) their answer.

Question 3

3 Find the first three terms of the binomial expansion of $\frac{1+2x}{(2-x)^3}$ in ascending powers of x.

State the set of values of x for which the expansion is valid. **[7]**

The vast majority of candidates correctly re-wrote the expression $\frac{1+2x}{(2-x)^3}$ as $(1+2x)(2-x)^{-3}$ and then went on to re-write $(2-x)^{-3}$ as $\frac{1}{8}\left(1-\frac{1}{2}x\right)^{-3}$. However, some candidates either expanded $(2-x)^3$ in the denominator of the original expression, stated that $(2-x)^{-3}=1+(-3)(-x)+...$ or tried (mostly without success) to express the original expression using partial fractions. While some candidates struggled with the factor of $\frac{1}{8}$, most correctly expanded $\left(1-\frac{1}{2}x\right)^{-3}$ (although the third term for some contained a binomial coefficient of $\frac{(-3)(-2)}{2!}$). Most candidates, after expanding $(2-x)^{-3}$, went on to successfully multiply their expansion for $(2-x)^{-3}$ with $(1+2x)$ and therefore obtained the first three terms of the required binomial expansion.

Finally, a number of candidates either did not state the set of values of x for which the expansion was valid or gave an answer of $|x|<\frac{1}{2}$ either on its own or together with the correct answer of $|x|<2$.

Question 4(i)

4 A curve has parametric equations $x = \sin 2\theta$, $y = 1 + 2\cos\theta - \cos 2\theta$, where $0 < \theta < \pi$.

(i) Find $\frac{dy}{dx}$ in terms of θ. **[3]**

Apart from the standard errors in trigonometric differentiation most candidates differentiated both terms correctly and then divided $\frac{dy}{d\theta}$ by $\frac{dx}{d\theta}$ to obtain the required $\frac{dy}{dx}$ in terms of θ. Some, however, began this part by attempting to expand the double-angles before differentiation (and many of these attempts were not successful). Finally, a number of candidates, after having stated the correct derivative, spent time unnecessarily 'simplifying' their expression and, in some cases, introducing unnecessary errors.

Question 4(ii)

(ii) Find the exact coordinates of the point on the curve where the gradient is zero. **[4]**

While it was pleasing to note that nearly all candidates recognised that only the numerator of their algebraic fraction from part (i) needed to be put equal to zero many struggled with solving the corresponding trigonometric equation with many writing $\sin 2\theta = \sin\theta \Rightarrow \theta = 0$ and hence gaining no marks in this part. Of those that applied the correct double-angle formula many did obtain the correct $\theta = \frac{\pi}{3}$ and hence the correct (exact) values of *x* and *y*. While it was pleasing to see many candidates considering the equation $\sin\theta(2\cos\theta - 1) = 0$ (rather than immediately dividing by $\sin\theta$) a number gave a solution of (0, 2) which came from a value of theta (that is $\theta = 0$) that was unfortunately not in the required range.

Question 5(i)

5 Fig. 5 shows the curve with equation $y = \sqrt{1 + x^3}$.

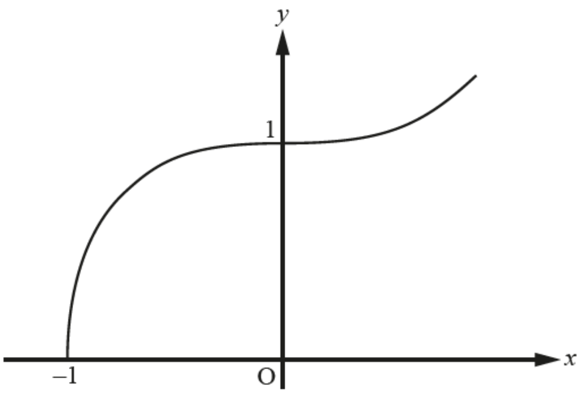

Fig. 5

(i) Use the trapezium rule with 4 strips to estimate the finite area enclosed by the curve and the *x*- and *y*-axes, giving your answer correct to 3 significant figures. **[3]**

Part (i) was answered extremely well with the vast majority of candidates giving the correct answer of 0.797. When errors occurred, it was usually due to an incorrect value for the width of the strips or with the omission of a value. It was very rare for candidates to use the *x* values or to not give the answer to the required 3 significant figures.

Question 5(ii)

(ii) Use a quarter circle of radius 1 to estimate this area, giving your answer correct to 3 significant figures. **[1]**

Nearly all candidates achieved the correct answer of 0.785 although some did not give the answer to the required 3 significant figures, instead giving it to at least 4 significant figures or as a multiple of π. There were a small proportion of candidates that used incorrect formulae for the area of a circle.

