AS LEVEL

Exemplar Candidate Work

FURTHER MATHEMATICS A

H235
For first teaching in 2017

Y534/01 Summer 2018
examination series

Version 1

www.ocr.org.uk/mathematics
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Introduction

These exemplar answers have been chosen from the summer 2018 examination series.

OCR is open to a wide variety of approaches and all answers are considered on their merits. These exemplars, therefore, should not be seen as the only way to answer questions but do illustrate how the mark scheme has been applied.

Please always refer to the specification https://www.ocr.org.uk/qualifications/as-a-level-gce/further-mathematics-a-h235-h245-from-2017/assessment/#as-level for full details of the assessment for this qualification. These exemplar answers should also be read in conjunction with the sample assessment materials and the June 2018 Examiners’ report or Report to Centres available from Interchange https://interchange.ocr.org.uk/Home.mvc/Index

The question paper, mark scheme and any resource booklet(s) will be available on the OCR website from summer 2019. Until then, they are available on OCR Interchange (school exams officers will have a login for this and are able to set up teachers with specific logins – see the following link for further information http://www.ocr.org.uk/administration/support-and-tools/interchange/managing-user-accounts/).

It is important to note that approaches to question setting and marking will remain consistent. At the same time OCR reviews all its qualifications annually and may make small adjustments to improve the performance of its assessments. We will let you know of any substantive changes.
## Question 1(i), (ii) and (iii)

1. Some jars need to be packed into small crates.

   There are 17 small jars, 7 medium jars and 3 large jars to be packed.
   - A medium jar takes up the same space as four small jars.
   - A large jar takes up the same space as nine small jars.

   Each crate can hold:
   - at most 12 small jars,
   - or at most 3 medium jars,
   - or at most 1 large jar (and 3 small jars),
   - or a mixture of jars of different sizes.

   (i) One strategy is to fill as many crates as possible with small jars first, then continue using the medium jars and finally the large jars.

   Show that this method will use seven crates. \[2\]

   The jars can be packed using fewer than seven crates.

   (ii) The jars are to be packed in the minimum number of crates possible.

   - Describe how the jars can be packed in the minimum number of crates.
   - Explain how you know that this is the minimum number of crates. \[3\]

   Some other numbers of the small, medium and large jars need to be packed into boxes.

   The number of jars that a box can hold is the same as for a crate, except that
   - a box cannot hold 3 medium jars.

   (iii) Describe a packing strategy that will minimise the number of boxes needed. \[1\]

### Exemplar 1

#### (i)

<table>
<thead>
<tr>
<th>(i)</th>
<th>0 marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(i)</td>
<td></td>
</tr>
<tr>
<td>Crate 1:</td>
<td>13 small jars and 2 medium</td>
</tr>
<tr>
<td>Crate 2:</td>
<td>3 small jars and 2 medium</td>
</tr>
<tr>
<td>Crate 3:</td>
<td>3 small jars and 2 medium</td>
</tr>
<tr>
<td>Crate 4:</td>
<td>8 small jars and 1 medium</td>
</tr>
<tr>
<td>Crate 5:</td>
<td>1 large jar</td>
</tr>
<tr>
<td>Crate 6:</td>
<td>1 large jar</td>
</tr>
<tr>
<td>Crate 7:</td>
<td>1 large jar</td>
</tr>
</tbody>
</table>
Examiner commentary

This question was a straightforward question about packing algorithms although it required candidates to cope with several constraints at the same time. In part (i), this candidate may at first appear to have tried to pack 13 small and 2 medium jars in crate 1, but in fact they have packed 3 small and 2 medium. Centres need to be aware that errors should be crossed out and not just erased as erased work does show through in the scan. However, although the amounts are consistent with the limits given in the question, they have not done what was asked in the question. The question said to fill crates with small jars first – so one crate filled with 12 small jars; a second crate with 5 small jars and 1 medium jar; a third crate with 3 medium jars; and so on.

In part (ii), an ad-hoc approach, such as full bin, could be used to find a ‘good’ packing, but candidates also needed to explain how they knew that there was no packing that used fewer crates. This candidate has given a model answer to this part, apart from the slightly awkward wording of the idea of the equivalent number of small jars.

For part (iii), ‘full bin’ was often given as the answer, but this did not address the restriction to a maximum of 2 medium jars in a crate. Some candidates gave a specific solution for the numbers of jars used in the previous parts, instead of a general strategy. A description was needed that both ensured that there were never 3 medium jars in a crate and that the number of crates was minimised subject to this additional restriction.
Question 2(i), (ii) and (iii)

2 Mo eats exactly 6 doughnuts in 4 days.

(i) What does the pigeonhole principle tell you about the number of doughnuts Mo eats in a day? [1]

Mo eats exactly 6 doughnuts in 4 days, eating at least 1 doughnut each day.

