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LEVEL 3 CERTIFICATE

Transition guide

CORE MATHS B (MEI)

H869
For first teaching in 2015

Conditional probability

Version 1

Mathematics Education Innovation

www.ocr.org.uk/maths

LEVEL 3 CERTIFICATE IN CORE MATHS B (MEI)

Welcome

Welcome to the KS4–Core Maths Transition Guide

Key Stage 4 to Core Maths Transition Guides focus on how a particular topic is covered at the different key stages and provide information on:

- Differences in the demand and approach at the different levels;
- Useful ways to think about the content at Key Stage 4 which will help prepare students for progression to Core Maths;
- Common student misconceptions in this topic.

Transition Guides also contain links to a range of teaching activities that can be used to deliver the content at Key Stage 4 and Core Maths and are designed to be of use to teachers of both key stages. Central to the Transition Guide is a Checkpoint Task which is specifically designed to help teachers determine whether students have developed deep conceptual understanding of the topic at Key Stage 4 and assess their 'readiness for progression' to Core Maths content on this topic. This Checkpoint Task can be used as a summative assessment at the end of Key Stage 4 teaching of the topic or by Core Maths teachers to establish their students' conceptual starting point.

Key Stage 4 to Core Maths Transition Guides are written by experts with experience of teaching at both key stages.

Mapping KS4 to Core Maths — Page 3

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Possible Extension Activities (Core Maths focus) — Page 11

Key Stage 4 Content

GCSE Content

Based on the GCSE subject content (DfE 2013), all learners joining a Core Maths* class should be able to:

- Construct theoretical sample spaces for single and combined experiments with equally likely outcomes and use these to calculate theoretical probabilities.
- Calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions.

In addition, learners who have been taught the Higher GCSE syllabus should be able to:

- Calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams.

Core Maths Content

Core Maths Content

The competence statements in Quantitative Reasoning (MEI) and Quantitative Problem Solving (MEI) specifications describe content required to cover during the course of study.

Introduction to Quantitative Reasoning (in both Quantitative Reasoning (MEI) and Quantitative Problem Solving (MEI)):

- Understand the difference between dependent and independent events and be able to calculate probability in simple cases. Contexts include games of chance, and risk of suffering from a disease.
- Be able to work with a tree diagram when calculating or estimating a probability, including conditional probability.

Critical Maths (only in Quantitative Reasoning (MEI)):

- Use notation "given that" and P(A) , $P(A|B)$.
- Know that a conditional probability of event A given event B is different from the conditional probability of event B given event A in relevant contexts. Questions will be asked in words in context, for example, recognising that the probability of testing positive for a disease given that the patient has the disease may not be the same as the probability of the patient having the disease given that he/she tests positive.
- Recognise common examples of incorrect reasoning in probability and be able to explain the errors. Recognise and explain the prosecutor's fallacy, the defendant's fallacy and the gambler's fallacy.

Comment

Difference between the level of demand at KS4 and Core Maths

At GCSE the examples used to teach and assess probability are often artificial. Core Maths develops learners' understanding of probability by using contexts from, for example, law, medicine and sport; contexts in which, every day, people have to understand chance and risk and make difficult decisions based on probabilities. This will be challenging for many learners as they will be required to model situations, to calculate and estimate probabilities and then to interpret these probabilities in context. It means that learners will need to understand conditional probability, rather than simply learning and applying rules, so most of the resources, activities and questions provided in this transition guide have been chosen because they encourage understanding of what can sometimes seem to be difficult concepts.

Pre-requisite knowledge for Core Maths

One issue for teachers is that learners may come to Core Maths with different prior knowledge and understanding of probability. All learners should be aware of the difference between dependent and independent events and be able to calculate probabilities of combined events involving independent and dependent probabilities. However, those learners who have followed the Foundation GCSE syllabus are unlikely to be familiar with the notation and vocabulary of conditional probability and will have been taught how to calculate P(A or B) and P(A and B) in an informal manner. In fact it will only be the most able learners who are completely secure in their understanding of the conditional probability subject content at GCSE. If introducing or revising conditional probability, it is advisable to begin with tables and Venn diagrams as these are far easier to interpret than Bayes rule and will encourage an intuitive and common sense approach.

Teachers who have not previously taught statistics at AS level may want to refresh and extend their knowledge in this area as probability is often an area of mathematics that people, including mathematicians, find difficult to grasp. This transition guide, therefore, includes links to good explanations and examples as well as to activities and teaching resources.

Comment

Common Misconceptions

Probability, beyond the basics of finding the probability of a simple outcome such as rolling a six on a fair die, can be a difficult area of mathematics and statistics to understand and to teach. It is certainly the case that people commonly make errors in interpreting evidence because they are unclear as to exactly what the numbers mean.

Some common misconception and misunderstandings follow.

