GCE

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

Report on the Units

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CONTENTS

Advanced GCE Further Mathematics (MEI) (7896)
Advanced GCE Further Mathematics (Additional) (MEI) (7897)
Advanced GCE Mathematics (MEI) (7895)
Advanced GCE Pure Mathematics (MEI) (7898)

Advanced Subsidiary GCE Further Mathematics (MEI) (3896)
Advanced Subsidiary GCE Further Mathematics (Additional) (MEI) (3897)
Advanced Subsidiary GCE Mathematics (MEI) (3895)
Advanced Subsidiary GCE Pure Mathematics (MEI) (3898)

REPORT ON THE UNITS

GCE Mathematics And Further Mathematics Certification
From the January 2008 Examination session, there are important changes to the certification rules for GCE Mathematics and Further Mathematics.

1 In previous sessions, GCE Mathematics and Further Mathematics have been aggregated using ‘least-best’ i.e. the candidate was awarded the highest possible grade in their GCE Mathematics using the lowest possible number of uniform marks. The intention of this was to allow the greatest number of uniform marks to be available to grade Further Mathematics.

From January 2008 QCA have decided that this will no longer be the case. Candidates certificating for AS and/or GCE Mathematics will be awarded the highest grade with the highest uniform mark. For candidates entering for Further Mathematics, both Mathematics and Further Mathematics will be initially graded using ‘least-best’ to obtain the best pair of grades available. Allowable combinations of units will then be considered, in order to give the candidate the highest uniform mark possible for the GCE Mathematics that allows this pre-determined pair of grades. See the next page for an example.

As before, the maximisation process will award a grade combination of AU above, say, BE. Where a candidate’s grade combination includes a U grade a request from centres to change to an aggregation will be granted. No other requests to change grading combinations will be accepted. e.g. A candidate who has been awarded a grade combination of AD cannot request a grading change that would result in BC.

2 In common with other subjects, candidates are no longer permitted to decline AS and GCE grades. Once a grade has been issued for a certification title, the units used in that certification are locked into that qualification. **Candidates wishing to improve their grades by retaking units, or who have aggregated GCE Mathematics or AS Further Mathematics in a previous session should re-enter the certification codes in order to ensure that all units are unlocked and so available for use.** For example, a candidate who has certificated AS Mathematics and AS Further Mathematics at the end of Year 12, and who is certificating for GCE Mathematics at the end of Year 13, should put in certification entries for AS Mathematics and AS Further Mathematics in addition to the GCE Mathematics.

Grading Example

A candidate is entered for Mathematics and Further Mathematics with the following units and uniform marks.

Unit	Uniform marks	Unit	Uniform marks
C1	90	M1	80
C2	90	M2	100
C3	90	M3	90
C4	80	S1	70
FP1	100	S2	70
FP2	80	D1	60

Grading this candidate using least-best gives the following unit combinations:

Mathematics		Further Mathematics	
Unit	Uniform marks	Unit	Uniform marks
C1	90	FP1	100
C2	90	FP2	80
C3	90	M1	80
C4	80	M2	100
S1	70	M3	90
D1	60	S2	70
Total	480 (Grade A)	Total	520 (Grade A)

Under the new system, having fixed the best pair of grades as two As, the mark for the Mathematics would be increased by combining the units in a more advantageous manner. The table below shows the allowable combination of units.

Option	Applied units used for Maths	Total uniform marks for Mathematics	Applied units used for Mathematics	Total uniform marks for Further Mathematics
1	M1, S1	500	M2, M3, S2, D1	500
2	M1, D1	490	M2, M3, S1, S2	510
3	S1, D1	480	M1, M2, M3, S2	520
4	M1, M2	530	M3, S1, S2, D1	470
5	S1, S2	490	M1, M2, M3, D1	510

Option 4 gives the highest uniform mark for Mathematics. However, this would only give a grade B in the Further Mathematics, and so is discarded. Option 1 is the next highest uniform mark for Mathematics and gives an A in Further Mathematics, and so this is the combination of units that would be used.

4751 Introduction to Advanced Mathematics (C1)

General Comments
The usual spread of candidates, from very good to extremely weak, was seen. Time did not appear to be a problem, even for weak candidates, with most parts of the last question usually being attempted.

Compared to some recent past papers, there were perhaps fewer parts in this paper that hindered good candidates from obtaining high marks, so that more candidates gained over 60 marks, for instance, than compared with last June. Some topics, such as using the discriminant to determine when the roots of a quadratic equation are real, remain poorly done by many candidates.

Some centres continue to issue graph paper to their candidates in the examination. This is to their disadvantage, when some with graph paper then spend time attempting plotted graphs (often with inappropriate scales) when all that is required is a sketch graph.

The examiners are also concerned that some candidates may not be sufficiently practised in non-calculator work. There were fewer fractions used in this paper than in last January's, but nonetheless poor arithmetic can lower the mark considerably for some candidates.

Comments on Individual Questions

Section A

1) This ought to have been an easy starter. Many picked up both marks, but some did not reach $4x > 6$ and some who did then gave $x > \frac{2}{3}$.

2) Most knew what to do and many gained full marks. To find the intercepts, those who rearranged to $y = -\frac{2}{3}x + 4$ before substituting $y = 0$ often gave themselves too hard a task in finding x. Some gave just the y- intercept. The usual errors were seen in the gradient, with $\frac{2}{3}$ and $-\frac{2}{3}x$ being seen more than occasionally. Thankfully, the gradient was rarely inverted.

3) Although many were able quickly to factorise and solve $2x^2 + 3x = 0$, weaker candidates often found this difficult, with the formula being frequently used and an error often made such as $4 \times 2 \times 0 = 8$ or in not knowing what to do with $\frac{0}{4}$. Some rearranged the equation then divided by x and found only one root.

   In the second part, many did not know the condition for real roots. A common error was to substitute k instead of $-k$ for c, often then making errors with the resulting negative coefficient in the inequality.

4) This was reasonably well-answered, with parts (i) and (iii) usually correct; part (ii) presented the most problems to candidates, with 'false' instead of 'either' being a common response. Many candidates showed no working.

5) Many candidates made a good start on rearranging the equation by correctly multiplying by (x − 2), with relatively few omitting the brackets. Those who knew the strategy to use often proceeded to gain the rest of the marks, but some floundered from here. Some weaker candidates started by multiplying the right-hand side by $\frac{x+2}{x+2}$, then multiplying out and often 'cancelling' the x^2 terms. Some candidates used spare time at the end of the examination to have another attempt at this question after failing to manage it first time round – some of these later attempts were successful.

6) In this question on indices, the correct answer of 5 in the first part was common, sometimes given with no working shown. Of those who did not gain 2 marks, many knew that a square root was involved, often reaching $\frac{1}{5}$ or $-\frac{1}{5}$, but not managing to cope correctly with the reciprocal. A few thought the index was also inverted and calculated 25^2.

In the second part, those who did not gain full marks often had partially correct answers, with the most common error being to think that $\left(x^2\right)^5 = x^7$ and so on, as expected.

7) Most of the candidates made an attempt at the first part and realised that they needed to multiply the numerator and denominator by 5 - √3. A few used 5 + √3. However many of the candidates made an error in determining the new denominator, with expressions such as 25 + 3 or 25 − 9 being used.

In the second part, most candidates obtained a mark for getting 9 – 6√7 – 6√7 ….. . However the final term was often wrong (such as 28, 14 or 4√7). Some errors were made in combining the terms involving √7.

8) Although most candidates had some idea about what was required here, many of them were unable to cope with the $(-2)^3$ element. It often appeared with the x included and in the form $-2x^3$; even if it was written as $(-2x)^3$, it was frequently evaluated as $-2x^3$. The negative sign was also often dropped. There were also a number of candidates who only included two of the required elements (such as 10 x 25, or 10 x -2^3). A few candidates tried to take a factor of 5 outside the brackets but they commonly then made errors. Very few candidates tried to multiply out $(5 - 2x)^5$, but of those who did most failed to do so correctly.

