OCR
Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4777**: Numerical Computation

Advanced GCE

Mark Scheme for June 2018

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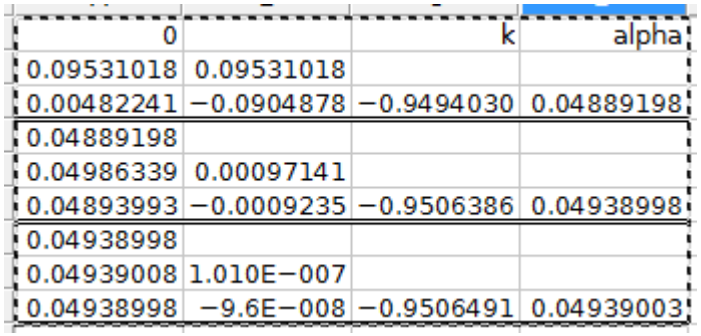
All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Question			Answer	Marks	Guidance
1	(i)		Approx error in x_2 is $k^2\varepsilon$ $x_0 = \alpha + \varepsilon$ $x_1 = \alpha + k\varepsilon$ $\quad x_1 - x_0 = (k-1)\varepsilon$ $x_2 = \alpha + k^2\varepsilon$ $\quad x_2 - x_1 = k(k-1)\varepsilon$ $\quad (x_2 - x_1)/(x_1 - x_0) = k$ Hence given expressions for α	B1 B1 B1 B1B1 B1 **[6]**	Expression for $x_{0,1,2}$ For Δx_0 For Δx_1, k
1	(ii)		Let f(x) = sin x + e^x. Then f '(x) = cos x + e^x . Note f(0) = 1, f '(0) > 0. f '(x) = 0 requires cos x = – e^x, but e^x >1 for x > 0. So f has no turning points for x > 0, and f(x) = c > 1 has a unique solution. Sketch of sin x + e^x to confirm this	B1 B1B1 B1 G1 **[5]**	
	(iii)		Iterations with c = 1.1, very slow e.g.:		
			0 0.09531 0.004822 0.090917 0.009167 0.086942	M1A1	
			0.1 0.000167 0.095159 0.004972 0.09078 0.009302	B1 for a	
			0.05 0.04881 0.049941 0.048866 0.049888 0.048916	2nd example	
			Set up and use method in part (i). E.g.:		
			 Root is 0.04939 to 4 sf	M2 for a correct spreadsheet A1A1A1 for three steps. A1 final answer **[9]**	

Question			Answer	Marks	Guidance	
1	(iv)		At least two examples with larger c, illustrating faster convergence. E.g. $c = 4$: 1 1.150106 1.127262 1.130356 1.129929 1.129988 E.g. $c = 8$: 2 1.958784 1.956472 1.956349 1.956343 1.956342	M1A1 M1A1 **[4]**	Assessment	
				[24]		

Question			Answer	Marks	Guidance
2	(i)		Set up as $\int_{-h}^{h} f(x)\,dx = a\,f(-\alpha) + a\,f(\alpha)$	M1A1	
			$f(x) = 1$ $\quad 2h = 2a$ $\quad a = h$	B1	Answer given
			$f(x) = x$, (and x^3) $\quad 0 = 0$	B1	
			$f(x) = x^2$ $\quad 2h^3 / 3 = 2a\,\alpha^2$ $\quad \alpha = h / \sqrt{3}$	B1	Answer given
			Rule is exact up to cubics	B1	No reason required
				[6]	
2	(ii)			Up to 9 marks for this spreadsheet:	m: B1 h: B1 L, R: B1 f(.): B1 G: B1 Total: B1 Subdivide, repeat: M1A1 And again: A1

