AS LEVEL

Exemplar Candidate Work

FURTHER MATHEMATICS A

H235
For first teaching in 2017

Y531/01 Summer 2018 examination series

Version 1

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Introduction

These exemplar answers have been chosen from the summer 2018 examination series.

OCR is open to a wide variety of approaches and all answers are considered on their merits. These exemplars, therefore, should not be seen as the only way to answer questions but do illustrate how the mark scheme has been applied.

Please always refer to the specification https://www.ocr.org.uk/qualifications/as-a-level-gce/further-mathematics-a-h235-h245-from-2017/assessment/#as-level for full details of the assessment for this qualification. These exemplar answers should also be read in conjunction with the sample assessment materials and the June 2018 Examiners’ report or Report to Centres available from Interchange https://interchange.ocr.org.uk/Home.mvc/Index

The question paper, mark scheme and any resource booklet(s) will be available on the OCR website from summer 2019. Until then, they are available on OCR Interchange (school exams officers will have a login for this and are able to set up teachers with specific logins – see the following link for further information http://www.ocr.org.uk/administration/support-and-tools/interchange/managing-user-accounts/).

It is important to note that approaches to question setting and marking will remain consistent. At the same time OCR reviews all its qualifications annually and may make small adjustments to improve the performance of its assessments. We will let you know of any substantive changes.
Question 1(i) and (ii)

(i) Find a vector which is perpendicular to both \(\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}\) and \(\begin{pmatrix} -3 \\ -6 \\ 4 \end{pmatrix}\). \[2\]

(ii) The cartesian equation of a line is \(\frac{x}{2} = y - 3 = 2z + 4\).

Express the equation of this line in vector form. \[3\]

Exemplar 1

2 marks

\[ \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \times \begin{pmatrix} -3 \\ -6 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} \]

\(\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}\) is perpendicular to both.

3 marks

\[ \frac{x}{2} = y - 3 = 2z + 4\]

\[\frac{x}{2} = \lambda \iff x = 2\lambda\]

\[y - 3 = \lambda \iff y = \lambda + 3\]

\[2z + 4 = \lambda \iff z = \frac{1}{2}(\lambda - 4) = \frac{1}{2}\lambda - 2\]

\[x = 0 + \lambda \times 2\]

\[y = 3 + \lambda \times 1\]

\[z = -2 + \lambda \times \frac{1}{2}\]

\[\mathbf{r} = \begin{pmatrix} 0 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}\]

\[\mathbf{r} = \begin{pmatrix} 0 \\ 3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}\]
Examiner commentary

The majority of candidates made efficient use of their calculators to answer the part (i), this candidate has made clear the method used.

A slightly different approach to the majority of the candidates has been used in part (ii), which works well and helps avoid the common mistakes other candidates have made. Setting each of the components equal to λ and then rearranging has a higher rate of success than trying to manipulate the components. The candidate gives two different versions of the final equation, both of which are correct. However, if their second version had been incorrect, then the final answer mark would have not been credited, as it is the candidate’s last answer which is marked. If candidates are providing more than one solution then they should clearly identify which one they want to be marked or the examiner will mark the last one.

Exemplar 2

Examiner commentary

This candidate showed clear working when answering part (i), which was not usual; most candidates just wrote down the cross product and then their answer.

Part (ii) highlights one of the most common mistakes in dealing with the z component of the equation. This candidate has divided the numerator by 2 but not the denominator.
Exemplar 3

1 mark

Examiner commentary

This candidate made a sign error when calculating the cross product in part (i), however since they had shown working they could be credited the method mark. In part (ii), the candidate again shows clear working but has made a mistake when calculating the \( x \) component of the line equation. This was an unusual mistake – usually the mistake was made with the \( z \) component.