(ii) Show that there must be either two consecutive days or three consecutive days on which Mo eats a total of exactly 4 doughnuts. [3]

Mo eats exactly 3 identical jam doughnuts and exactly 3 identical iced doughnuts over the 4 days.

The number of jam doughnuts eaten on the four days is recorded as a list, for example 1, 0, 2, 0. The number of iced doughnuts eaten is not recorded.

(iii) Show that 20 different such lists are possible. [3]

Exemplar 1

(i) 1 mark

(ii) 1 mark
Exemplar 2

(i) 0 marks

(ii) 1 mark

(iii) 3 marks
Examiner commentary

This question was about counting methods. Part (i) required the interpretation of the pigeonhole principle in this situation. The pigeonholes were the days and the pigeons were the doughnuts. Instead of having at least one pigeonhole containing at least two pigeons, there was at least one day on which at least two doughnuts were eaten. The extra doughnuts could all have been eaten on one day, however.

In part (ii), 6 doughnuts in four days with at least one doughnut each day meant that either there was one day on which 3 doughnuts were eaten (with 1 on each of the other days) or there were two days on which 2 doughnuts were eaten (with 2 on each of the other days). The different permutations could then be discussed or listed to show that there were either two consecutive days on which exactly 4 doughnuts were eaten or three consecutive days on which exactly 4 doughnuts were eaten. The first candidate has listed ways in which exactly 4 doughnuts can be eaten in two days or three days, but has not shown that there are no other possibilities for 6 doughnuts in four days. The second candidate has not considered the different arrangements of 3, 1, 1, 1 and of 2, 2, 1, 1.

In part (iii), some candidates explained why there were 20 possibilities using a reasoned argument, as the first candidate has done. Most candidates tried to enumerate the 20 possibilities, as the second candidate has done.
In the pay-off matrix below, the entry in each cell is of the form \((r, c)\), where \(r\) is the pay-off for the player on rows and \(c\) is the pay-off for the player on columns when they play that cell.

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>Q</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>(1, 4)</td>
<td>(5, 3)</td>
<td>(2, 6)</td>
</tr>
<tr>
<td>Y</td>
<td>(5, 2)</td>
<td>(1, 3)</td>
<td>(0, 1)</td>
</tr>
<tr>
<td>Z</td>
<td>(4, 3)</td>
<td>(3, 1)</td>
<td>(2, 1)</td>
</tr>
</tbody>
</table>

(i) Show that the play-safe strategy for the player on columns is P. \([2]\)

(ii) Demonstrate that the game is not stable. \([2]\)

The pay-off for the cell in row Y, column P is changed from \((5, 2)\) to \((y, p)\), where \(y\) and \(p\) are real numbers.

(iii) What is the largest set of values \(A\), so that if \(y \in A\) then row Y is dominated by another row? \([1]\)

(iv) Explain why column P can never be redundant because of dominance. \([1]\)

Exemplar 1

(i) \(2\) marks

The maximum pay-off for the player on columns playing column P is 2, for column Q it is 1 and for R it is 1. \(2 > 1\) therefore the play-safe strategy for column is column P.

(ii) \(0\) marks

The largest of the maximum for the player on rows is 5, and the smallest of the maximum for the player on columns is 3. \(3 \neq 5\) - indeterminate.
Examiner commentary

Some candidates assumed that the game must be zero-sum, this would have meant that the total points for each cell was a constant, whereas here the total varies from 1 (in row Y column R) to 8 (in row X columns Q and R). In part (i), considering just the pay-offs for the player on columns, the minimum possible outcome in column P is 2, in column Q is 1 and in column R is 1, the maximin is 2 in column P. This candidate has explained why P is the play-safe choice for the player on columns.

Similarly, for the player on rows, the play-safe strategy is to play row Z. For part (ii), one way to show that a game is not stable is to consider what happens if each player uses their play-safe strategy and whether the other player would want to use their play-safe strategy or not. In this case, if the player on columns plays strategy P, then the player on rows does best by playing row Y, rather than the play-safe of row Z. Alternatively, the pay-offs when both players play-safe (row Z, column P) are 4 for the player on rows and 3 for the player on columns, but these are not the maximin values (which are both 2). This candidate has tried to resort to row maximin = column minimax, but that would have required the game to be zero-sum.

For part (iii), row Y is dominated by row X provided 1 ≥ y and by row Z provided 4 ≥ y, so A = {y : y ≤ 4}. This candidate has not quite used set notation correctly but clearly knows what the solution set is. For part (iv), irrespective of the value of p, column P cannot be dominated by column Q because 4 > 3 in row X or 3 > 1 in row Z, and cannot be dominated by column R because 3 > 1 in row Z. This candidate has identified that in row Z column P gives the best pay-off for the player on columns and deduced that column P cannot be dominated by either column Q or column R.
Question 4(i), (ii), (iii)(a) and (iii)(b)

4  The complete bipartite graph $K_{3,4}$ connects the vertices \{2, 4, 6\} to the vertices \{1, 3, 5, 7\}.

(i) How many arcs does the graph $K_{3,4}$ have? \([1]\)

(ii) Deduce how many different paths are there that pass through each of the vertices once and once only. The direction of travel of the path does not matter. \([3]\)

The arcs are weighted with the product of the numbers at the vertices that they join.