a	b	m	h	L	R	f(L)	F(R)	G	Total
0	1	0.5	0.5	0.211325	0.788675	0.999004	0.849138	0.924071	0.924071
0	0.5	0.25	0.25	0.105662	0.394338	0.999938	0.988124	0.497016	
0.5	1	0.75	0.25	0.605662	0.894338	0.938828	0.78093	0.429939	0.926955
0	0.25	0.125	0.125	0.052831	0.197169	0.999996	0.999245	0.249905	
0.25	0.5	0.375	0.125	0.302831	0.447169	0.995821	0.980588	0.247051	
0.5	0.75	0.625	0.125	0.552831	0.697169	0.956334	0.899391	0.231966	
0.75	1	0.875	0.125	0.802831	0.947169	0.840535	0.744356	0.198111	0.927033
0	0.125	0.0625	0.0625	0.026416	0.098584	1	0.999953	0.124997	
0.125	0.25	0.1875	0.0625	0.151416	0.223584	0.999737	0.998753	0.124906	
0.25	0.375	0.3125	0.0625	0.276416	0.348584	0.997094	0.992698	0.124362	
0.375	0.5	0.4375	0.0625	0.401416	0.473584	0.987265	0.97576	0.122689	
0.5	0.625	0.5625	0.0625	0.526416	0.598584	0.963683	0.941395	0.119067	
0.625	0.75	0.6875	0.0625	0.651416	0.723584	0.920549	0.885917	0.112904	
0.75	0.875	0.8125	0.0625	0.776416	0.848584	0.856425	0.811498	0.104245	
0.875	1	0.9375	0.0625	0.901416	0.973584	0.776095	0.725772	0.093867	0.927037

Integral is 0.9270 to 4 dp. A1 **[10]**

Ratios of differences for these totals are 0.027212 and 0.047136 M1A1

Theory indicates 1/16 = 0. 0625 for a 4^{th} order method. Not close (but approaches in the limit). E1E1 **[4]**

Question			Answer	Marks	Guidance	
2	(iii)		Evidence of trial and improvement towards a solution. E.g. k 1.1 1.2 1.11 1.109 Int 0.9942 1.0546 1.0006 0.99993 1.11 to 2 dp	M1 A1 at least one A1 others A1 answer **[4]**		
				[24]		

3 (i)

Euler's method with $h = 0.1$:

h	x	y	y'
0.1	0	0	0
0.1	0.1	0	0.001
0.1	0.2	0.0001	0.007996
0.1	0.3	0.0009	0.026919
0.1	0.4	0.003592	0.06343
0.1	0.5	0.009935	0.122565
0.1	0.6	0.022191	0.208296
0.1	0.7	0.043021	0.323136
0.1	0.8	0.075334	0.467899
0.1	0.9	0.122124	0.641681
0.1	1	**0.186292**	0.841947

M2
Set up

A1 1st run
A1 2nd
A1 3rd

Differences and ratio of differences:

h	y(1)	diffs	ratio of diffs
0.1	0.186292		
0.05	0.203801	0.017509	
0.025	0.212647	0.008845	0.505192

M1A1

Ratio of 0.5 indicates first order method.

E1

8

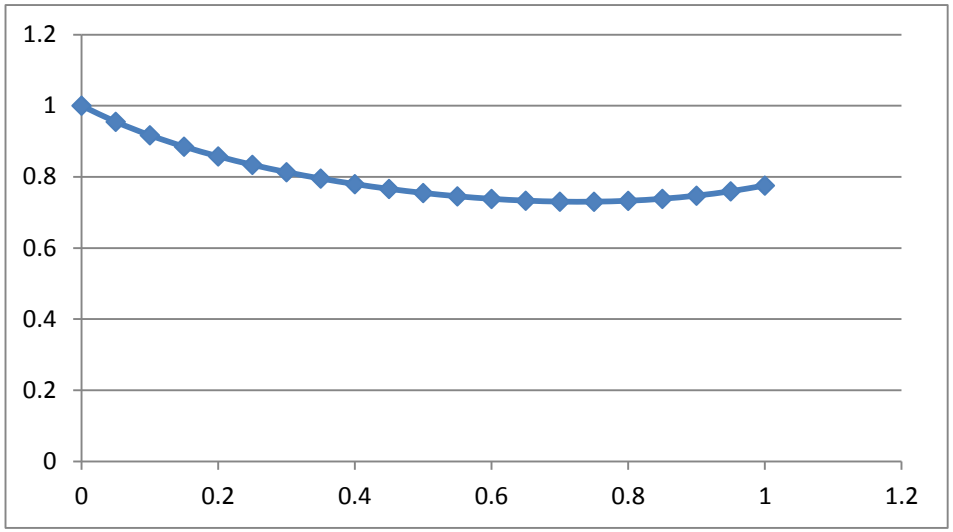
3 (ii)