Exemplar 4

1 mark

Examiner commentary

This candidate has started by dividing the Cartesian equation by 2 throughout, which was an unusual, but effective, strategy. This has made the \( z \) component easier to deal with, however a sign slip when writing down the component means that this candidate did not gain the last mark.
Question 2

2 In this question you must show detailed reasoning.
The cubic equation \(2x^3 + 3x^2 - 5x + 4 = 0\) has roots \(\alpha, \beta, \gamma\). By making an appropriate substitution, or otherwise, find a cubic equation with integer coefficients whose roots are \(\frac{1}{\alpha}, \frac{1}{\beta}\) and \(\frac{1}{\gamma}\). [3]

Exemplar 1 3 marks

\[
2x^3 + 3x^2 - 5x + 4 = 0
\]

\[
\frac{1}{x} \quad \text{will give an equation in } \frac{1}{u} \text{ with roots } \frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}
\]

\[
x = \frac{1}{u}
\]

\[
2\left(\frac{1}{u}\right)^3 + 3\left(\frac{1}{u}\right)^2 - 5\left(\frac{1}{u}\right) + 4 = 0
\]

\[
\frac{2}{u^3} + \frac{3}{u^2} - \frac{5}{u} + 4 = 0
\]

\[
2u^3 + 3u^2 - 5u + 4 = 0
\]

\[
4u^3 - 5u^2 + 3u + 2 = 0
\]

Examiner commentary

A very well laid out answer here. This method tended to be used by the higher achieving candidates and was more successful (as well as being much simpler to apply). Using this method meant that the equation automatically had integer coefficients.
Examiner commentary

Here the candidate uses the alternative method in the mark scheme and has shown their working clearly. However, they fail to gain the final A mark, as their final answer does not satisfy the requirements of the question. Firstly, it is given as an expression rather than an equation (there is no “=0”) and secondly the coefficients are not integers. Either one of these would result in a candidate not gaining full marks. A common mistake from candidates using this method was to not write their coefficients as integers.
Question 3(i), (ii), (iii) and (iv)

3 In this question you must show detailed reasoning.
The complex numbers $z_1$ and $z_2$ are given by $z_1 = 2 - 3i$ and $z_2 = a + 4i$ where $a$ is a real number.

(i) Express $z_1$ in modulus-argument form, giving the modulus in exact form and the argument correct to 3 significant figures. [3]

(ii) Find $z_1z_2$ in terms of $a$, writing your answer in the form $c + id$. [2]

(iii) The real and imaginary parts of a complex number on an Argand diagram are $x$ and $y$ respectively. Given that the point representing $z_1z_2$ lies on the line $y = x$, find the value of $a$. [2]

(iv) Given instead that $z_1z_2 = (z_1z_2)^k$ find the value of $a$. [2]

Exemplar 1

3(i) $z_1 = 2 - 3i$

| $|z_1| = \sqrt{(2)^2 + (-3)^2}$ |
| --- |
| $\sqrt{13}$ |

\[ \theta = \tan^{-1}\left(\frac{-3}{2}\right) = 0.9328 \text{ rad} \]

\[ x(3) = 2\pi - 0.9328 = 5.30 \text{ rad} \]

\[ = \sqrt{13} \approx 3.61 \]

2 marks

3(ii) $\left(2 - 3i\right)(a + 4i)$

\[ = 2a + 8i - 3ai - 12i^2 \]

\[ = 2a + 8i - 3ai + 12 \]

\[ = 2a + 12 + (8 - 3a)i \]

2 marks

3(iii) $\text{arg}\left(\frac{z_1z_2}{4}\right) = \frac{11}{4}$

\[ 2a + 12 = 8 - 3a \]

\[ a = -\frac{4}{5} \]

\[ L = \frac{8}{5} \]

2 marks
Examiner commentary

This candidate has shown excellent reasoning throughout this question and fully supported their answers with clear working. They have used a diagram in part (i) to justify their value for the argument – this was a technique used well by many candidates. Candidates who did not draw a diagram often obtained a wrong argument of 0.983 (i.e. neglecting the negative sign). This is a good example of what is required by the instruction “In this question you must show detailed reasoning.”

Exemplar 2

2 marks

Exemplar Candidate Work

2 marks

Exemplar Candidate Work

2 marks

Exemplar Candidate Work

2 marks
Examiner commentary

This candidate has made a good effort on this question, but carelessly dropped a couple of marks. In part (i), the candidate has calculated the modulus and argument correctly (degrees is fine here as the question did not specify radians). However, the candidate does not gain the final mark as the final answer uses an approximate value for the modulus rather than the exact value found earlier. In part (iii), there is an error in signs in the algebraic manipulation.
Examiner commentary

Part (i) is a good example of the level of working we were expecting to see detailed for full credit. Showing how the modulus was found to be equal to $\sqrt{13}$ was not actually necessary on this occasion, but it is good practice to show this.