Question 5(iii)

(iii) State, with a reason, which of these estimates is closer to the true area. **[1]**

While approximately half of the candidates correctly stated that the trapezium rule was the closer estimate of the two this therefore meant that approximately half of the candidates stated that the quarter circle was closer to the true area. Of those that did state that the trapezium rule was closer many did not give sufficient detail for why it provided a better estimate for the true area. Examiners needed to see mention of the fact that although the trapezium rule gives an underestimate of the area it is still greater in value than the quarter circle estimate.

Question 6(i)

6 In Fig. 6, triangle ADC is right-angled at C, with $CD = h$. The point B on AC is such that $AB = x$, angle $ADB = \alpha$ and angle $BDC = \beta$.

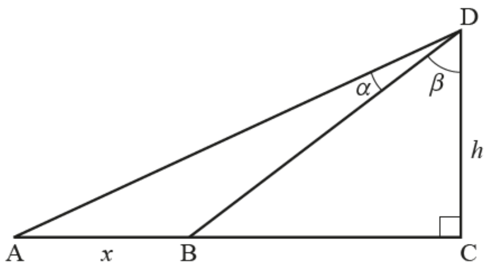

Fig. 6

(i) Find BC and AC in terms of h, α and β.

Hence show that $x = \dfrac{h\tan\alpha\sec^2\beta}{1-\tan\alpha\tan\beta}$. **[5]**

While many candidates did state correct expressions for BC and AC in terms of h, α and β, a number gave an incorrect answer of $BC = \dfrac{h}{\tan\beta}$ (together with a similar incorrect expression for AC), or did not give explicit expressions for these two lengths. It was also relatively common to see an expression for AC given in terms of *x*. The majority of candidates stated that $x = h\left(\tan(\alpha+\beta) - \tan\beta\right)$ and most correctly expanded $\tan(\alpha+\beta)$. While most went on to correctly combine both fractions only the most accurate of candidates obtained full marks for correctly obtaining (without errors) the given answer for *x*.

Question 6(ii)

(ii) Given that $x = h$ and $\beta = 30°$, find α, giving your answer correct to 1 decimal place. **[3]**

In questions such as this, candidates are strongly advised to immediately substitute the given values before attempting to re-arrange as this makes the resulting re-arrangement and simplification a lot easier to complete. Also, many candidates did not see the natural link between the answer given in part (i) and the demand in part (ii). While the majority of candidates correctly stated the value of α correct to 1 decimal place it was slightly worrying the number of candidates who either explicitly stated (or implied) that $\sec^2\beta = \dfrac{1}{\tan^2\beta}$.

Section B overview

Section B has two longer questions with a more problem solving focus. These questions proved challenging for many candidates, but it was pleasing to see candidates making a good attempt at the majority of parts ensuring partial credit, even if the response did not progress to a complete solution.

Question 7(i)

7 Three points A, B and C have coordinates A (2, 1, 1), B (1, –3, –1) and C (–4, –1, 0).

(i) Find the lengths AB and AC, and use a scalar product to calculate the angle BAC.

Hence find the area of triangle ABC. **[7]**

It was pleasing to note that the majority of candidates scored extremely well on this part with many scoring full marks, even though this was a multi-step solution. A number of candidates didn't state explicitly the lengths of AB and AC as requested and there were a significant number of candidates made transpose errors copying the coordinates from the question. The question specifically required candidates to use the scalar product, but a significant number of responses used the cosine rule to find this angle. The most common method error in this part was for those candidates who used incorrect direction vectors or simply used two of the three points as direction vectors in their scalar product. While most used $\frac{1}{2}ab\sin C$ correctly to find the area of triangle ABC a number either used an incorrect form of this equation (for example, some used cosine instead of sine and some forgot the half) while some tried (mostly unsuccessfully) to use the formula half base times height.

Question 7(ii)

The lines with vector equations

$$\mathbf{r} = 2\mathbf{i}+\mathbf{j}+\mathbf{k}+\lambda(2\mathbf{i}+\mathbf{j}-\mathbf{k}), \quad \mathbf{r} = \mathbf{i}-3\mathbf{j}-\mathbf{k}+\mu(-\mathbf{i}+3\mathbf{j}+3\mathbf{k}) \quad \text{and} \quad \mathbf{r} = -4\mathbf{i}-\mathbf{j}+\nu(4\mathbf{i}+\mathbf{j}+2\mathbf{k})$$

pass through the points A, B and C respectively.

