Qualification Accredited

Oxford Cambridge and RSA

AS/A LEVEL GCE

*Examiners' report*

# *MATHEMATICS (MEI)*

**3895-3898, 7895-7898**

## 4757/01 Summer 2018 series

Version 1

# Contents

# Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

# Paper 4757 series overview

This was the final assessment series for the unitised 3895,3898/7895,7898 GCE Mathematics (MEI) specification. There will be a resit opportunity in the summer 2019.

Further Applications of Advanced Mathematics (FP3) 4757 is an A2 GCE unit taken as part of the Mathematics (MEI) specification, occasionally used as the second optional unit in AS Further Mathematics (3896), but generally as one of the four optional units in A level Further Mathematics (7896). A few candidates are also studying AS Further Mathematics (Additional) (3897) or A level Further Mathematics (Additional) (7897), and in this case the grading optimisation process will determine which qualification this unit will contribute towards.

Further Applications of Advanced Mathematics (FP3) 4757 is made up of five optional questions from which candidates choose three. The work on this paper was generally of a high standard, with most candidates producing good answers to all of their chosen three questions. Presentation was variable, but most candidates did set out their solutions clearly.

Several questions asked for a proof or for a given result to be obtained. When answering such questions, candidates are advised to show every detail of their working. The examiner should not be expected to fill in gaps in calculations or reasoning, however 'obvious' they might seem. No credit can be given for just copying a given result, without explaining how it has arisen.

# Option overview

Q2 and Q4 were the most popular two questions (77% and 74% of the candidature respectively), Question 3 was the least frequently chosen question (with only 17% of the candidates attempting this question).

There was very little difference between the average score achieved on each of the five questions, although Q3 and Q4 were answered slightly better than the other questions.

There were a small percentage of candidates that answered more than the required three questions: in these cases all questions were marked and the best three scores used. Generally, candidates are better advised to focus all their time on three specific questions rather than attempting more questions in a time designed for three full answers; any time at the end would be better used to refine their earlier answers rather than attempting an extra question.

## Question 1 (i)

**1** The equations of two planes, $P$ and $Q$, are as follows.

$$\begin{aligned} P: &\quad x + 2y - 2z = 11 \\ Q: &\quad 2x - y + z = 7 \end{aligned}$$

The planes intersect in the line $L$.

**(i)** Find a cartesian equation for $L$. **[5]**

Most candidates found the direction of the line of intersection as the vector product of the two normal vectors, then found a point on the line. Others began by eliminating a variable and worked algebraically. A very large number gave the equation of the line in vector or parametric form instead of the cartesian form requested. The correct answer was $x = 5$, $y = z + 3$ seen together, although answers written in the form $\frac{x-5}{0} = \frac{y}{1} = \frac{z+3}{1}$ were condoned.

## Question 1 (ii)

**(ii)** The point with coordinates $(a, 1, 1)$ is equidistant from $P$ and $Q$. Find the two possible values of $a$. **[4]**

The only successful approach here was to find the distances of the point from the two planes, and then equate them. Most candidates used the standard formula and obtained a correct linear equation for *a*. To find the second value of *a* it was necessary to appreciate that the formula gives the distance as the modulus of an expression; this leads to a second linear equation, or, by squaring, to a quadratic equation. Many candidates gave the second value as minus the value already obtained.

## Question 1 (iii)

The points B and C have coordinates (1, 2, 7) and (1, 0, −5) respectively.

**(iii)** Show that B lies on $Q$ but not $P$ and that C lies on $P$ but not $Q$. Explain why this means that the lines BC and $L$ are skew. **[2]**

For the first mark, candidates were required to show four substitutions. This was usually done correctly, although there were many careless arithmetic errors. For the second mark, candidates were expected to explain convincingly why BC and *L* do not intersect and are not parallel. Few candidates considered both of these requirements.

## Question 1 (iv)

**(iv)** Find the shortest distance between the lines BC and $L$. **[5]**

The method for finding the shortest distance between skew lines was well understood and it was usually applied accurately.

## Question 1 (v)

The point E is the mirror image of C in the plane $Q$, and O is the origin.