In part (ii), the candidate has found the correct product, but they have not written their answer in the required form $c + id$. In order to gain full marks they would have had to factorise out "i" from the imaginary terms.

The candidate has simply repeated the question in part (iii) and so credited no marks. In part (iv), the candidate perhaps did not realise that they had almost solved this problem, rearranging their $6ai-16i=0$ to find $a$. 

---

0 marks

---

1 mark
Examiner commentary

Here the candidate has mistakenly assumed that $(z_1, z_2)^*$ is on the line $y = x$. Candidates should not assume that information given in one part of the question is still valid in subsequent parts unless the question says so (here the question explicitly says “instead”). Any information given in the “stem” of the question (the bit of the question before parts (i), (ii) etc.) can be assumed to be true for the entire question.
Question 4(i), (ii), and (iii)

4 The matrix \( A \) is given by \( A = \begin{pmatrix} 2 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & 2 & a \end{pmatrix} \).

(i) Show that \( \det A = 6 - 3a \). [2]

(ii) State the value of \( a \) for which \( A \) is singular. [1]

(iii) Given that \( A \) is non-singular find \( A^{-1} \) in terms of \( a \). [4]

Exemplar 1

Examiner commentary

This candidate has produced an excellent answer to this question. Working is clearly shown throughout. In part (i), they have expanded the determinant via the third column (rather than the more usual first row) which has the advantage of there only being one \( a \) term to consider.
Exemplar 2

Examiner commentary

Here the candidate has used an alternative method finding the determinant where they have subtracted the sum of the "backwards" diagonals from the sum of the "forwards" diagonals.

Mistakes with part (ii) were rare, but occasionally candidates would make a sign error when solving the equation, as in this case. A quick sense check, substituting back into the original would have flagged up the mistake quickly.

Part (iii) did not gain any marks; candidates are expected to be able to find the inverse of a non-singular 3 x 3 matrix with and without a calculator.
Examiner commentary

The candidate has not found the correct cofactors here, as they have been multiplied by the corresponding element of the matrix. The second method mark is dependent on the first method mark being credited, so the candidate gains no credit for this attempt.
Question 5(i), (ii), and (iii)

5  In this question you must show detailed reasoning.

(i) Express \((2 + 3i)^3\) in the form \(a + ib\).  

\[
(2 + 3i)^3 = \frac{3C_0(2)^0(3i)^3}{3C_2(2)^2(3i)^1} + \frac{3C_1(2)^1(3i)^2}{3C_3(2)^3(3i)^0} \\
= (3i)^3 + 6(3i)^2 + 12(3i) + 8 \\
= -27i - 54i + 36i + 8 \\
= -10i + 8 \\
= -4i + 9i \\
= -4 + 9i
\]

3 marks

(ii) Hence verify that \(2 + 3i\) is a root of the equation \(3z^3 - 8z^2 + 23z + 52 = 0\).

\[
3(2 + 3i)^3 - 8(2 + 3i)^2 + 23(2 + 3i) + 52 = 0 \\
= 3(-10i + 8) - 8(-5 + 12i) + 46 + 60i + 52 = 0 \\
= -30i + 24i - 96i + 46 + 60i + 52 = 0 \\
= -98 + 98 - 60i + 60i = 0 \\
0 + 0i = 0 \\
0 = 0
\]

\(2 + 3i\) is a root of the equation.

3 marks

(iii) Express \(3z^3 - 8z^2 + 23z + 52\) as the product of a linear factor and a quadratic factor with real coefficients.

\[
(2 + 3i)^2 = 4 + 12i, \quad -9 = 9 + 12i.
\]
Examiner commentary
These are excellently presented and explained answers, to these question parts.