9) Most candidates were able to factorise (or use the formula) and of these most went on to arrive at the values of y. Only a minority realised that there was a connection between this equation and the next. As a consequence few candidates gained full marks on this question as they were unable to determine four roots from the quartic equation. Of those who did realise that x^2 was equal to 3 or 4 many only gave the two positive roots. 2 was sometimes left in the form √4.

Section B

10 (i) Completing the square was done better than expected, aided by the fact that no fractions were involved this time. Many candidates had clearly learned a formula for this. Some got as far as $(x - 3)^2$ but did not know how to proceed from there.

(ii) Although some realised the relevance to part (i) and just wrote down the answer as expected, a surprising number started again and used calculus to obtain the result, sometimes making errors in the process.

(iii) In sketching the graph, most knew the general shape of a parabola but many omitted the fact that it went though (0, 2), often having a graph with a negative *y*-intercept from estimating the general direction of the curve. Some did not use their turning point from part (ii) but instead had the minimum at (0, 2) or (3, 0).

(iv) Most candidates equated the two expressions for *y* and many then rearranged successfully and went on to obtain $x = 4$ as the only root. Forgetting to then go on to obtain the *y* value was a frequent error. Many did not realise (or failed to state) that the equal roots implied that the line was a tangent. Instead, some successfully used calculus to show that the gradients of the curve and the line were the same at (4, −6). Weaker candidates who had got this far sometimes thought that showing that (4, −6) satisfied both equations was sufficient to imply the line was a tangent. Candidates who started this part by substituting $\frac{y+7}{2}$ for *x* rarely made progress.

11 In general, this was the section B question which caused most difficulty to candidates, particularly part (iv).

(i) Many gained both marks, with calculating f(−4) being the most common method. Working out 7 × 16 proved beyond some, whilst the other main error was in −7(−4) becoming −28. More able candidates often divided by $(x + 4)$ and positioned themselves well for part (ii).

(ii) Many coped correctly with the division and achieved $2x^2 - x - 3$, with fewer sign errors being seen than in some past papers. Some obtained this by inspection. Although many proceeded correctly from here, some made errors in factorising the quadratic factor, or resorted to using the formula, often failing to cope with the fraction in factorising after this.

(iii) Most candidates realised the link with part (ii), with follow-through marks from their factors or roots allowing some to obtain full marks in spite of errors in part (ii). Omission of the *y*-intercept label was the most common error. Those who used graph paper often struggled with the scales, with the *y*-intercept of −12 producing a graph that they then found difficult to draw with a smooth curve.

(iv) The easiest way of showing the required result was to realise that the roots found in previous parts were increased by 4 and to work with $x(x + 3)(2x - 11)$. Some of the most able candidates did this, although some did not cope correctly with the 2 and used $(x - 5.5)$ or $(2x - 7)$. The majority who attempted a correct method went down the route of substituting $(x - 4)$ for *x* in the non-factorised form of f(*x*). This then meant they had to work out an involved algebraic expression, including expanding $2(x - 4)^3$. Some achieved this correctly, but often sign errors at various stages stopped progress. Weaker candidates often did not know what to do, with some attempting to divide the given expression by (x − 4) or substituting 4 or − 4 in it, making no progress and gaining no marks.

12 (i) Finding the equation of the line joining the two points was done well by most candidates.

(ii) This was done less well than part (i), with many candidates failing to give complete reasoning. Most knew the condition for the gradients of perpendicular lines, but many failed to appreciate that the line went through the midpoint of AB. Many candidates worked backwards from the given equation in order to determine its gradient and used the perpendicularity condition, which afforded only a partial solution. Those using the 'backwards' method who went on to show that the intersection of the lines was the midpoint of AB were able to gain full marks.

(iii) Many candidates successfully showed that $k = 40$ and that B was on the circle. There was some confusion between 40 and $\sqrt{40}$. Some used long methods such as expanding $(-1 - 5)^2$ term by term.

(iv) Only the better candidates gained full marks here. Many candidates realised that they needed to put $y = 0$ into the equation of the circle (although a few put $x = 0$). Some thought that $(0 - 3)^2 = 0$. Candidates did not always use the equation in its simplest form, or did not realise the easy route of solution from having reached $(x - 5)^2 = 31$ and as a consequence they then needed to use the quadratic formula.

4752 Concepts for Advanced Mathematics (C2)

General Comments
The paper was accessible to nearly all candidates, but there was enough to challenge even the best. There was no evidence that candidates struggled for time. Some candidates set their work out fully and clearly. Nevertheless, many marks were lost by failing to show sufficient detail of the method, using a method different to the one specified in the question or, in a small number of cases, by contriving a new question and answering that instead. Some also lost marks by failing to leave answers in their simplest form, or by rounding prematurely.

Comments on Individual Questions

Section A

1) This question was generally very well done. A few candidates simply converted $\frac{7}{6}\pi$ to a decimal, and some were unsure what to do with "π radians = 180°".

2) This question was very well done, with the majority scoring full marks. Some candidates evaluated S_4 in part (i), and a few decided the series was arithmetic and gave the fourth term as 5.7. Even so, they often went on to score both marks in part (ii).

3) This was done very badly, even by very strong candidates. Many correctly obtained y = 3f(*x*), but did not make any further progress. Those who did describe the transformation correctly often spoiled their answer by adding a second one – usually a vertical translation of +10.

4) Most candidates obtained the first two marks, but only a few went on to give the correct answer. Common errors were "$x > 2$ or $x < -2$" and "$-2 > x < 2$". Some weaker candidates ignored the instruction to use calculus, and simply evaluated the function for various values of *x*, which usually led to the wrong conclusion (and scored zero anyway). A few others thought that "increasing function" meant "$f(x) > 0$" and did quite a bit of work but failed to score.

5) Most did well on this question. Problems in part (i) usually arose from premature rounding. A minority obtained 5.7 and then changed this to 5.70 in response to the instruction to give the answer to two decimal places. Similarly, in part (ii), a correct method was often spoiled by premature rounding leading to a *less* accurate answer. A few candidates used new *x*-values such as 3.02 and 3.05, which didn't score, and a few others simply used the midpoint of the original chord, obtaining the same answer as in part (i). A handful completely missed the point, and wrote answers such as $x2^{x-1}$.

6) This was often done very well. A few encountered difficulty in evaluating 6 / 1.5, or simply left it unsimplified. A significant minority thought $6\sqrt{x}$ was *x* to the power $\frac{1}{6}$, and then integrated. Another minority took the gradient to be "$6\sqrt{x}$" and then substituted in y = m*x* + c.

7) Most used ½$r^2\theta$ correctly, but a few candidates converted θ to degrees and then used the formula. A handful used the formula ½$r\theta^2$ and scored zero. In the second part, almost everyone knew what was required, but some candidates wrongly assumed the triangle was right angled, and used ½ base × height. Some candidates wasted much time by using the cosine rule to find the "base" and then dropping a perpendicular for the "height", and occasionally penalised themselves even more by rounding prematurely.

8) Many candidates were able to write down two correct equations, but a significant minority then made errors with the algebra when eliminating a or d. Weaker candidates wrote "11 = a + 10d" and "120 = 5(2a + 119d)" was seen occasionally. Some of the better candidates went straight to 121 = 5.5(a + 1) and reached the correct solution quickly. Some weaker candidates could not adopt an algebraic approach, but contrived the solution correctly by trial and error.

9) Nearly all candidates scored full marks on this question. It seemed that many had the facility on their calculator to evaluate $\log_5 235$ directly. No marks were given for an unsupported answer. Some carelessly rounded to 3.4 and lost a mark; a few gave the answer as 3.40 and lost both marks. Weaker candidates ignored the instruction to use logarithms, and used trial and error, which scored zero.