**(v)** Find the volume of the tetrahedron OBCE. **[8]**

The first step was to find the point E. Some candidates considered the equation of the line from C perpendicular to *Q*, finding where it meets *Q* and then going the same distance on the other side. Others used the formula to find the perpendicular distance from C to *Q*, then CE is a vector normal to *Q* with length twice this distance. Many candidates did not know how to find E, and several just wrote down that E is $(-1, 0, 5)$, which is the reflection of C in the origin O. Most candidates gave a correct scalar triple product for calculating the volume of the tetrahedron, and were able to evaluate it using the coordinates of their point E.

## Question 2 (i)

**2** The surface $S$ has equation $z = x + 4x^2y - 2y^2 + 2$.

**(i)** Show that the tangent plane to the surface at the point (1, 1, 5) has equation $z = 9x - 4$. **[5]**

Almost all candidates found the partial derivatives correctly and evaluated them at the given point. Most candidates then obtained the equation of the tangent plane correctly.

## Question 2 (ii)

**(ii)** Show also that if a tangent plane to $S$ has equation $z = 9x + k$ then the only possible value for $k$ is −4. **[3]**

Most candidates realised that the partial derivatives needed to take the same values as in part (i). It could then be shown that this occurs only at the point given in part (i) and hence *k* must be −4. Some candidates did not mention *k* in their answer, even though this was the aim of the question.

## Question 2 (iii)

**(iii)** A point on the surface has coordinates $(1 + a, 1 + a, 5 + b)$ where $a$ and $b$ are small. Show that $b \approx \lambda a$, where $\lambda$ is a constant to be determined. **[3]**

This was very well answered. Most candidates used the values of the partial derivatives, some substituted the coordinates into the equation of the tangent plane, and others substituted the coordinates into the equation of the surface.

## Question 2 (iv)

**(iv)** Find the coordinates of the points on the surface at which the normal line is parallel to the vector $\mathbf{i} + 16\mathbf{j} - \mathbf{k}$. **[4]**

Most candidates knew that this required $\frac{dz}{dx} = 1$and $\frac{dz}{dy} = 16$, and used these to find the coordinates of the three points. Some started with $\frac{dz}{dx} = -1$ and $\frac{dz}{dy} = -16$; and others with just $\frac{dz}{dx} = \lambda$ and $\frac{dz}{dy} = 16\lambda$.

## Question 2 (v)

**(v)** Show that the only stationary point, A, on $S$ has coordinates $\left(-\frac{1}{2}, \frac{1}{4}, \frac{13}{8}\right)$.

By finding the cross-sections through A parallel to $x = 0$ and $y = 0$ respectively, determine the nature of this stationary point. **[9]**

Most candidates were able to use $\frac{dz}{dx} = \frac{dz}{dy} = 0$ to obtain the stationary point. As this point is given on the question paper, candidates should show all their working, including the calculation of the $z$-coordinate. Very many candidates did not explain why it is the only stationary point. There were 4 marks for finding the cross-sections and determining the nature of the stationary point, and many candidates did not score any of these marks, usually by considering the sections given by $x = 0$ and $y = 0$ (which do not contain the stationary point) or by using methods which do not involve cross-sections. Candidates who considered the correct sections $x = -\frac{1}{2}$ $x = -\frac{1}{2}$ and $y = \frac{1}{4}$ were very often successful in showing that the stationary point is a saddle point.

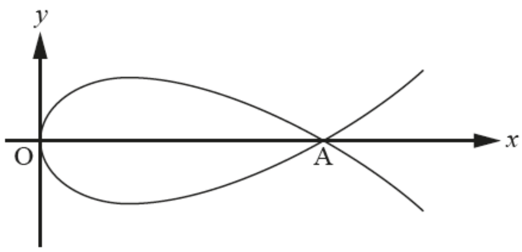
## Question 3 (a)

3 (a) Prove by integration that the surface area of a sphere with radius $a$ is given by $S = 4\pi a^2$. **[6]**

Most candidates attempted to find the surface area of revolution of a circle, taken as $x = a\cos\theta$, $y = a\sin\theta$ slightly more often than $y^2 = a^2 - x^2$ Most candidates successfully manipulated $\int 2\pi y \, ds$ into an integrable form, and often completed the proof correctly. A significant number of candidates did not choose appropriate limits of integration and so were unable to progress to a fully correct solution, with a small minority scoring zero.

## Question 3 (b) (i)

(b) A curve has parametric equations $x = 6t^2$, $y = 4t - 3t^3$. The curve crosses the $x$-axis at the origin O and at the point A, as shown in the diagram.