Exemplar 2

3 marks

2 marks

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Examiner commentary

This candidate has produced a good answer to part (i) with working clearly shown. We don’t see \( i^2 = -1 \) explicitly, but there is enough evidence to show that this has been used twice. In part (ii), the candidate has not shown enough detailed working to show us that \( 2+3i \) is a root convincingly. To gain full marks the candidate would have needed to show that both the real and imaginary parts are equal to zero, either by gathering the real and imaginary parts separately or by stating that their expression is equal to \( 0+0i \). In part (iii), the candidate has used the sum of the roots in order to find what the third root of the equation is. This method is fine, but the candidate has not realised that their factorisation gives \( z^2 \) rather than \( 3z^3 \), and they have mistakenly written their linear factor as \( \left( z - \frac{4}{3} \right) \) rather than \( \left( z + \frac{4}{3} \right) \). To gain full credit for this part the candidate would have had to either use a linear factor of or use a factor of \( (3z+4) \) outside the two brackets. Note that using the solve function on the calculator would have confirmed the roots as \( \frac{4}{3}, 2+3i \) and \( 2-3i \); care is needed if attempting to reverse engineer the factors from roots, especially on questions which have the instruction “In this question you must show detailed reasoning.”
Exemplar 3  

**3 marks**

\[(2 + 3i)^3 = -46 + 9i\]

\[
(2 + 3i)(2 + 3i) = 4 + 12i - 9
\]

\[
2 + 3i = -5 + 12i
\]

\[
24 + 6i
\]

\[
2 + 3i \quad 9i^2 = -9
\]

\[
(-5 + 12i)(2 + 3i) = -10 + 24i + 36i - 36
\]

\[
= -5 + 12i
\]

\[
2 - 10 + 24i
\]

\[
+3i - 15i + 36i^2 = -36
\]

**Examiner commentary**

The full and detailed working here is a very good example of a response gaining full marks. Tables can be a very clear way for a candidate to show their reasoning when expanding brackets.

Exemplar 4  

**1 mark**

\[(2 + 3i)(2 + 3i)(2 + 3i)\]

\[
(2 + 3i)(2 + 3i) = -46 + 9i
\]

**Examiner commentary**

Here the candidate has not shown enough detailed reasoning to gain full credit (the question stated “In this question you must show detailed reasoning”). Detailed reasoning should have included evidence of \(i^2\) becoming -1 twice (which could have been in a table, or using \(i^2 = -i\) in a binomial expansion). This candidate gains B1 under the Special Case rules given in the mark scheme for using \((2 + 3i)^2 = -5 + 12i\).
Exemplar 5

This is another example of a candidate who has not shown sufficient working to gain full credit. If we had seen evidence of \( i^2 = -1 \) and \( i^3 = -i \) they would have gained the first answer mark. However, on the third line there is a \( (2)^3 \) term where there should be a \( (2)^2 \) term so this candidate could not have gained full credit due to incorrect working seen.

Exemplar 6

This candidate has shown excellent detailed reasoning in their answer to this question.
Exemplar 7

Examiner commentary

A final answer of 0+0i is acceptable for full credit here. This candidate has gathered the real and imaginary terms and shown that in each case they sum to 0.
Question 6(i), (ii), and (iii)

The matrices A and B are given by $A = \begin{pmatrix} t & 6 \\ -2 & t \end{pmatrix}$ and $B = \begin{pmatrix} 2t & 4 \\ t & -2 \end{pmatrix}$ where $t$ is a constant.

(i) Show that $|A| = |B|$. [2]

(ii) Verify that $|AB| = |A||B|$. [3]

(iii) Given that $|AB| = -1$ explain what this means about the constant $t$. [2]

Examiner commentary

In part (i), the candidate has clearly found the determinants and then explicitly stated that $|A| = |B|$ for full credit. In part (ii), the candidate has found $|AB|$ and $|A||B|$ to verify that these are equal. Part (iii) was answered using a written calculation to provide a concise explanation.
Exemplar 2

2 marks

\[ A = \begin{pmatrix} 1 & 6 \\ -2 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 2t & 4 \\ t & -1 \end{pmatrix} \]

\[ |A| = -2t - 6t + 8t = -8t \]

\[ |B| = -4t - 4t \]

\[ |A| = |B| \]

3 marks

\[ AB = \begin{pmatrix} 1 & 6 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 2t & 4 \\ t & -1 \end{pmatrix} \]

\[ = \begin{pmatrix} 2t^2 + 6t & 4t - 12 \\ 2t^2 - 2t & 4t + 4 \end{pmatrix} \]

\[ |AB| = (2t^2 + 6t)(4t + 4) - (2t^2 - 2t)(4t - 12) \]

\[ = 32t^2 + 24t + 32t^2 - 24t \]

\[ = 64t^2 \]

\[ |AB| = (-8t)(-8t) = 64t^2 \]

\[ |AB| = |A||B| \]