10) This was generally not well done, with only a minority scoring full marks. Many knew to use Pythagoras, but made errors in substitution, such as "$\cos\theta = 1 - \sin\theta$", "$\sin^2\theta = \cos^2\theta - 1$", "$2 - \cos^2\theta = \cos\theta + 2$" or even "$\sin^2\theta = 1/\cos^2\theta$". Those that did manage to produce the correct quadratic often made errors in factorising to obtain $\cos\theta = ½$.

Section B

11) (i) This was generally very well done.

(ii) Most candidates correctly expanded the brackets from part (i), but some simply evaluated f(-5), f(-2) and f(2), which didn't score.

(iii) Most candidates attempted to solve $\frac{dy}{dx} = 3x^2 + 10x - 4 = 0$, but many spoiled their solutions by simply writing down the answer as 0.4. Candidates need to be reminded that full working is expected for answers which are given. A significant minority of candidates wasted time by considering the second derivative to demonstrate the nature of the turning points. This was not asked for, and gained no credit. A good number failed to give the co-ordinates of the maximum point to the required accuracy.

(iv) Many failed to use the answer to part (iii) and wasted much time re-working the equation – frequently making errors in the process. Those who did use the answers from part (iii) often doubled *x* or *y* or both.

12) (i) This was often very well done, with many candidates scoring full marks. However, those who were unable to quote the formula correctly were heavily penalised, as it is provided in the formula booklet. Common errors were the omission of a pair of brackets, which was often irrevocable, and "h = 2.5" or " h = 0.5". Most knew to multiply their answer by 2.5 to obtain the volume, but a few multiplied by 100. Most gave the correct units.

(ii) The integral was often correctly done. Where slips were made, by far the most common was to give the last term as "$0.15^2/2$". A few candidates found $\frac{dy}{dx}$ and substituted in 0.5, and a few others simply found f(0.5). Many candidates were not able to say what the integral represented, and made comments about over-estimates and under-estimates.

13) (i) This was often well done, but those who tried to derive the result often made the error "$\log at^b = b \log at$", and a significant minority failed to state the intercept.

(ii) The table of results was largely correct, with just the odd slip, and most were able to plot the points correctly. A common mistake was to plot (0.078, 1.64) instead of (0.78, 1.64). Lines of best fit were generally ruled and sensible.

(iii) Most knew how to find the gradient of their line, but frequently made arithmetical errors – usually by being out by a factor of 10 in the numerator. "a" was often found correctly, but only the better candidates were able to produce a correct version of the equation. Marks were often lost at this stage for inaccurate work on the insert leading to answers outside the acceptable range.

(iv) Those who had done well in the earlier parts had no difficulty here. In addition, many candidates scored zero in parts (i) and (iii), yet still managed to obtain this mark by extrapolating from the table of values.

4753 Methods for Advanced Mathematics (C3) (Written Examination)

General Comments

This paper proved to be a fair and accessible test, and many candidates achieved over 50 marks. Section A was often very well done – well prepared candidates were able to tackle these questions confidently and lost few marks. Question 8 of section B also scored well, but the final question tested all candidates and discriminated quite well. All but few also had ample time to complete all the questions.

In general, there is a considerable amount of calculus tested by this paper, and students are usually well prepared in calculus techniques such as chain, product and quotient rules, and integration by parts and substitution. There is some confusion between differentials and integrals of trigonometric, logarithmic and exponential functions. The language of functions, and work on the modulus function, are perhaps less secure.

The paper proved to be quite forgiving about notational weaknesses, and there were quite a few generous 'E' marks. Any candidate scoring less than 20 marks on this paper should arguably not have been entered, and it is pleasing to report that this occurred relatively infrequently. However, algebraic fragility continues to mar the work of candidates, although we often, with reluctance, use the principle of ignoring subsequent working ('isw') in marking correct formulations of answers which are then corrupted by poor algebra. The number of candidates who, for example, leave expressions such as x^2/x unsimplified, or omit essential brackets, or write $(1 + \cos x)^2 = 1 + \cos^2 x$, is disappointing!

Several questions involved sketches of graphs – these are far better done *without* graph paper – it is indeed extremely unlikely that graph paper is required to be used in this paper.

Comments on Individual Questions

Section A

1) Most candidates correctly derived $x \le 2$ and scored 2 marks out of 4, but many had difficulties in handling the left hand boundary of the inequality, for example arguing that $-2x \le 2 \Rightarrow x \le -1$. Writing $|2x| \le 4$ was penalised. Some candidates squared both sides, but some of these made the error of arguing that $(x + 1)(x - 2) \le 1 \Rightarrow x \le -1$ and $x \le 2$. The best solutions worked with each side simultaneously. Very few students used a graphical approach.

2) Although there were many completely correct solutions, the constant of integration was frequently missing (though this equally gave an easy mark to some candidates who made a mess of the integration). Those who failed to do the integration correctly often confused integration by parts with the product rule, or integrated e^{3x} as $3e^{3x}$.

3) The algebraic and geometric conditions for an even function were generally well known, though some candidates omitted the axis of symmetry from the latter. Most classified at least one of f, g or h correctly (albeit perhaps fortuitously); the function g giving the most problems – many thought this was 'odd'.

4) Many candidates scored full marks for this question, with a pleasing number using inspection rather than substitution. We needed to see the step ln 18 − ln 3 = ln(18/3) which was occasionally incorrectly written ln 18 / ln 3. Although sloppy working (for example, omitting 'dx' or 'du', limits inconsistent with these, etc.) was generally condoned here, this has been penalised in some recent papers, and it would be pleasing to see less of this.

5) The product rule and the derivative of ln x were well known, and most candidates found the derivative correctly. However, quite a few failed to score the last two marks – it is important with a given answer that enough working is shown, and they needed to show the step from ln $x = -½$ to $x = e^{-½}$, or, if verifying but substituting $x = e^{-½}$ into the derivative, that $\ln(e^{-½}) = -½$. The latter method was quite common, and accepted. A number of candidates left x^2/x unsimplified – as the derivative had not been asked for explicitly, this was not penalised.

6) This exponential decay question proved to be a 'banker' for even the weakest candidates. The first three marks were almost always awarded, and solving the exponential equation in part (ii) using logarithms is very well done. Some candidates are still plotting graphs – we much prefer to see a sketch *not* on graph paper, and this in effect makes the asymptotic behaviour *easier* to credit than a graph which 'stops' at a plotted point, especially if the asymptote $m = 20$ is shown. Occasionally the vertical axis was erroneously marked as an asymptote.

7) This provided the most challenging test in section A. In particular, differentiating xy using the product rule was achieved by better candidates only. The error of starting an implicit differentiation by stating dy/dx = … is still quite common, and sometimes compounded by some candidates then proceeding to collect the extraneous dy/dx term in with the other terms in dy/dx! Another common error was to 'forget' to equate the derivative of the constant 12 to zero.

Section B

8) This question was in general done well, with most candidates gaining well over half marks. There is, however, quite a lot of confusion between differentials and integrals of sin x and cos x, perhaps encouraged by the reasoning required in part (iii).

(i) This was an easy mark for all and sundry. However, some weaker candidates used calculators in degree mode when substituting $x = \pi/3$.

(ii) Quite a few candidates used a quotient rule rather than a chain rule, and omitted the 'zero' derivative du/dx. Sign errors were common, as were omitted brackets.

(iii) Most candidates attempted a quotient rule, but again this was often marred by poor algebra, for example sign errors, incorrect formulae, $(1 + \cos x)^2 = 1 + \cos^2 x$, and indiscriminate cancelling. However, competent candidates often achieved full marks. In the second part, weaker candidates missed the point, and often started integrating $\sin x / (1 + \cos x)$ as $-\ln(1 + \cos x)$. Some also failed to express the final answer exactly.

(iv) Most candidates managed to invert the function correctly, but very few gained the mark for the correct domain. The sketches were often well done – most candidates attempted a reasonable reflection in $y = x$, and we condoned domain and range errors in this instance.

9) This question was less successfully answered than question 8, especially the final part. The language of functions is a topic which requires sound notation and conceptual understanding, and this often found out less secure candidates.