Find

(i) the values of $t$ at the point A, **[2]**

This was well answered. Just a few candidates included the incorrect value $t = 0$

## Question 3 (b) (ii)

(ii) the length of the arc OA for which $t$ is positive, **[6]**

Most candidates understood how to find the arc length, and carried out the process accurately.

## Question 3 (b) (iii)

(iii) the radius and centre of curvature at the point where $t = \frac{1}{3}$. **[10]**

There were several steps to be completed here: finding the second derivatives and hence the radius of curvature; finding the coordinates of the point on the curve; finding the unit normal vector, and hence the coordinates of the centre of curvature. Most candidates applied the relevant techniques confidently, and there were very many completely correct solutions.

## Question 4 (i)

**4** You are given that the set $\{1, 2, 4, 7, 8, 11, 13, 14\}$ together with the binary operation of multiplication modulo 15 forms a group G.

**(i)** Find the order of each element of G. **[4]**

Almost every candidate gave the orders of all the elements correctly.

## Question 4 (ii) (*A*)

**(ii)** (*A*) A subgroup of G has order $n$. Write down the possible values of $n$. **[2]**

Most candidates gave all the factors of 8 correctly. Some discounted $n = 1$ or $n = 8$ or both, presumably because these correspond to trivial or improper subgroups, although the question did not ask for these to be excluded.

## Question 4 (ii) (*B*)

(*B*) State all the proper cyclic subgroups of G. **[4]**

In this part the question does ask for only proper cyclic subgroups. Most candidates gave the three subgroups of order 2, and the two cyclic subgroups of order 4, correctly. Many also gave the non-cyclic subgroup {1, 4, 11, 14}.

## Question 4 (iii)

**(iii)** For each of the following three cases, determine whether the set together with the binary operation forms a group. If the set does form a group, state whether or not it is isomorphic to G, justifying your answer. (You may assume that each of the binary operations is associative.)

(*A*) The set $\{0, 1, 2, 3, 4, 5, 6, 7\}$ together with the binary operation of addition modulo 8.

(*B*) The set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ together with the binary operation of multiplication modulo 9.

(*C*) The set of matrices

$$\left\{\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}, \begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}, \begin{pmatrix}-1 & 0\\0 & -1\end{pmatrix}, \begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}, \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}, \begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix}, \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}, \begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}\right\}$$

together with the binary operation of matrix multiplication. (You may assume that the set is closed under matrix multiplication.) **[14]**

Most candidates understood the properties which they should investigate, and presented their work clearly. Many lost marks by omitting details (for example, asserting that every element has an inverse without listing the inverses or explaining why) and by not stating their conclusion (whether or not the set is a group). For (*A*) and (*C*), which are groups, candidates needed to consider whether the group is isomorphic to *G*, and some did not attempt this. Most based their arguments on the orders of the elements, for example (*A*) has one element of order 2, (*C*) has five and *G* has three. There was some confusion between self-inverse elements (which include the identity) and elements of order 2 (which do not include the identity), but this was not penalised if the candidate's intention was clear. Other arguments used were whether or not the group is cyclic (for (*A*)), or commutative (for (*C*)). For (*B*), which is not a group, most candidates showed that the set is not closed. Others observed that, for example, the element 3 does not have an inverse. Some considered the orders of the elements (for example, 2 has order 6, which is not a factor of 8).

## Question 5 (i)

**5** At a factory there are four security lights, $A$, $B$, $C$ and $D$, only one of which is on at any time; which light is on at any time can be randomised by control equipment. A number of programs are devised so that the lights switch from one to another every minute with certain probabilities. For example, if $A$ is on then at the next minute one of $B$, $C$ or $D$ will come on, with probabilities determined by the particular program being used.

The time after the start of a program is denoted by $t$ minutes. Light $A$ comes on when $t = 0$.

For program 1 the transition matrix is as follows.

$$\mathbf{P}_1 = \begin{pmatrix} 0 & 0.2 & 0.1 & 0.4 \\ 0.2 & 0 & 0.6 & 0.4 \\ 0.3 & 0.4 & 0 & 0.2 \\ 0.5 & 0.4 & 0.3 & 0 \end{pmatrix}$$

The four rows and columns correspond to lights $A$, $B$, $C$, $D$ in that order.