(iii) (a) Use an appropriate algorithm to find a minimum spanning tree for this network. \([3]\)

(b) Give the weight of the minimum spanning tree. \([1]\)

Exemplar 1

(i) \([1\text{ mark}]\)

\[
\begin{array}{c|c}
4(i) & 3 \times 4 = 12 \\
\end{array}
\]

(ii) \([2\text{ marks}]\)

\[
\begin{array}{c|c}
4(ii) & 4 \times 3 \times 2 \times 2 = 3 \times 3 \times 2 \times 2 \times 1 = 36 \\
\end{array}
\]

(iii)(a) \([3\text{ marks}]\)

\[
\begin{array}{c|c}
4(iii)(a) & \\
\end{array}
\]

Start at 1. Prim's algorithm

1 → 2 = 7
1 → 4 = 6
1 → 6 = 6
2 → 3 = 1
2 → 5 = 6
2 → 7 = 14
Examiner commentary

This question was about graphs and networks. Part (i) just required candidates to understand the meaning of the notation $K_{3,4}$. In part (ii), the path needs to alternate between the two sets, starting and finishing in the larger set. The multiplication principle could then be used to count the number of possible paths, as this candidate has done apart from forgetting that any path can be travelled in either direction so the total is half of $144 = 72$ paths.

In part (iii)(a), a network is formed by weighting the arcs. This candidate has forgotten that arcs always connect a vertex labelled with an odd number to one labelled with an even number, so they have ended up with 21 arcs instead of 12. The candidate has then used Prim's algorithm on their table and has drawn a correct minimum spanning tree, but it does not match their working. For part (iii)(b), the weights of the six arcs used in the minimum spanning tree needed to be summed, this candidate has made this part much more difficult than it really is.
Question 5(i), (ii)(a), (ii)(b), (iii) and (iv)

5 Greetings cards are sold in luxury, standard and economy packs.

The table shows the cost of each pack and number of cards of each kind in the pack.

<table>
<thead>
<tr>
<th>Pack</th>
<th>Cost (£)</th>
<th>Handmade cards</th>
<th>Cards with flowers</th>
<th>Cards with animals</th>
<th>Other cards</th>
<th>Total number of cards</th>
</tr>
</thead>
<tbody>
<tr>
<td>Luxury</td>
<td>6.50</td>
<td>10</td>
<td>5</td>
<td>5</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>Standard</td>
<td>5.00</td>
<td>5</td>
<td>10</td>
<td>5</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>Economy</td>
<td>4.00</td>
<td>0</td>
<td>10</td>
<td>10</td>
<td>20</td>
<td>40</td>
</tr>
</tbody>
</table>

Alice needs 25 cards, of which at least 8 must be handmade cards, at least 8 must be cards with flowers and at least 4 must be cards with animals.

(i) Explain why Alice will need to buy at least two packs of cards. [2]

Alice does not want to spend more than £12 on the cards.

(ii) (a) List the combinations of packs that satisfy all Alice’s requirements. [2]

(b) Which of these is the cheapest? [1]

Ben offers to buy any cards that Alice buys but does not need. He will pay 12 pence for each handmade card and 5 pence for any other card.

Alice does not want her net expenditure (the amount she spends minus the amount that Ben pays her) on the cards to be more than £12.

(iii) Show that Alice could now buy two luxury packs. [2]

Alice decides to buy exactly 2 packs, of which $x$ are luxury packs, $y$ are standard packs and the rest are economy packs.

(iv) Give an expression, in terms of $x$ and $y$ only, for the number of cards of each type that Alice buys. [4]

Exemplar 1

(i) 0 marks

(ii)(a) 2 marks
(ii)(b) 1 mark

Examiner commentary

Candidates needed to unpick constraints presented in tables and words. In part (i), they were asked to explain why Alice would need at least two packs. This required them to explain why a single pack of each type did not meet at least one of the requirements (in particular, the total needed to be at least 25 and there needed to be at least 8 handmade cards). Far too many candidates just repeated the wording of the question, as this candidate has done. Part (ii) required finding all the combinations that satisfied the requirements and cost at most £12, since 3 economy packs do not have any handmade cards there needed to be two packs. This gave the three combinations listed by this candidate. The cheapest of these is to buy two standard packs.