(i) Most candidates realised that this was about squaring and comparing with the circle equation, and some did it in reverse by square rooting. However, explanations about why the full circle was not represented by the function were often unconvincing, mixing up the ideas 'square roots cannot be negative' and 'you cannot square root negative numbers'. We also allowed well expressed arguments about functions being single valued.

(ii) More assured candidates followed the three steps correctly, with correct notation. However, many missed the point, failing at the first hurdle to write down the gradient of OP as b/a. Many, although spotting a chain rule, missed the negative sign in differentiating y, and failed to substitute $x = a$ in their derivative to give $f'(a)$.

(iii) The stretch and the translation can of course be done in either order. We wanted to see 'stretch' and 'translation', and penalised other descriptions such as 'move', 'shift', 'multiply the y-coordinate by 3', which were common. Sketches of the curve which went through (0, 0), (4, 0) and (2, 6) usually gained full marks, notwithstanding weaknesses in their elliptical shape.

(iv) Success in this part proved to be the preserve of more assured candidates. The lynch pin was to write $y = 3\sqrt{(4 - (x - 2)^2)}$ – without this, little progress was made. Expanding the bracket and handling the negatives then caused some candidates to lose their way.

4754 Applications of Advanced Mathematics (C4)

General Comments

Many candidates found this paper was easier than that of last summer but more difficult than that in January this year. Section A included sufficient questions to challenge the good candidates but also many questions that were accessible to weaker candidates. Thus, there were very few low marks overall. The Comprehension scored highly with a number of straightforward numerical questions. Few, however, scored full marks here. Although candidates often appeared to have some understanding of the answers for the worded questions, they often did not give sufficient information to achieve the marks involved.

The presentation was generally better than in previous years.

Comments on Individual Questions

Paper A

Section A

1) Many candidates scored only one mark from the three available here when attempting to add algebraic fractions. The majority of candidates knew and used the correct method for adding two fractions. However, only the minority factorised and cancelled their expression. In most cases the denominator $(x^2-4)(x-2)$ was used and the numerator $3x^2+2x-8$ was found but then candidates failed to continue. Only a small number realised that the simplest denominator was $(x+2)(x-2)$.

2) Most candidates correctly found the volume of revolution. Errors included failing to substitute the lower limit, failing to include the π in the term for the second limit and errors in integrating e^{2x} .

3) There were many completely correct solutions to this question. Some started badly by making an incorrect substitution for $\cos 2\theta$. $2\sin\theta\cos\theta$ and $1-\sin^2\theta$ were commonly seen. Those that substituted correctly were usually successful provided that they realised that their equation was a quadratic equation to be solved. Almost all candidates gave their answers in the required exact form in terms of π although some missed the solution $5\pi/6$.

4) There were some good, clear concise answers to this question, but they were in the minority. There was some very poor setting out of a logical argument. Candidates frequently failed to 'show that' sufficiently to achieve the mark for establishing the result. Common errors included $2\sec\theta = 1/2\cos\theta$ and $(3\sec\theta)^2=3\sec^2\theta$. Candidates need to improve their presentation of coherent arguments in 'proof' questions.

5) Almost all candidates obtained full marks on this question. Using the chain rule was the most common approach. The most frequent error was to substitute u=5 in part (ii). Occasionally a final answer of ±6 was seen.

6) Most candidates used the Binomial expansion correctly in (i). Occasionally the powers of ½ or -1 were used instead of -1/2. There were a few numerical errors in evaluating the third term. The commonest error, however, in (i) was the validity. This was often omitted and frequently wrong. Errors included, $|4x|<1$, $|4x^2|\leq 1$, $-1/2\leq x\leq 1/2$ and $\sqrt{-1/4}<x<\sqrt{1/4}$. It was disappointing, although not penalised, to find so many candidates failing to simplify √1/4 to 1/2.
Most candidates correctly used their expression from (i) in (ii) although weaker candidates failed to recognise the usefulness or relevance of part (i). A few tried to expand $(1-x^2)$ in (ii) using the Binomial expansion again.

7) Many candidates approached the solution to this question by learning the process rather than from first principles often leading to $\tan\alpha=\sqrt{3}$ instead of$1/\sqrt{3}$. Sign errors, commonly $-1=R\sin\alpha$ were seen and again, disappointingly, √4 left unresolved was often seen for *R*- although most candidates did obtain $R=2$. The final part was usually either completely correct or omitted.

Section B

8)

(i) A wide variety of methods were seen. These included substituting the co-ordinates of the points in the equation of the plane, using a vector product and a combination of substituting points and the scalar product of vectors in the plane with the normal vector. Finding the Cartesian equation of the plane from the vector equation was also often used. Many candidates (even the most able students) failed to realise that more than one point substituted was needed and also failed to realise that one scalar product, or two, alone is insufficient to verify the Cartesian equation of the plane.

(ii) This was usually successful and full working was shown. The last part was sometimes again found from the vector equation of the plane. Some gave the final part as $2x-y+20z=0$ or $2x-y+20z=800$.

(iii) The scalar product was usually correct provided the correct vectors were used although there were numerical errors.

(iv) The correct vector equation was usually given although '*r* =' was often omitted. The final answer was also often found but some failed to realise that they needed to substitute into the equation of the plane. A frequent error was '$413+413\lambda=0,\ \lambda=1$'.

9)

(i) Most candidates separated the variables and attempted to integrate. Most did not include a constant. Common incorrect answers being $v=\pm 20e^{-\frac{1}{2}t}$, $10e^{-\frac{1}{2}t}$ and $5e^{-\frac{1}{2}t}$.

(ii) For good candidates who had found 'c' in (i), (ii) was usually successful. Many failed to realise that $e^{-\frac{1}{2}t}\to 0$, choosing ∞ or 1 instead. Some substituted a large value for *t* which was condoned in (ii) but not in (v).

(iii) The partial fractions were almost always correct.

(iv) Some candidates made a poor start here, failing to separate the variables and only working with their partial fractions from (iii). For those that separated the variables correctly, their initial integration was usually correct and they combined their logarithms correctly but for those that included a constant it was rarely found successfully. The correct final answer was given and often appeared without derivation. For those evaluating a constant a **frequent error**, as in the past, was to separate $\exp(-4.5t+c)$ as $\exp(-4.5t)+\exp c$.

(v) Few found that $w\to 4$ as $t\to\infty$. $w\to 0$ was fairly common.

Paper B - The Comprehension

1) The tables were almost always filled in correctly.

2) Many candidates failed to make it clear that they were discussing 2x2 blocks in their explanations. Some referred to 3x3 blocks or blocks in general or said that a Sudoku had to be 9x9. For those that did indicate 2x2 blocks, they usually gave the required reason for the Latin Square not being a Sudoku puzzle which is because there were repeated entries in the individual blocks.

3) This table was usually completed correctly although some gave Fig. 10.

4) This was usually correct.

5) This was not well done. Some said that because there were 9 squares it must be 9! rather than explaining this was the number of ways to fill in a row of 9. Some said if 3x3 was 3! And 4x4 was 4! then this must be 9!. Others failed to explain that the 9! would need to be multiplied by the number of ways for the rest of the grid to get the total number.

6) This was often completely correct. Most found 25x25 correctly but there were errors in the second column. In (ii) common errors were $M=b^4-b^2$ and using *s* instead of *b*.

7) (i) Those that realised that all the 3s and 5s were missing from the givens were usually successful. Others missed the point and said there was no unique solution because there were less than 77 givens.

(ii) There were many confused explanations here often referring, again, to the total number of givens rather than discussions about the numbers 1,2 and 3 forming an embedded square with 12 different solutions possible.

4755 Further Concepts for Advanced Mathematics (FP1)

General comments

Candidates generally performed well, demonstrating a good knowledge of the syllabus. The paper enabled the candidates to demonstrate their knowledge, but also differentiated well between them.