0 marks

Examiner commentary

This candidate’s responses to the first two parts of the question show clear working and the required conclusion is stated clearly in both cases. In part (iii), the candidate has not answered the question (which asks what can be said about the constant \( t \)), but instead says what they know about the matrix \( AB \). This highlights the importance of reading the question carefully to provide the required answer.
Exemplar 3

1 mark

Examiner commentary
This candidate has shown that both determinants are equal to \(-8t\), but has not stated the conclusion \(|A| = |B|\), so does not gain the answer mark. This was a fairly common response from candidates.

Exemplar 4

2 marks

Examiner commentary
This candidate knows how to answer the question but makes some basic arithmetical mistakes, which mean that they did not show the equality between \(|AB|\) and \(|A||B|\). A surprising number of candidates lost marks through basic mistakes with negative numbers and powers.
Question 7

Prove by induction that \(2^{n+1} + 5 \times 9^n\) is divisible by 7 for all integers \(n \geq 1\). [6]

Exemplar 1

1. \(2^{n+1} + 5 \times 9^n\) is divisible by 7 for all integer \(n \geq 1\).

2. \(\text{When } n = 1: \quad 2^{n+1} + 5 \times 9^n = 2^2 + 5 \times 9 = 4 + 45 = 49 = 7 \times 7\)

3. If true for \(n = k\):

\[2^{k+1} + 5 \times 9^k = 7A\]

Then also true for \(n = k+1: \quad 2^{k+2} + 5 \times 9^{k+1} = 7B\)

\[\text{LHS: } 2^{k+2} + 5 \times 9^{k+1}\]
\[= 2(2^{k+1}) + 5 \times 9^k = 2 \times (7A + 5 \times 9^k) + 9 \times 9^k\]
\[= 14A + 2 \times 5 \times 9^k + 9 \times 9^k\]
\[= 14A + 10 \times 9^k + 45 \times 9^k\]
\[= 7(2A + 5 \times 9^k)\]
\[= 7(2A + 5 \times 9^k)\]

Since statement is true for \(n = 1\) (base case), if true for \(n = k\), then also true for \(n = k+1\). Hence:

\[2^{n+1} + 5 \times 9^n\] is divisible by 7 for all integers \(n \geq 1\).

Examiner commentary

A good clear proof by induction, with a concise concluding statement to ensure full credit.
Exemplar 2

5 marks

Examiner commentary

This is a good answer; however, the candidate's conclusion is not good enough to gain the final E mark for a clear conclusion to the induction process. The candidate needs to say that if the statement is true for $n=k$ then it is true for $n=k+1$ and since it is true for $n=1$ then it is true for all positive integers $n$. 
Examiner commentary

There were several candidates who tried this “f(k+1) – f(k)” method. Usually (as in this case) they found it difficult to show that f(k+1) – f(k) is divisible by 7. Others could show that the difference was divisible by 7 but then did not explain clearly why this meant that f(k+1) was divisible by 7.

Candidates who stated “is divisible by 7” for the n=k case usually did not gain more than the first three marks, whereas those who stated $2^{k+1} + 5 \times 9^k = 7A$ tended to be more successful.
Exemplar 4

2 marks

Examiner commentary

This candidate has chosen to start by showing that the statement is true when $n=2$. If they had completed the induction method correctly, they could have gained three method marks, and potentially the answer mark for showing divisibility for the $n = k+1$ case, but could not have gained the first and last marks unless they showed that the statement is true when $n=1$ (the question asks you to prove the statement for all integers greater than or equal to 1).

Errors in trying to remove common factors (as seen here) were fairly common.
Examiner commentary

This candidate has shown that the statement is true when $n=1$, and so gains the first method mark, but gains no further credit as they have not stated that they are assuming that when $n=k$ the expression is divisible by 7 (which could be done in words or by setting the expression equal to $7A$).
Question 8(i), (ii) and (iii)

8 The $2 \times 2$ matrix $A$ represents a transformation $T$ which has the following properties.

- The image of the point $(0, 1)$ is the point $(3, 4)$.
- An object shape whose area is 7 is transformed to an image shape whose area is 35.
- $T$ has a line of invariant points.

