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AS/A LEVEL GCE

*Examiners' report*

# *MATHEMATICS (MEI)*

**3895-3898, 7895-7898**

## 4756/01 Summer 2018 series

Version 1

# Contents

# Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

# Paper 4756/01 series overview

Candidates found parts of question 1 (calculus) and question 2 (complex numbers) to be particularly challenging, and the marks on these two questions were generally low. Candidates performed much better on question 3 (matrices) and question 4 (hyperbolic functions).

There are several places where candidates are asked for a proof, or where the answer is given on the examination. In such cases candidates are advised to show every detail of their working, so that they do not lose marks unnecessarily. Examiners will not fill in any gaps in the calculations or the reasoning, however ‘obvious’ the steps might appear to be. Just copying the given answer will not be given any credit unless it follows from the previous working. Candidates often obtain an answer which differs from the given one; it is then perfectly legitimate to look back, discover the error and make corrections. However, great care is needed, as forgetting to correct a single line could invalidate the whole argument.

 

## Question 1 (a) (i)

**1** **(a)** The polar equation of a curve is $r = a\sin^2\theta\cos\theta$ for $0 \leqslant \theta \leqslant \frac{1}{2}\pi$.

**(i)** Find the value of $\theta$ for which the curve has the maximum $x$-coordinate. **[3]**

The essential first step was to use $x = r\cos\theta$ and the polar equation of the curve to express $x$ in terms of $\theta$. Candidates who did this were very often able to complete this part; some differentiated the expression, and others rewrote it as $x = \frac{1}{4}a\sin^2 2\theta$. Most candidates did not start in the right way, and so earned no credit. Many differentiated the polar equation, as if they were maximising $r$ instead of $x$. Many obtained the cartesian equation of the curve, but were unable to progress beyond this.

## Question 1 (a) (ii)

**(ii)** Prove that the maximum $y$-coordinate on the curve is $\frac{3\sqrt{3}}{16}a$ and state the value of $\theta$ at which this is attained. **[4]**

To earn any credit in this part it was necessary to write $y$ in terms of $\theta$ and obtain $\frac{\mathrm{d}y}{\mathrm{d}\theta}$. Only a small minority of candidates did this. Some tried to use trigonometrical identities to express $y$ in a form where the maximum value could be seen, but this proved to be unsuccessful. Most candidates either omitted this part altogether, or repeated the misunderstandings they had shown in part (i).

## Question 1 (b) (i)

**(b)** **(i)** Sketch the graph of $y = \arcsin x$ for $-1 \leqslant x \leqslant 1$. **[1]**

Most candidates drew a graph with the correct shape. The most common reason for not earning the mark was not indicating the $y$-coordinates $\left(\pm\frac{1}{2}\pi\right)$ of the end-points.

## Question 1 (b) (ii)

**(ii)** Prove that $\frac{\mathrm{d}}{\mathrm{d}x}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$. **[4]**

This was very well answered, with most candidates differentiating $\sin y = x$ and then using $\cos^2 y = 1 - \sin^2 y = 1 - x^2$. As the answer is given, a fully detailed explanation is expected, including why the positive square root is taken. Most cited that the gradient of the graph is always positive; and others stated that $\cos y > 0$ because $-\frac{1}{2}\pi < y < \frac{1}{2}\pi$.

## Question 1 (b) (iii)

**(iii)** Using integration by parts and a suitable substitution, show that

$$\int_0^1 x^2 \arcsin x \, dx = \frac{3\pi - 4}{18}. \quad [6]$$

Almost all candidates applied the formula for integration by parts correctly. Most then selected the substitution $u = 1 - x^2$ or $u^2 = 1 - x^2$ to obtain an integrable form. Many selected $x = \sin u$ but some could not deal with the resulting integral $\int \sin^3 u \, du$. Many solutions were quite difficult to follow, with numerous corrections and changes of sign; and missing d$x$'s and d$u$'s making it uncertain when the change of variables had been completed. Many candidates did not explain clearly how they were dealing with the limits of integration. The answer is given, so to earn full marks the final version presented needs to be accurate and fully explained.

## Question 2 (a) (i)

**2** **(a)** **(i)** Use de Moivre's theorem to prove that

$$\cot 4\theta = \frac{1 - 6\tan^2\theta + \tan^4\theta}{4\tan\theta(1 - \tan^2\theta)}. \quad [5]$$

Most candidates understood how to apply deMoivre's theorem to obtain this result. With the answer given, a full explanation was expected, and very many candidates did not state that they were dividing the numerator and denominator by $\cos^4\theta$ to convert the terms into powers of $\tan\theta$. Some candidates ignored the instruction to use deMoivre's theorem and used the formula for $\tan 2\theta$ to obtain the result; this was not given any credit.

## Question 2 (a) (ii)

**(ii)** Hence express the roots of the equation

$$x^4 + 4x^3 - 6x^2 - 4x + 1 = 0$$

in exact trigonometrical form. **[4]**

Most candidates did not see how to relate this equation to part (i), and many omitted this part altogether. Many did use $x = \tan\theta$ to transform the equation to $\cot 4\theta = 1$, but only some of these went on to complete the solution.