Some candidates dropped marks through careless algebraic manipulation, and a smaller number by failing to label diagrams and graphs clearly.

Comments on Individual Questions

1) There were many good answers, but some surprising errors, including vectors given as answers. In (iii) candidates must show the multiplication of the two matrices to avoid loss of a method mark if either of their earlier answers was incorrect.

2) (i) A circle of radius 2 was usually clearly shown. An incorrect centre was the most common error.

(ii) A full line was often shown, instead of a half line, and many lines went through -2j instead of 2j. Most candidates correctly drew lines parallel to the real axis.

(iii) If (i) and (ii) were correct, (iii) was usually correct too, though not all candidates showed the points clearly on their Argand diagrams.

3) This was very well done by about half the candidates. Others either omitted it or made errors such as failing to use an invariant vector, or setting the transformed point ($-x - y$, $2x + 2y$) to (0, 0).

4) There were many good answers, but also many careless mistakes. Often $(1-x)^3$ was expanded without multiplying all terms by *A* (especially the -1). There was also evidence of careless solving of the equation leading to *B*.

5) (i) This was well done by almost all candidates.

(ii) This was often badly done, indicating that many students did not properly understand inverse matrices. Many tried to use an algorithm to invert the matrix; some did this correctly, but wasted a lot of time in doing so; others tried to apply "rules" for inverting a 2×2 matrix to the 3×3 case.

6) The most popular method involved using sums and products of roots, but there were quite a few careless errors involving signs. Those who substituted $x=\frac{w}{2}$ were usually successful. Several candidates omitted the '= 0' from their equation and so lost the final mark.

7) (i) This was well done by almost all candidates.

(ii) The factor of $\frac{1}{3}$ was very often lost following successful expansion and cancellation of terms. A few candidates multiplied by 3 instead of dividing. A few left answers in terms of *r* not *n*.

8) (i) This was well answered, but *y* = 1 and *y* = 0 were quite frequent errors for the asymptote parallel to the *x*-axis.

(ii) Most candidates showed their method, usually by substituting large positive and negative values of *x*. The results were not always put to correct use in (iii).

(iii) The left-hand branch was often incorrect. Many candidates seemed to hold the misconception that the graph cannot cross an asymptote. Even where graphs were shown crossing the asymptote, they often did not show a clear minimum.

(iv) Many correct answers, but $x \neq 0$ was often omitted. A fairly common error was $x < -2, x > 3$, presumably from a misinterpretation of the "< 0" in the question. Several candidates used algebraic methods when the answer could be most easily found directly from the graph.

9) (i) and (ii) were both well done by almost all candidates.

(iii) Use of the sums and products of roots was the most common method, but candidates found difficulty in correctly finding $\sum \alpha\beta$ and $\sum \alpha\beta\gamma$. They also had problems in assigning the correct signs to four coefficients, and got muddled by *A B C D* and *a b c d* and *e*.

(iv) Multiplying factors with conjugate pairs was a less popular but more successful method, apart from occasional sign errors.

A few candidates tried to substitute roots, equating the real and imaginary parts to 0 and then solving simultaneous equations. This was the least successful method.

10) (i) This was usually started well, but a surprising number did not take out the common factors to ease the working. As a result there were algebraic errors and sometimes fudging of the final factorisation of the quartic.

(ii) This was well done in many cases, but there were signs that some candidates were under time pressure. The basic structure was well done in many cases, but the presentation and notation were often poor, with r, k and n often used in the wrong places. Summation signs were often omitted so that statements effectively meaning “last term = sum of k terms + last term” frequently appeared.

(iii) Candidates often failed to take out common factors in the algebraic manipulation, and their proofs faltered as a result. The final words of explanation were not always convincing.

(iv) However, it was encouraging that many perfect solutions were seen, showing that candidates understood the proof and were able to communicate their arguments clearly.

4756 Further Methods for Advanced Mathematics (FP2)

General Comments

Most candidates for this paper appeared to be well prepared across the range of syllabus topics, although a significant number were unfamiliar with converting from cartesian to polar coordinates. The marks were higher than last year, with about a quarter of the candidates scoring 60 marks or more (out of 72), and about 10% scoring fewer than 30 marks. The presentation of the candidates' work was generally very good, and most candidates appeared to have sufficient time to complete the paper. Some wasted time by using inefficient methods, or by deriving results which could be found in the formula book, such as the series for $\ln(1+x)$ and the logarithmic form of $\operatorname{arsinh} x$. Again, very many candidates lost marks for not showing sufficient working when the answer was given on the question paper. In Section B, the overwhelming majority of candidates chose the hyperbolic functions option.

Comments on Individual Questions

1) *(Polar coordinates, integration and Maclaurin series)*
The first and last parts of this question posed significant difficulties for a large number of candidates. The average mark for the question was about 11 (out of 18).

(a)(i) Many candidates found this to be an unconventional and surprising beginning, with even high-achieving candidates omitting this part; but there were many fully correct answers, some of which started with the polar equation and worked backwards. The most common error was to start with $x = \cos\theta$, $y = \sin\theta$.

(a)(ii) There were many fully correct graphs. The most common errors were to draw loops in the second and third quadrants, or to begin their loops too far away from the origin along the positive *x*-axis. Others exhibited cardioids and curves with many loops.

(b) This integration was efficiently done and was a good source of marks across the ability range, although there were some mistakes with the constants.

(c)(i) The Maclaurin series for $\ln(1+x)$ was usually produced correctly from the formula book, although some ignored the instruction to 'write down' the answer and obtained it by multiple differentiation. However, the series for $\ln(1-x)$ eluded many; candidates often did not realise that they were expected merely to replace *x* by $-x$ in the series already obtained, and began to differentiate (sometimes correctly), or just changed the signs randomly.

(c)(ii) The great majority of candidates subtracted here, although errors in part (i) prevented many from obtaining the correct answer, and a few tried dividing the two series. Some obtained the correct series by observing the logarithmic form of $\tanh x$ and the Maclaurin series for $\tanh x$, both given in the formula book; but this was not given full credit as the question required a derivation from part (i).

(c)(iii) This part was found to be difficult, and many candidates left it out. Those who made an attempt generally wrote out a few terms of the given series; slightly fewer observed that $\ln 3$ could be linked with the logarithmic expression when $x = \frac{1}{2}$, and still fewer convincingly reconciled the two series. Many tried in vain to produce a convergent geometric series.

2) *(Complex numbers)*
The average mark on this question was about 12.

(i) The principles involved here were understood well, although many got off to a bad start by thinking that the modulus of $\sqrt{32}(1+\mathrm{j})$ was $\sqrt{32}$.

(ii) Most candidates realised that they could use the modulus and argument they had found in part (i).

(iii) Again, this was very well done and the method understood well. A few forgot to divide the argument by 3.

(iv) This part proved to be a considerable challenge, and few candidates achieved full marks, although the mark for k_1 was scored reasonably regularly. Candidates who achieved correct matchings sometimes had the *k* on the wrong side and produced the reciprocal of the required value, or left out minus signs.

3) *(Matrices)*
This was the best answered question, with an average mark of about 14. Many candidates chose to start with this one.

(i) The method for finding the inverse of a (3×3) matrix was very well known and often carried out correctly. A few candidates set $k = 4$ at the start of the question and left out *k* altogether.

(ii) Again, most candidates were familiar with what was required here and full marks were common. Virtually all could produce the matrices **P** and **D**. Then most knew how to find **M** as $\mathbf{PDP}^{-1}$, although a few used $\mathbf{P}^{-1}\mathbf{DP}$. A few attempted to find **M** from $\mathbf{P}^{-1}\mathbf{MP} = \mathbf{D}$ and simultaneous equations, which took a great deal of time and was rarely successful.

(iii) Candidates were expected to 'write down' the characteristic equation in factorised form, as the eigenvalues were given, but very many went straight to $\det(\mathbf{M} - \lambda\mathbf{I}) = 0$, which wasted time, although here it was often successful. The Cayley-Hamilton theorem was very well known and many were able to produce an expression for $\mathbf{M}^4$ in terms of $\mathbf{M}^2$, **M** and **I**, although there were quite a few slips here.