(i) Find a possible matrix for $A$. [8]

The transformation $S$ is represented by the matrix $B$ where $B = \begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}$.

(ii) Find the equation of the line of invariant points of $S$. [2]

(iii) Show that any line of the form $y = x + c$ is an invariant line of $S$. [3]

Exemplar 1 8 marks

\[
\begin{align*}
4a - 3b &= 5 \\
ax + 3y &= x \\
bx + 4y &= y \\
3y = x(1-a) &\quad y = \frac{x(1-a)}{3} \\
3y = -bx &\quad y = -\frac{bx}{3} \\
\frac{x(1-a)}{3} = -\frac{bx}{3} &\quad x(1-a) = -bx \\
\frac{1-a}{3} = \frac{-b}{3} &\quad 1-a = -b
\end{align*}
\]

hence 2 simultaneous equations

\[
\begin{align*}
4a - 3b &= 5 \\
1-a &= -b \\
a &= (b-1)x - 1 \\
\end{align*}
\]

\[
\begin{align*}
4(b+1) - 3b &= 5 \\
4b + 4 - 3b &= 5 \\
b + 4 &= 5 \\
b &= 1
\end{align*}
\]

\[
\begin{align*}
a &= b + 1 & b+1 &= 2 & a &= 2
\end{align*}
\]

hence matrix $A$ is $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$.
Examiner commentary

Nice presentation of work to secure full marks on all three parts.
Exemplar 2

6 marks

\[ \begin{align*}
35 &= 5 \quad \text{det} = 5 \\
A &= \begin{pmatrix} a & b \\ c & d \end{pmatrix} \\
\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} &= \begin{pmatrix} 3 \\ 4 \end{pmatrix} \\
b &= 3 \\
a &= 4 \\
\det &= ad - bc = 5 \\
4a - 3c &= 5 \\
\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} &= \begin{pmatrix} u \\ v \end{pmatrix} \\
uu + bv &= u \quad \Rightarrow \quad au + 3v = u \\
uu + dv &= v \quad \Rightarrow \quad cu + 4v = v \\
3u &= -cu \\
3v &= u - au \\
-cu &= u - au \\
-c &= -a \quad \text{(Correct)} \\
c &= a \\
\text{Possible matrix} \quad A &= \begin{pmatrix} 1 & 3 \\ 1 & 4 \end{pmatrix} \\
\text{Check:} \quad \begin{pmatrix} 1 & 3 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} &= \begin{pmatrix} 3 \\ 4 \end{pmatrix} \\
\det &= 4a - 3c = 5 \\
\text{Sub:} \quad 4a - 3a &= 5 \\
\text{Possible matrix} \quad A &= \begin{pmatrix} 5 & 3 \\ 5 & 4 \end{pmatrix}
\end{align*} \]
Examiner commentary

This is a very good response to this question. The candidate has formed all the equations which can be found from the information given in the question and has solved them simultaneously. The only mistake the candidate has made is in part (i) when cancelling \(-cu = u - au\) the \(u\) term becomes 0 rather than 1 (so they obtain \(-c = -a \) rather than \(-c = 1 - a\)).
Examiner commentary

This is a fairly typical response. Here the candidate has been credited the first two B marks for using the first two pieces of information given in the stem of the question. However, they have misunderstood what is meant by “a line of invariant points” and hence make no further progress in finding the matrix. The distinction between “Invariant points”, “invariant lines” and “lines of invariant points” was one that many candidates struggled with.
Examiner commentary

Quite a few candidates tried to use this method to answer this question. Most did not gain full credit, but this is a good example of how this method might be used convincingly to show that $y = x + c$ is an invariant line. Here the candidate has clearly shown that the coefficient of $x$ implies that $m$ is either $-2$ or $1$, and that the constant term implies that either $c = 0$ or $m = 1$. They then clearly state what this means in terms of invariant lines.

This method is considerably longer, and more prone to errors, than the alternate method shown. To gain full credit using this method the candidate’s argument had to be convincing. Very few candidates managed to gain full credit using this method.
Examiner commentary

This candidate has confused the idea of an invariant line and an invariant point (or possibly a line of invariant points). They do not gain any credit for this attempt, however if they had used $X$ and $mX+c$ in their second vector they would have gained a method mark.
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