In part (iii), most candidates realised that Alice could sell up to 12 handmade cards to Ben, and some showed that if she did this she would end up with a net expenditure of £11.56. However, many candidates forgot that Alice needed 25 cards, so that most that Ben could buy was 15 cards, often they tried to sell Ben 12 handmade cards and 8 cards with flowers or animals, as this candidate has done.

In part (iv), this candidate has forgotten that Alice only buys 2 packs, so if she buys x luxury packs and y standard packs then she must buy (2 – x – y) economy packs, and these need to be included in the net income for flowers, animals and other. At this point the question rewarded candidates who persevered, even with incomplete expressions for the net income. This candidate has given up on the question, although they went on to attempt question 6.
Sheona and Tim are making a short film. The activities involved, their durations and immediate predecessors are given in the table below.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Duration (days)</th>
<th>Immediate predecessors</th>
<th>S</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Planning</td>
<td>2</td>
<td>–</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>B Write script</td>
<td>1</td>
<td>A</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>C Choose locations</td>
<td>1</td>
<td>A</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>D Casting</td>
<td>0.5</td>
<td>A</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>E Rehearsals</td>
<td>2</td>
<td>B, D</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>F Get permissions</td>
<td>1</td>
<td>C</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>G First day filming</td>
<td>1</td>
<td>E, F</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>H First day edits</td>
<td>1</td>
<td>G</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>I Second day filming</td>
<td>0.5</td>
<td>G</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>J Second day edits</td>
<td>2</td>
<td>H, I</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>K Finishing</td>
<td>1</td>
<td>J</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

(i) By using an activity network, find:
   - the minimum project completion time
   - the critical activities
   - the float on each non-critical activity. [7]

(ii) Give two reasons why the filming may take longer than the minimum project completion time. [2]

Each activity will involve either Sheona or Tim or both.

- The activities that Sheona will do are ticked in the S column.
- The activities that Tim will do are ticked in the T column.
- They will do the planning and finishing together.
- Some of the activities involve other people as well.

An additional restriction is that Sheona and Tim can each only do one activity at a time.

(iii) Explain why the minimum project completion is longer than in part (i) when this additional restriction is taken into account. [2]

(iv) The project must be completed in 14 days. Find:
   (a) the longest break that either Sheona or Tim can take, [2]
   (b) the longest break that Sheona and Tim can take together, [2]
   (c) the float on each activity. [2]
Exemplar 1

(i) 7 marks

Minimum project completion time = 10 days

Critical activities: A - B - E - G - I - S - K

<table>
<thead>
<tr>
<th>Activity</th>
<th>A</th>
<th>C</th>
<th>D</th>
<th>F</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Float (days)</td>
<td>1</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td></td>
</tr>
</tbody>
</table>

(ii) 2 marks

Because sometimes they must both work on the same project and so cannot do two things at the same time.

Because, 1 person may have to do two things at the same time so one may have to be delayed. Like when Shea has to do both B and D. Also, some activities may be delayed and take longer than expected. The critical path would be improve.
(iii) Because it means that if there are two activities that need to happen at the same time and are done by the same person, then one of them must be delayed because only one can be done at a time. An example of this is B and D that both need to be done by Sheena.

(iv)(a) Turn can take a break of 3-6 days after doing B & F because this can be done by 1 day and H will not start until 6-5 days, 6-5-6 = 2-5 days. Doesn't need to start until 10-10-10 = 6.

(iv)(b) 4 days together at least after doing A is when they 3-5 days together after A occur between any activities.

(iv)(b) A: 3.5  D: 1.5  G: 3.5  J: 3.5
B: 4  E: 3.5  H: 3.5  K: 3.5
C: 5  F: 5  I: 4
Examiner commentary

The specification states that activity networks need to be drawn with activity-on-arc. This candidate has used activity on node, but despite this the candidate has correctly identified the minimum project completion time. The question is relatively unstructured, requiring candidates to make their own decisions about carrying out a forward pass to find the minimum completion time and a backward pass to find the critical activities. This candidate has successfully done all of this.

For part (ii) candidates needed to give a reason why activity durations may be extended and a resourcing issue, usually concerning the availability of Sheona and Tim. For part (iii), the problem arises at the start when Sheona would need to be doing activities B and D at the same time, and because she cannot this delays the start of activity E. The candidate has dropped a mark here, needing to be more explicit about the impact on critical activity E.

Part (iv) tested the idea of float, the end of the last question will often test the extremes of the specification. Here the question is relatively unstructured, even though it is split into three parts. Candidates could identify that each person was busy for 8 of the 14 days, giving a break of up to 6 days. Using an ad-hoc approach it can be seen that Sheona can take a 6-day break between activities I and K, and Tim can take a 6-day break between activities F and H. Because Sheona cannot do activities B and D at the same time, it takes them a minimum of 10.5 days to complete the project, so they can have at least 3.5 days free together which leads to the listed floats. This is well answered by this candidate.
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