4) *(Hyperbolic functions)*
The average mark on this question was about 11. Candidates who wrote everything in terms of exponentials at the earliest opportunity were considerably less successful than those who realised that part (i) could usefully be applied in the following parts.

(i) Most candidates approached this confidently and produced a convincing proof.

(ii) For many candidates this part provided 6 quick and easy marks: obtaining and solving a quadratic equation for $\sinh x$, then using the logarithmic formula for arsinh (from the formula book). Some wasted time solving $\sinh x = \frac{3}{4}$ and $\sinh x = -3$ by forming quadratic equations in exponentials. Candidates who ignored the hint given by part (i), and began by converting the original equation to exponential form, produced a quartic equation which they could not solve.

(iii) Convincing explanations that $\mathrm{d}y/\mathrm{d}x = 0$ has only one solution were quite rare, and a surprisingly common error was differentiating $9\sinh x$ to give $9\sinh x$. Many candidates found the value of *x*, although this was not required; using part (i) the value of *y* can easily be found from the value of $\sinh x$.

(iv) Although many candidates integrated the expression efficiently and correctly, the great majority lost the last two marks through failure to show sufficient working, particularly when evaluating $\sinh(2\ln 2)$. This working must be seen for the marks to be awarded when the answer is given.

5) *(Investigation of curves)*
Very few candidates (less than 1%) chose this question, and most of the attempts were fragmentary.

4757 Further Applications of Advanced Mathematics (FP3)

General Comments

Most of the candidates for this paper were well prepared and demonstrated a sound understanding of their chosen three topics. Candidates appeared to have sufficient time to do all they could, and their presentation was generally very good. However, there were parts which proved difficult for all but the most able candidates, and overall the marks were again disappointingly low. About 15% of candidates scored 60 marks or more (out of 72) and about 10% scored fewer than 30 marks. Questions 1 and 2 were about twice as popular as the other three questions, and the most frequent selection this year was questions 1, 2 and 4.

Comments on Individual Questions

1) *(Vectors)*

This was the most popular question, attempted by about 90% of the candidates, and the average mark was about 16 (out of 24). Appropriate vector products were evaluated confidently and accurately by most candidates, although sign errors were fairly common. Another error which occurred quite frequently was misinterpreting a scalar product as a vector; after quoting a formula involving, say, $|\mathbf{p}.\mathbf{q}|$, this was evaluated as $\sqrt{(p_1q_1)^2+(p_2q_2)^2+(p_3q_3)^2}$ instead of $|p_1q_1+p_2q_2+p_3q_3|$.

Parts (i) to (iv) were very well understood and often answered correctly. Parts (v) and (vi) were quite often omitted, but a good number of candidates did find the values of λ and μ. When explaining why E is between A and C, most mentioned $\lambda < 1$ bur failed to mention $\lambda > 0$. Candidates who attempted part (vi) usually made good progress with it, although some wasted time by calculating the volume of the tetrahedron ABEF from the coordinates of E and F, instead of using the values of λ and μ.

2) *(Multi-variable calculus)*

This was the worst answered question, with an average mark of about 11; parts (iii) and (v) posed significant difficulties for a large number of candidates.

Most candidates found the partial derivatives correctly in part (i), although quite a number made algebraic slips when simplifying their expressions; this was not penalised here, but it led to problems later on. Most also knew how to find the equation of the normal line in part (ii), although some just gave the normal vector, and some gave the equation of the tangent plane.

In part (iii) most candidates tried to use $h \approx (\partial g/\partial x)\delta x + (\partial g/\partial y)\delta y + (\partial g/\partial z)\delta z$, but few could combine this with PQ being parallel to the normal vector they had found in part (ii).

Part (iv) was done quite well. After manipulating $\partial g/\partial x = \partial g/\partial y = 0$ to obtain $z = 0$, a good proportion of the candidates could use the equation of *S* to explain why this is impossible. Many also incorrectly stated that $\partial g/\partial z = 1$, but provided this was not used in the subsequent argument, they were not penalised.

In part (v), those who started with $\partial g/\partial y = 5\partial g/\partial x$ and $\partial g/\partial z = 0$ were often successful. However, a large number of candidates assumed that $\partial g/\partial x = 1$ and $\partial g/\partial y = 5$; they could then find (incorrect) values of *x, y* and *z* without substituting into the equation of *S*, and scored only 2 marks out of the 8.

3) *(Differential geometry)*
This was by far the best answered question, with an average mark of about 19. The techniques required in all four parts were very well known. The differentiations and integrations were almost always carried out accurately, but some candidates had difficulties with the algebra when deriving the given results in parts (i) and (iii).

4) *(Groups)*
The average mark on this question was about 15. Candidates attempting it generally demonstrated a good basic knowledge of groups, and parts (i), (ii), (v), (vi) and (vii) were answered very well.
In part (iii), closure was often not shown satisfactorily, with many candidates showing in effect that $x * y \neq -½$ rather than $x * y > -½$. The identity and inverse were usually found correctly, but many omitted to show that $x^{-1} > -½$.
Part (iv) was often omitted, but there were also many correct answers.

5) *(Markov chains)*
This was the least popular question, attempted by about 30% of the candidates, and the average mark was about 16. Most of those who attempted the question demonstrated a sound knowledge of the techniques and used their calculator effectively to evaluate powers and products of matrices. Parts (i), (ii), (v) and (vi) were answered very well.
In part (iii), a very large number of candidates started by finding the probabilities for the 8th day, then proceeded as if the 7th and 8th days were independent. Similar errors were made in parts (iv) and (vii).

4758 Differential Equations (Written Examination)

General Comments

Many candidates demonstrated a good understanding of the specification and high levels of algebraic competency, but some candidates struggled to apply the correct methods.

Candidates should note that if a general solution only is required, then this will be specified. When conditions have been given and a solution is asked for (without the word 'general' preceding it), then the conditions should be used to find the particular solution.

Candidates are again reminded that sketch graphs should show the initial or boundary conditions. Although follow through marks are often awarded when the solution to the differential equation is wrong, if the graph is not consistent with known information, marks will usually be lost. Detailed graph-sketching skills are not required, but sketches are often assessed on a 'beginning, middle and end' approach – i.e. do they 'begin in the right place' (showing the conditions) do they 'end in the right place' (e.g. behaviour for large values of the independent variable) and is their shape 'in the middle' approximately correct.

Comments on Individual Questions

1) (i) Many candidates were able to write down a correct equation of motion, although some omitted the weight. However, few candidates scored full marks as most either omitted to justify the signs (as requested) or gave incorrect reasons for the sign of the resistance force.

(ii) This was often done very well, although a few candidates made errors in their general solution.

(iii) Finding the general solution was often done very well. The sketch graph often omitted important details, in particular the initial velocity. Some graphs did not make it sufficiently clear that the curve was oscillating. Some did not make it clear that the graph tends to zero.

(iv) Most candidates knew to use the discriminant to determine the type of damping, but some candidates used the wrong sign. Sketch graphs were often good, but some were oscillating. A small minority of candidates seemed not to know what was required here.

2) (i) This was often done very well, but some candidates omitted to use the condition on x.

(ii) This was also often done well, some using the complementary function and particular integral method, others using the integrating factor method. Only a small minority incorrectly tried to separate variables.

(iii) Most candidates gave insufficient working to justify that y was positive. However sketch graphs were generally correct.

(iv) Many candidates either omitted to show that $x + y + z = 8$, or tried to solve the differential equation, despite the instruction not to solve it. Many argued from their knowledge of physics about conservation of mass. Only a minority took the correct approach of summing the differential equations to show that $x + y + z$ must be constant, and then using the initial conditions to find the value of the constant.

(v) Many correct solutions for the time were seen, although some made the solution harder by multiplying out the given expression which had been deliberately given in factorised form to help candidates.