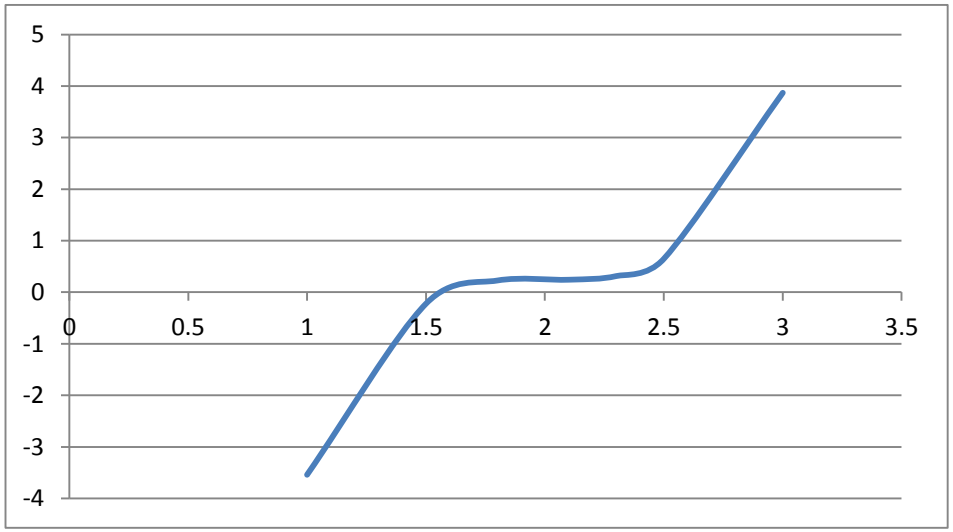
Modified Euler method with $h = 0.1$:

h	x	y	y'	x+	y+	y+'
0.1	0	0	0	0.1	0	0.001
0.1	0.1	0.00005	0.001	0.2	0.00015	0.007994
0.1	0.2	0.0005	0.00798	0.3	0.001298	0.026884
0.1	0.3	0.002243	0.0268	0.4	0.004923	0.063222
0.1	0.4	0.006744	0.062939	0.5	0.013038	0.121823
0.1	0.5	0.015982	0.121128	0.6	0.028095	0.206337
0.1	0.6	0.032355	0.204946	0.7	0.05285	0.318911
0.1	0.7	0.058548	0.31651	0.8	0.090199	0.460047
0.1	0.8	0.097376	0.456342	0.9	0.14301	0.628644
0.1	0.9	0.151625	0.623389	1	0.213964	0.822021
0.1	1	**0.223896**	0.81501	1.1	0.305397	1.03558

M3 set up

A1 1st run
A1 2nd
A1 3rd

Question			Answer	Marks	Guidance
			Differences and ratios of differences:		
			h **y(1)** **diffs** **ratio of diffs** 0.1 0.223896 0.05 0.22212 -0.00178 0.025 0.221687 -0.00043 0.243497 Ratio of 0.25 indicates second order method.	B1 E1 **[8]**	
3	(iii)		 Minimum points for reducing *h*: h x y 0.1 0.7 0.731079 0.05 0.75 0.729895 0.025 0.725 0.729388 0.0125 0.725 0.729321 0.00625 0.73125 0.729297 (0.73, 0.73) seems safe to 2 dp. *x* = *y* at the minimum is to be expected because that makes *y*' zero	M1A1 G2 M1A1 A1 E1	New values soi NB: the curve rises slightly if too large a value of *h* is used.
				[8]	
				[24]	