- All events are considered equally likely. So, for example, the probability of me winning the lottery is considered $\frac{1}{2}$ because either it will happen or it won't. Or the probability of David winning a race against Barry and Chris is only simply $\frac{1}{3}$ because there are three people in the race.
- Events are not equally likely when in fact they are. So, for example, the six numbers in the lottery draw are perceived to be less likely to be 1, 2, 3, 4, 5, 6 than the numbers 2, 8, 13, 27, 32, 45, when in fact both combinations are equally likely.
- If events are random, then the results of a series of independent events are equally likely. So for example, if you flip two fair coins, getting two heads (actually $\frac{1}{4}$) is as likely as getting one head and one tail (actually $\frac{2}{4}$).
- The Gambler's Fallacy: if you are flipping a **fair** coin and get six heads in a row, the next flip is more likely to be a tail "by the law of averages". Or if a family has four boys, then the next baby is most likely to be a girl, again "by the law of averages".
- The Prosecutor's Fallacy: for example, the probability that an animal is a donkey, given that it has four legs, is the same as the probability that an animal has four legs, given that it is a donkey.
- The Defendant's Fallacy: for example, the four legged animal with long ears reportedly seen on the beach giving rides to children is unlikely to be a donkey because donkeys form such a small proportion of all the animals with four legs.
- P(A and B) = P(A) × P(B), which is commonly taught at GCSE and is only true when events are independent.
  In fact P(A and B) = P(A) × P(B| A).
- P(A or B) = P(A) + P(B), which is also commonly taught at GCSE and is only true when events are mutually exclusive.
  In fact P(A or B) = P(A) + P(B) – P(A and B)

Activities

Centre for Innovation in Mathematics Teaching

CIMT Probability

This chapter from an online pupil text book takes the reader through Foundation KS4 probability with clear explanations, worked examples, exercises (with answers) and investigations. It covers sample space diagrams (systematic listing, tables, tree diagrams, Venn diagrams), finding simple probabilities (theoretical and experimental) and calculating the probability of two or more events (dependent and independent). It also covers events which are not mutually exclusive. Learners who have studied Foundation GCSE should be familiar with this content.

CIMT Stats Ch1

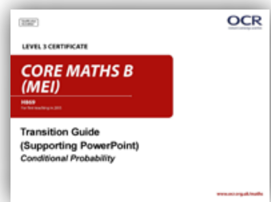
This chapter briefly covers some of the same ground and also introduces "reverse" conditional probability as well as set notation, for example, $P(A \cap B)$ and $P(A')$, and the rule that $P(A|B) = P\left(\frac{A \cap B}{P(B)}\right)$.

Set notation will not be used in Core Maths but this concept of conditional probability will be. Again there are explanations, worked examples and exercises, the answers to which can be found here (CIMT Probability answers 2). Learners who have previously studied Higher GCSE should be familiar with this content but their understanding may not be secure.

All learners will need to be comfortable with basic probability and with finding and distinguishing between $P(A|B)$ and $P(B|A)$ before moving on to the context rich problems and applications to decision making they will encounter in Core Maths (although it is not required from learners to use the notation it would be beneficial for them to familiarise themselves with it).

Ball Game

(Original activity pack © DIME Projects 1988, previously published by Tarquin) [DIME Project cards & DIME Project solutions]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CORE MATHS B (MEI)
Transition Guide
(Supporting PowerPoint)
Conditional Probability

These resources encourage learners to think about and discuss different ways of finding and representing probabilities. The original activity pack, which included sets of coloured beads in sealed tubes for learners to experiment with, is no longer available, but this paper-based resource replicates the questions that were asked to stimulate learners to discuss and explain probability. The cards are designed to be used with small coloured counters for learners to manipulate, although some will prefer to draw diagrams on paper. The answers suggest alternative informal approaches and the arguments can be understood without knowledge of notation and formal calculations. An extension might be to ask learners to formalise the representations and calculations using tree diagrams, multiplication and addition.

A computer simulation of one of the DIME problems for use with learners can be found at Nrich. This is a computer simulation encouraging learners to think about and discuss probabilities and expectations. 7 balls are shaken in a container. You win if the two blue balls touch and lose if they don't. Learners can look at the random results and calculate 'What is the probability of winning?'

Louis' Ice cream Business

Nrich

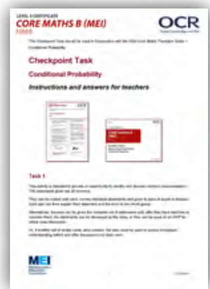
This is a practical activity which provides a context for probability and risk as an aspect of decision-making in the real world and so is a good activity to do at the transition between KS4 and Core Maths. The final section provides a context for using a probability tree and sampling with replacement to consider the issue of dependence and independence. As usual with Nrich activities, teacher's resources are provided.

The Dog Ate My Homework

Nrich

This is an introduction to conditional probability, a practical experiment using dice and multi-link cubes gives data which can be used to answer questions like 'What is the chance that a learner who is not lying about their homework is nevertheless accused?' The practical representation of data using the multi-link cubes helps learners to appreciate that this is not the same question as 'What is the chance that a learner who is accused is not lying about their homework?'