(ii) Show that these three lines meet at a point D. **[6]**

While some candidates gave textbook answers to this part and showed conclusively that all three lines would meet at the point D most did not show sufficient working. In fact, examiners commented that in many scripts the candidates' work was unclear making it extremely difficult to read and understand at times. The most common way of showing this required result was to first show that two of the lines did indeed meet at a point. This should have been done by first showing that there exists unique scalar parameters such that all three corresponding equations for those two lines are consistent. While many candidates started this method correctly by taking two of the equations and solving to find the required parameters they never showed consistency in the third equation, therefore never showing that two of the lines did indeed meet at a point. Of those that did show conclusively that two of the lines did meet at a point many did go on to either solve for the third parameter and then show that this value gave the same point as before or they proceeded to set the third line equal to D and showed that all three equations in the final parameter were consistent. The most common error was from those candidates who thought that simply finding D was equivalent to showing that all three lines meet at a point (and in many cases this only gained three of the six marks available).

Question 7(iii)

You are given that the plane ABC has equation $\mathbf{r}.(\mathbf{j}-2\mathbf{k}) = -1$. The normal through D to the plane ABC meets the plane at E.

(iii) Find the coordinates of E. **[3]**

Part (iii) and part (iv) were the only questions where a significant number of candidates did not provide any response. Of those candidates that did attempt part (iii); very few correctly substituted the line through D normal to the plane into the equation of the plane. Therefore, the majority did not solve a linear equation to find the scalar parameter which then needed to be substituted back into the line normal to the plane to obtain the coordinates of E as (0, 0.6, and 0.8).

Question 7(iv)

The volume of a tetrahedron is $\frac{1}{3} \times$ area of base $\times$ height.

(iv) Find the volume of the tetrahedron ABCD. **[3]**

Although a significant number of candidates made no attempt on this part, a good number of candidates were able to score at least the method mark for correctly calculating $\frac{1}{3}(\text{their DE})(\text{their area from part (i)})$.

Question 8(i)

8 The speed $v\,\mathrm{m\,s^{-1}}$ of an object at time t seconds is modelled by the differential equation

$$\frac{dv}{dt} = -kv(4+v^2),$$

where k is a positive constant. Initially, $v = 4$.

(i) Find constants A, B and C such that $\dfrac{1}{v(4+v^2)} = \dfrac{A}{v} + \dfrac{Bv+C}{4+v^2}$. **[5]**

This part was answered extremely well with many candidates scoring full marks. However, a number of candidates obtained the correct values of *A*, *B* and *C* from incorrect working which then had a knock-on effect with regards to the accuracy marks in part (ii).

Question 8(ii)

(ii) Hence show by integration that $v = \dfrac{4}{\sqrt{5e^{8kt} - 4}}$. **[9]**

This part was the probably the most demanding part of the paper and many candidates made very little progress after correctly separating the variables and writing $\int \frac{\mathrm{d}v}{v\left(4+v^2\right)} = -\int k \,\mathrm{d}t$. Of those that did use part (i) many incorrectly wrote $\frac{1}{4}\int \frac{1}{v} - \frac{1}{4+v^2}\mathrm{d}v = -kt + c$ or could not deal with the required integration. Of those that obtained a correctly integrated expression, for example, $\frac{1}{4}\ln v - \frac{1}{8}\ln\left(4+v^2\right) = -kt + c$ some either forgot the constant of integration or assumed it was zero. Many candidates who did work out their constant correctly and had a correct equation, for example, $\frac{1}{4}\ln v - \frac{1}{8}\ln\left(4+v^2\right) = -kt + \frac{1}{4}\ln 4 - \frac{1}{8}\ln 20$ then struggled with the algebra required to either combine the log terms or to remove the logs correctly from all terms. Very few candidates obtained a correct equation without logs, for example, $\frac{5v^2}{4\left(4+v^2\right)} = e^{-8kt}$ and of those that did many then struggled to make v^2 and then v the subject.

Question 8(iii)

(iii) After 1 second the speed of the object is $2\,\mathrm{m\,s^{-1}}$. Find the value of k. **[3]**

It was pleasing to note that many candidates who had struggled with part (ii) realised that they could still access the marks in part (iii). Many correctly substituted $t = 1$ and $v = 2$ into the given answer from part (ii) and most could then re-arrange correctly to make e^{8k} the subject. From this point, most then went on to correctly state k either exactly or correct to at least three decimal places.

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