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th colspan="2">Guidance</th></tr>
<tr><td>4</td><td>(i)</td><td></td><td>Sketch of data points;
 

Appears roughly cubic</td><td>G2

B1
[3]</td><td colspan="2"></td></tr>
<tr><td></td><td>(ii)</td><td></td><td>Ordinary differences require equal spacing.
Lagrange’s method is computational intensive.
Divided differences allow unequal spacing and increasing the degree is easy</td><td>E1
E1
E1
[3]</td><td colspan="2"></td></tr>
<tr><td></td><td>(iii)</td><td></td><td>Divided difference table:
<table>
<tr><th>x</th><th>f</th><th>1DD</th><th>2DD</th><th>3DD</th><th>4DD</th></tr>
<tr><td>1</td><td>-3.542</td><td></td><td></td><td></td><td></td></tr>
<tr><td>1.5</td><td>-0.224</td><td>6.636</td><td></td><td></td><td></td></tr>
<tr><td>1.8</td><td>0.226</td><td>1.5</td><td>-6.42</td><td></td><td></td></tr>
<tr><td>2.1</td><td>0.241</td><td>0.05</td><td>-2.41667</td><td>3.639394</td><td></td></tr>
<tr><td>2.3</td><td>0.312</td><td>0.355</td><td>0.61</td><td>3.783333</td><td>0.110723</td></tr>
<tr><td>2.5</td><td>0.647</td><td>1.675</td><td>3.3</td><td>3.842857</td><td>0.059524</td></tr>
<tr><td>3</td><td>3.867</td><td>6.44</td><td>6.807143</td><td>3.896825</td><td>0.044974</td></tr>
</table>
Similar 3DDs suggests cubic should be a good fit.</td><td>B1
B1
B1

E1
[4]</td><td>1DD
2DD
3DD</td><td></td></tr>
</table>

Question			Answer	Marks	Guidance
4	(iv)	**(A)**	To estimate f(2):		
			x **f** **1DD** **2DD** **3DD** **4DD** 2.1 0.241 1.8 0.226 0.05 2.3 0.312 0.172 0.61 2.5 0.647 1.675 2.147143 3.842857 1.5 -0.224 0.871 1.005 3.807143 0.0595238	Modify order of data in table B1	
			x = 2 f(x) = 0.241 0.236 0.2238 0.246857 0.2466786 linear quadratic cubic quartic	A1A1A1A1	
			0.247 or 0.25 seem safe estimates	A1	
		(B)	To estimate f(2.7):		
			x **f** **1DD** **2DD** **3DD** **4DD** 2.5 0.647 3 3.867 6.44 2.3 0.312 5.078571 6.807143 2.1 0.241 0.355 5.248413 3.896825 1.8 0.226 0.05 0.61 3.865344 0.0449735	Modify order of data in table A1	
			x = 2.7 f(x) = 0.647 1.935 1.526571 1.433048 1.4324 linear quadratic cubic quartic	A1A1	
			1.43 seems safe	A1 **[10]**	
	(v)		To estimate *x* such that f(*x*) = 0, intelligent trial and error leading to something like:		
			x **f** **1DD** **2DD** **3DD** **4DD** 1.5 -0.224 1.8 0.226 1.5 2.1 0.241 0.05 -2.41667 1 -3.542 3.439091 -4.23636 3.639394 2.3 0.312 2.964615 -2.37238 3.727972 0.1107226	Modify order of data in table A1	
			x = 1.594 f(x) = -0.224 -0.083 -0.0362 -0.00054 0.0001002 linear quadratic cubic quartic	Try values of *x* M1	
			Answer: 1.59 (or 1.60)	A1 A1 **[4]** **[24]**	

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