Checkpoint task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Checkpoint Task
Conditional Probability
Instructions and answers for teachers

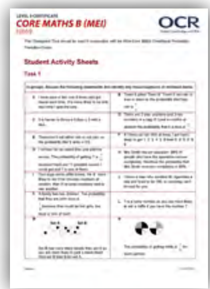
Task 1

This activity is intended to provide an opportunity to identify and discuss common misconceptions. The statements given are all incorrect.

They can be copied onto card, cut into individual statements and given to pairs of pupils to discuss. Each pair can then explain their statement and the error to the whole group.

Alternatively, learners can be given the complete set of statements and, after they have had time to consider them, the statements can be discussed by the class, or they can be used on an OHP for whole class discussion.

Or, if another set of similar cards were created, the sets could be used to assess individuals' understanding before and after discussion and class work.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Student Activity Sheets
Task 1

Task 2

These questions test learners' ability to find simple probabilities using different types of diagram, to find the probabilities of compound events and to use calculate conditional probabilities both forward and, in the last two questions, in reverse.

Activities

Introductory videos

A good introduction to "reverse" conditional probability (P(A|B) rather than P(B|A) can be found at Conditional probability video.

Chapter 1 uses Venn diagrams and Chapter 2 uses tree diagrams. Chapter 3 applies this to medical testing. The lesson is well paced and allows for pauses for discussion.

An introduction to the real life implications and importance of conditional probability is given by Peter Donnelly in his TED talk - the relevant part runs from 11.00 and covers testing for disease and miscarriage of justice.

Great Expectations: Probability Through Problems

This article from Nrich and the links in it offer contexts and an approach to teaching probability which aligns closely to the philosophy and assessment objectives for Core Maths. The task "Who Is Cheating?" would be useful for investigating the context of drugs testing. This practical activity models the statistical interpretation of test results for cancer, HIV, pregnancy, DNA at a crime scene, and many other similar situations, including drug testing in sport.

Prosecutors Fallacy and Defendant's Fallacy (for learners following Quantitative Reasoning (MEI) specification)

Clear explanations of these can be found at

Prosecutor's fallacy 1

Prosecutor's fallacy 2

Defendant's fallacy

Activities

Monty Hall problem

This problem is based on the game show where there are three doors. Behind two doors are goats and behind the other is a car. The host asks you to pick one of the three doors. After you have picked a door, the host will open one of the doors you did not pick to reveal a goat. You are then asked if you wish to switch your choice to the other unopened door or stick with the one you already chose. The question is, "What is the probability that you win the car if you stick with the door you have already chosen? What if you swap doors?"

This problem involves conditional probabilities and is counter-intuitive: most people assume it is of no advantage to switch doors but mathematically it is. This allows for a lot of debate, discussion and reasoning in the classroom. It uses aspects of conditional probability and can be solved by drawing tree diagrams and calculating probabilities of successive events. An explanation of the problem and informal solution can be found at Monty Hall 1. Other explanations, including a tree diagram and the direct calculation using conditional probability, can be found at Monty Hall 2

A computer simulation can be used. This one needs Internet Explorer version 4 or later. However, it is more powerful if learners simulate the game show themselves using, for example, black and red playing cards or upturned cups with coloured counters representing the car and goats.

Drugs testing

Nrich

This task might be used as an extension to the "Who Is Cheating?" activity. It starts considering a similar situation but compares the payoff matrix for two athletes depending on the testing regime and whether or not they are each guilty.

Activities

Conditional probability extension questions

Additional questions with answers.

The following are two interesting articles about the role of statistical evidence in miscarriages of justice. The second is about the Sally Clarke case referred to by Peter Donnelly in the TED video.

1. **One in Millions, Billions and Trillions: Lessons from People V. Collins (1968) for People V. Simpson (1995)**

On June 18, 1994, newspaper headlines across the world trumpeted the arrest of sports superstar O. J. Simpson in Los Angeles for the murder of Nicole Brown Simpson and Ronald Goldman. After a contentious nine-month trial, the jury took only three and a half hours to find Mr. Simpson not guilty of both murders. For years to come, learners entering law school will know the basic facts of the Simpson case and will have opinions about the incriminating value of the DNA evidence that linked Mr. Simpson to the crime scene. But few among them will know the significance of certain events that took place just a few miles from the Simpson/Goldman crime scene exactly thirty years earlier.

Jonathan J. Koehler
Northwestern University - School of Law
Journal of Legal Education, Vol 47, pp. 214-223, 1997

2. **Reflections on the cot death cases**

Sally Clark, Angela Cannings and Trupti Patel not only suffered the tragic sudden deaths of two or more babies. They were then accused, and in the first two cases convicted, of their murder. The misuse of statistics at Sally Clark's trial had profound consequences, especially for her, but also for the subsequent cases. Ray Hill reflects on the statistical issues involved.

Ray Hill
University of Salford, Lancaster
Med Sci Law. 2007 Jan;47(1):2-6.

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