3) (i) This was often done extremely well. Only a small number of candidates did not recognise the need to use the integrating factor method. A surprising minority did not use the laws of indices correctly – in particular dividing t^{k+1} by t^k to obtain t^k was a common error.

(ii) Sketch graphs often lacked the given condition being marked and the behaviour for large *t* being clear, but many good sketches were seen.

(iii) This was often completely correct, but some incorrectly used their answer to (i).

(iv) The numerical solution was often done well, although a sizeable minority made numerical errors in the second step.

(v) Many candidates knew that reducing the step size is likely to increase accuracy. Reasons whether the estimates were over- or underestimates were often unclear, lacking in evidence or omitted. Some gave very good explanations.

4) (i) Most candidates completed this correctly. Some made algebraic errors, but only very few did not know how to do the elimination.

(ii) This was often correct.

(iii) Many candidates correctly used their solution for x in the first of the displayed differential equations. It was good to see that very few candidates attempted to construct a differential equation for y.

(iv) Few candidates gave a complete argument. Many candidates' arguments liberally used ∞ as a number without much success, often wrongly claiming that $y = \infty$ meant that y was bounded. A few gave very clear and concise arguments.

(v) The particular solutions were often hampered by candidates not realising that one of their arbitrary constants had to be zero, but follow through marks were given when appropriate. The expressions for x and y as $t \to \infty$ were sometimes correct, but often were not consistent with the previous solutions, e.g. positive exponential terms disappearing without comment.

4761 Mechanics 1

General Comments

Although many of the candidates obtained high scores and many scored full marks to several of the questions, there were quite a few who struggled to make much progress with any questions other than Q8 (i), (ii) and (iii), which were done correctly by most. All of the questions were answered well by many of the candidates but Q4, Q5 Q6 and Q7 tended to attract very good or quite poor answers, very often from whole centres.

As in previous sessions, there were quite a few candidates who, when stuck, seemed to be trying almost everything they knew about mechanics instead of trying to identify the principle that might apply to the situation. It is always a pity when this happens as the candidate may well know much more about mechanics than is being passed on to the examiner.

There were two examples this time of questions that many candidates didn't quite finish, presumably because they forgot exactly what they had been asked to do. In Q4 (ii), many candidates found the time taken for the particles to collide but didn't go on to find the distance travelled. In Q8 (v), many candidates showed that the velocity was zero at certain times but did not go on to find the position of the particle at those times. Candidates should always re-read a question when they think they have finished it to be sure they really have done so.

It was a pleasure to see so many well presented and clearly argued solutions to the questions from candidates who had clearly developed a very sound understanding of the principles and techniques required for this unit.

Comments on Individual Questions

Section A

1) **Newton's second law applied to vertical motion**

(i) This was done well by many candidates. The most common error was to omit the weight term. A few candidates used $F = mga$.

(ii) Again, this was done well by many candidates, the most common error being to omit the weight term.

(iii) Quite a few candidates 'started again' instead of using their result from (ii). The most common error, apart from omitting the weight term, was to resolve the acceleration term instead of the tension term.

It was interesting that many candidates omitted the weight term from only one or two of the three parts. Many of those who 'started again' in part (iii) did not include or omit the weight term consistently with their attempt at part (ii).

2) **Magnitude, direction and a scalar multiple of a vector**

(i) Very few candidates scored this mark. The most common mistake(s) were to omit arrows or not to indicate unit vectors **i** and **j**.

(ii) Most candidates correctly calculated the magnitude of the vector. Calculation of the bearing proved more challenging but there were very many correct answers. Relatively few candidates used an incorrect trigonometric ratio but many, working without using a diagram, did not do the right thing with the angle they found; 90 + 38.7, 180 + 38.7,
180 – 51.3 and 360 – 38.7 etc were commonly seen.

(iii) Most candidates wrote down the answer correctly but quite a few spent a long time establishing the answer approximately by working out the components of a vector with three times the magnitude and the same direction as the given position vector.

3) **Newton's second law and kinematics with constant acceleration, both in vector form**

(i) Most candidates applied Newton's second law correctly but quite a few made arithmetic slips and many gave the magnitude of the force instead of the vector as their answer.

(ii) There were many correct answers but also a lot of mistakes were seen. Some candidates did not use an appropriate constant acceleration formula to find a displacement and of those that did, many then forgot to take account of the initial position. Many made arithmetical errors in evaluating their expressions. The candidates who used a calculus approach very often started off integrating the velocity at $t = 0$ once instead of the acceleration twice.

4) **A kinematics problem in a line where one particle catches up with another**

(i) Most candidates managed to obtain expressions for the distances travelled by P and Q but some clearly didn't understand what is meant by 'an expression …… at time *t* seconds'. Many candidates gave the position of Q instead the distance travelled.

(ii) Most candidates did attempt an equation connecting the distances travelled but many omitted the original separation of 125 m. Many candidates found the roots of a three term quadratic accurately. A common mistake was to give the time at which the particles collide as the answer instead of the distance travelled by P. Some candidates did not know what to do and quite a few of them wasted a *lot* of time on an iterative approach. They first found out how long it took P to get to the starting point of Q and then calculated how much further Q had gone in that time. They then repeated the process until they gave up.

5) **Tension in the coupling between two accelerating connected boxes**

This question was not generally done well although many perfect solutions were seen. Most of the candidates who knew what to do first found the acceleration by considering the overall motion and then applied Newton's second law again to the motion of one of the boxes. A smaller number set up the equations of motion for each of the boxes in terms of the common acceleration and the tension in the coupling and then solved their equations simultaneously; with this approach there were sometimes inconsistencies with signs or incorrect solution of the equations. With either approach there were misuses of Newton's second law, the most common being to write it as $F - mg = ma$, a formula presumably learned to solve a problem involving vertical motion and misapplied here. As always with connected body problems, quite a few candidates do not realize that Newton's second law is involved and do complicated things with the weights and the resistances.

6) **The range of a projectile**

(i) Most candidates made a good attempt at this part. The most common mistakes were to take the initial vertical component of velocity to be 40 instead of 40 × 0.6 = 24. Few candidates confused sine and cosine. The definition of the angle as $\sin\theta = 0.6$,
$\cos\theta = 0.8$ seemed unfamiliar to some candidates.

(ii) Most candidates had a suitable plan. Those who tried to find when $y = 0$ were usually accurate. Most of those found the time to the highest point did this accurately but many then forgot to double this time to get the time of flight. Many candidates who tried to use the formula for the range either misremembered it or struggled to find 2θ or wrongly used the form for the maximum horizontal range. Candidates are usually better advised to work the given problem than to try to apply formulae of this sort.

Section B

7) **The static equilibrium of a box supported by two strings**

It is pleasing to be able to report that there were many complete solutions to this question. The candidates from many centres maintained a high level of accuracy and, generally, there were fewer instances of sine and cosine being used wrongly in resolution.

(i) Most candidates found at least two of: 'light string', 'continuous string' and 'smooth pulley'. The most common error was to think that the system being in equilibrium was a reason.

(ii) This part was often not done as well as part (iii). The most common errors came from candidates producing circular arguments that started by assuming the symmetry that they were trying to establish. The most common method was to resolve horizontally but many candidates failed to say where their equation had come from. There were a few nice arguments based on the triangle of forces.

(iii) This was done accurately and efficiently by many candidates. Quite a common mistake was to consider only one string and then try to fudge finding an angle that has a sine greater than 1. A few obtained the complementary angle.

(iv) Most candidates recognised that in this situation there could be different tensions in the two strings. Quite a few candidates did not understand how to set up the equations for horizontal and vertical equilibrium and instead tried to write down expressions for the tension in one string that did not involve the tension in the other (these expressions were based on no stated principle). Some candidates tried to resolve in directions that included only one string; most of these seemed to think that this was achieved by resolving parallel to each string instead of perpendicular to each string.