## Question 2 (b) (i)

**(b)** The vertices of a square with sides of length 1 unit lie on the axes of an Argand diagram. The vertices represent the complex numbers $z_1$, $z_2$, $z_3$ and $z_4$ and the midpoints of the sides of the square represent the complex numbers $z_5$, $z_6$, $z_7$ and $z_8$.

**(i)** Express $z_5$, $z_6$, $z_7$ and $z_8$ in modulus-argument form, and hence determine a polynomial equation of degree 4, with integer coefficients, whose roots are $z_5$, $z_6$, $z_7$ and $z_8$. **[4]**

Most candidates gave the four arguments correctly. Many candidates took the vertices of the square to be at 1, j, –1, –j so that the sides had length $\sqrt{2}$ instead of 1, and therefore gave the wrong modulus for the midpoints. Those who realised that the midpoints represented the fourth roots of a number were able to obtain a polynomial equation; but others tried to expand $(z-z_5)(z-z_6)(z-z_7)(z-z_8)=0$ usually without success.

## Question 2 (b) (ii)

Let $\mathrm{P}(z)=0$ be a polynomial equation of degree 8, with integer coefficients, whose roots are $z_1$, $z_2$, $z_3$, $z_4$, $z_5$, $z_6$, $z_7$ and $z_8$.

**(ii)** Explain why $\mathrm{P}(z)$ cannot be of the form $az^8+b$ where $a$ and $b$ are integers. **[1]**

Most candidates who offered a correct explanation stated that the eight complex numbers did not all have the same modulus.

## Question 2 (b) (iii)

**(iii)** Find $\mathrm{P}(z)$. **[4]**

Candidates who recognised that the eight complex numbers consisted of two sets of fourth roots were usually able to make progress.

## Question 3 (i)

**3** **(i)** Find the inverse of the matrix $\begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & k \\ k & 1 & 6 \end{pmatrix}$. **[5]**

Most candidates understood how to find the inverse matrix. The only common mistakes were careless errors in the calculation of the determinant or the cofactors.

## Question 3 (ii)

The matrix $\mathbf{M}$ has eigenvalues 1, 2 and 3. The corresponding eigenvectors are $\begin{pmatrix} 2 \\ 0 \\ 6 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ respectively.

**(ii)** Write down the matrix $\mathbf{P}$ such that $\mathbf{M}=\mathbf{PDP}^{-1}$ where $\mathbf{D}=\begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. **[2]**

Almost all candidates wrote down the correct matrix. Just a few had the columns in the wrong order.

## Question 3 (iii)

**(iii)** Hence find **M**. **[5]**

Most candidates found the inverse of **P** and carried out the matrix multiplications confidently, although careless errors were fairly common.

## Question 3 (iv)

**(iv)** Find constants $a$, $b$ and $c$ such that $\mathbf{M}^{-1} = a\mathbf{M}^2 + b\mathbf{M} + c\mathbf{I}$. **[6]**

Most candidates approached this by finding the characteristic equation of **M**, applying the Cayley-Hamilton theorem, and then multiplying by $\mathbf{M}^{-1}$. This was very often completed accurately, with the first step being the most likely to go wrong. The characteristic equation was usually obtained correctly when considered as the equation with the eigenvalues (1, 2, 3) as its roots. Many candidates attempted to find it by expanding $\det(\mathbf{M} - \lambda\mathbf{I})$ and this was very rarely successful.

## Question 4 (i)

**4** **(i)** Prove, using definitions in terms of exponential functions, that

$$\cosh 2A = 1 + 2\sinh^2 A.$$ **[3]**

This was answered very well by most candidates. Some lost marks by missing out steps in the working, for example, writing down $2\sinh^2 A = \frac{1}{2}\left(e^{2A} + e^{-2A} - 2\right)$ without giving the exponential form of $\sinh A$. Some gave the correct working in terms of exponentials, but never related their conclusion to $\cosh 2A$ and $\sinh A$.

## Question 4 (ii)

**(ii)** Find $\int \sinh^2 x \, dx$. **[3]**

This was also answered well, either by using the identity from part (i) or by converting to exponential form. Marks were often lost through careless slips, or by omitting the arbitrary constant.

## Question 4 (iii)

**(iii)** Let $z = \text{arsinh}(1)$. Form an equation involving $z$ and solve it to find the exact value of arsinh(1) in logarithmic form. **[4]**

This was well understood, with almost all candidates obtaining the correct answer.

## Question 4 (iv)

**(iv)** Using a substitution of the form $ax = b\sinh u$, find the exact value of

$$\int_0^{\frac{2}{3}} \frac{x^2}{\sqrt{4+9x^2}}\,dx,$$

giving your answer in the form $p(q - \ln r)$, where $p$, $q$ and $r$ are constants. **[8]**

Most candidates used the right substitution $3x = 2\sinh u$ and transformed the integral to $\int \frac{4}{27}\sinh^2 u \, du$, although some did not carry out the change of variable correctly. This integral was given by part (ii), and then the most challenging step was to find the exact value of $\sinh 2u$ when. $\sinh u = 1$. Many candidates did this efficiently by writing it as $2\sinh u \cosh u$ or by converting to exponential form.

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