**(i)** Interpret the values in the leading diagonal, stating the run length for each light. **[2]**

Most candidates understood that the zeros in the leading diagonal indicated that no light would stay on for two consecutive minutes, and went on to state the run lengths. Either 0 or 1 was acceptable as the value for the run lengths.

## Question 5 (ii)

**(ii)** Find the probabilities that $A$ comes on at $t = 1$, 2, 3 and 4. **[4]**

Most candidates calculated the four probabilities correctly.

## Question 5 (iii)

**(iii)** The equilibrium probability for $A$ is $a$. From your working in part **(ii)**, write down a range within which $a$ lies. **[2]**

The probabilities in part (ii) are oscillating below and above the limiting value, and many candidates correctly deduced that *a* lies between the probabilities at $t = 3$ and $t = 4$ Some gave less efficient limits such as the probabilities at $t = 0$ and $t = 1$.

## Question 5 (iv)

**(iv)** Find the probability that the light that comes on at $t = 5$ is different from the light that comes on at $t = 1$. **[5]**

A common error here was to find the probabilities at $t = 5$ and at $t = 1$, and then proceed as if these were independent; this approach earned just 1 mark out of 5. Those who considered the correct conditional probabilities (which are the diagonal elements of $P_1^4$) usually knew how to apply them, and there were a very good number of fully correct solutions.

## Question 5 (v)

For program 2 the following rules apply.

- The light following $A$ is always $B$.
- The light following $B$ is never $D$ and is equally likely to be $A$ or $C$.
- The light following $C$ is never $A$ and is equally likely to be $B$ or $D$.
- The light following $D$ is always $C$.

**(v)** Write down the transition matrix, $\mathbf{P}_2$, for program 2. **[2]**

Almost all candidates wrote down the correct transition matrix.

## Question 5 (vi)

**(vi)** For program 2 identify any absorbing states and reflecting barriers. **[3]**

The definitions of absorbing states and reflecting barriers were generally well understood.

## Question 5 (vii)

**(vii)** Find the proportions of times that each light is on over a long period. Give your answers as exact fractions. **[6]**

Candidates who used an invariant column vector (and the sum of the probabilities being one) usually set up and solved the equations accurately. Some candidates who considered instead a high power of $P_2$ were unable to progress when they discovered that this was not of the form they were expecting. The limits for large odd and even powers are different, and candidates who realised this, and averaged the two limiting matrices, were usually able to obtain the correct limiting probabilities.

# Supporting you

For further details of this qualification please visit the subject webpage.

## Review of results

If any of your students' results are not as expected, you may wish to consider one of our review of results services. For full information about the options available visit the OCR website. If university places are at stake you may wish to consider priority service 2 reviews of marking which have an earlier deadline to ensure your reviews are processed in time for university applications.

activeresults

Active Results offers a unique perspective on results data and greater opportunities to understand students' performance.

It allows you to:

- Review reports on the **performance of individual candidates**, cohorts of students and whole centres
- **Analyse results** at question and/or topic level
- **Compare your centre** with OCR national averages or similar OCR centres.
- Identify areas of the curriculum where students excel or struggle and help **pinpoint strengths and weaknesses** of students and teaching departments.

http://www.ocr.org.uk/administration/support-and-tools/active-results/

CPD Hub
Your route to OCR's teacher training

Attend one of our popular CPD courses to hear exam feedback directly from a senior assessor or drop in to an online Q&A session.

https://www.cpdhub.ocr.org.uk

The small print

We'd like to know your view on the resources we produce. By clicking on the 'Like' or 'Dislike' button you can help us to ensure that our resources work for you. When the email template pops up please add additional comments if you wish and then just click 'Send'. Thank you.

Whether you already offer OCR qualifications, are new to OCR, or are considering switching from your current provider/awarding organisation, you can request more information by completing the Expression of Interest form which can be found here: www.ocr.org.uk/expression-of-interest



## Looking for a resource?

There is now a quick and easy search tool to help find **free** resources for your qualification:
www.ocr.org.uk/i-want-to/find-resources/

**www.ocr.org.uk**

OCR Customer Contact Centre

**General qualifications**
Telephone 01223 553998
Facsimile 01223 552627
Email general.qualifications@ocr.org.uk

OCR is part of Cambridge Assessment, a department of the University of Cambridge. *For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored.*



Cambridge Assessment

CERTIFIED ISO 9001

UKAS MANAGEMENT SYSTEMS 001