However, it was a pleasure to see very many candidates writing down two correct equations from resolution and then solving them correctly. There were many more who obtained the correct equations but could not solve them correctly; many of these had not spotted or used the simplification that cos 45° = sin 45°.

8) **The kinematics of a particle travelling along a straight line with non-constant acceleration.**

Most candidates realized that calculus was required and did well on parts (i), (ii) and (iii). As expected, only the stronger candidates could see how they might apply the answers to the earlier parts to help them with parts (vi) and (vii).

(i) This was answered correctly by almost every candidate.

(ii) Most candidates differentiated and did so correctly.

(iii) Most candidates differentiated but a surprising number obtained $a = 6 - 6t$ or $a = 6 - 16t$ instead of $a = 6 - 12t$. Some candidates who had correctly differentiated x to get v for part (ii) now integrated x in this part.

(iv) Although many candidates recognized this as a standard problem others did not and simply produced a table of some values of v in the interval; the conclusion of these candidates was almost always that the maximum value was 36 m s^{-1}, the value at $t = 0$ and $t = 1$. Quite a few candidates only gave the time at which the maximum value of v is obtained, not the value.

(v) Most candidates knew what to do and many did it correctly. Quite a few candidates who used the factorization method made sign errors; very few of those who used the substitution method established that there are *only* two times at which $t = 0$. A surprisingly large number could not correctly evaluate x when $t = -2$. Quite a few candidates did not make an attempt to evaluate x at the given times – presumably they forgot that they had been asked to do so.

(vi) Very few candidates realized that they were looking for a *distance travelled* in a situation where the particle changed direction and so many found the displacement instead. Quite a few candidates did not seem to have a plan.

(vii) A few candidates produced complete and well argued solutions. However, many candidates did not make it clear what they were trying to do; those who considered a sketch graph usually made at least some progress. Some candidates tried to solve the cubic equation $x = 0$ with false methods. Others used information obtained earlier in the question and looked for sign changes; many of these were successful. Many candidates thought that the answer must be three because x against t is a cubic curve. Some thought that the answer is zero because the graph of x against t does not pass through the origin.

4762 Mechanics 2

General Comments

Many good scripts were seen in response to this paper with the majority of candidates able to make some progress worthy of credit on every question. One disappointing aspect was the poor algebraic manipulation of equations and the inability of a large number of candidates to manipulate surds. The majority of candidates seemed to understand the principles and could use them efficiently. However, some did not clearly identify the principle or process being used; as has happened in previous sessions, those parts of the questions that were least well done were those that required an explanation or interpretation of results or that required the candidate to show a given answer. Those candidates who drew good diagrams were generally more successful than those who didn't. The majority of candidates appeared to have no problem in finishing the paper but a small number gave the impression of having had to rush the last part of question 4.

Comments on Individual Questions

1 Many candidates obtained significant credit on this question, particularly in part a). However, those who failed to draw a diagram or failed to indicate the positive direction in some other way did not always include an adequate indication of direction in their answers.

(a)(i) This part was well done by many candidates. However, only a minority remembered to indicate the direction of the velocity of B.

(ii) This part posed few difficulties with the majority of candidates gaining full credit.

(iii) Well done on the whole but the direction of B was not always clearly indicated.

(b) Many candidates found this part difficult. Some failed to appreciate that they had been given the components of a velocity vector; tried resolving (which then led to complicated algebraic expressions that could not easily be simplified); others took *u* to be the initial velocity and *v* the final velocity and thus lost the thread of the argument because they were using the same notation to represent different things. It was common to see $\tan\alpha = \frac{v\mathbf{j}}{u\mathbf{i}}$ indicating a lack of understanding of vectors.

2) Many candidates scored highly on this question. They appeared to understand the principles and processes required and could structure their answers well.

(i) The majority of candidates obtained full credit for this part of the question. Those who did not, but used column vector notation, generally made fewer errors than those who calculated the co-ordinates separately.

(ii) It was pleasing to see many good responses to this part with almost all candidates able to draw a diagram showing the centre of mass vertically below D. However, some diagrams were too small to be helpful to the candidate; the lengths were not indicated clearly or correctly and so, some of the triangles used to find the required angle had little connection with either CD or the vertical.

(iii) This part posed few difficulties.

(iv) Most candidates approached this part well, appreciating that the *x* co-ordinate of the centre of mass had to be zero and then taking moments using their answer to part i). A small number of candidates went back to square one and attempted to find the moments of all of the individual elements again. Unfortunately, many who did this omitted one element.

3) This question was found hard by many candidates; some found the framework difficult, others had difficulty with part b). Few had difficulty with both parts.

(a)(i) The quality of the diagrams offered for this part was poor. In many cases labelling was incomplete or omitted altogether and spurious reaction forces often appeared at A and B. Arrows were not always as clear as they could be as some candidates adjusted their diagram in the light of the answers they obtained for the next part of the question.

(ii) The vast majority of candidates understood that they had to consider equilibrium at the pin joints to obtain the internal forces. However, there were many inconsistencies seen between the equations offered and the diagram drawn in (i). These inconsistencies were compounded by sign errors in solving the equations and, indeed, in following through. Algebraic manipulation of equations was poor, for instance many cases of $T \sin 30 = L$ leading to $T = ½ L$ were seen.

(b) This part of the question required candidates to *show* a given answer. Those candidates who gained some credit for it usually drew a fully labelled and clear diagram showing all of the forces correctly, including the direction of the frictional force and the forces due to the hinge at Q. However, some candidates assumed that the weight of the structure (i.e. $2W$) acted at Q; others showed a weight W acting in the centre of the structure. In order to show the given result candidates had to consider one beam, take moments and resolve. Many chose to take moments about Q but forgot the normal reaction force at R. Full explanations of the processes and principles being employed were rare. On some scripts no consistent sign conventions were adopted and on others algebra was not used properly to establish the result.

4) Some excellent answers were seen to this question with few candidates attempting to use, inappropriately, the constant acceleration formulae and Newton's second law in part (a).

(a)(i) Many candidates had difficulty with this part. Few could explain clearly that the tension was perpendicular to the motion of the sphere and hence did no work on it.

(ii) This part posed few difficulties to the majority of candidates.

(iii) Few had problems with this part.

(iv) Most candidates could go some way to obtaining a solution to this part. The main errors were in the calculation of the work done, with some candidates not appreciating the circular nature of the motion. It was common to see 0.6×0.4769 used as the work done.

(b) The majority of candidates could obtain some credit for this part. However, marks were lost in many cases because the candidates did not work precisely enough. They used decimals throughout but then stated the answer as a surd without any attempt to justify the agreement.

4763 Mechanics 3

General Comments

Most candidates for this paper were able to demonstrate a sound understanding of the topics being examined. They generally seemed to have sufficient time to complete the paper, and they presented their work clearly. They certainly found this paper to be considerably more difficult than last year's; about a quarter of the candidates scored more than 60 marks (out of 72) and about 20% scored fewer than 30 marks.

Comments on Individual Questions

1) *(Dimensional analysis and elastic energy)*
This was the best answered question, with an average mark of about 15 (out of 18).

(a)(i) Almost every candidate gave the dimensions correctly.

(a)(ii) Almost all candidates knew how to find the dimensions of λ, although there were a few slips here.

(a)(iii) This was generally well done. Just a few candidates did not seem to realise that *g* was an acceleration.

(a)(iv) The method for finding the indices was well understood, and very many candidates carried it out accurately. Most obtained $\beta = -2$ correctly, but a very common error was to equate the power of *L* to zero instead of one. Some made slips when solving the simple simultaneous equations which they had obtained.

(b) Candidates who considered energy were quite often successful, although some forgot to include the kinetic energy. Many obtained the correct vertical displacement but did not calculate the required distance OA from it. However, a surprising number of candidates did not consider energy, and there were many failed attempts involving the tension.

2) *(Circular motion)*
The average mark on this question was about 10.

(i) Almost all candidates found the angle correctly.

(ii) Most candidates found the speed correctly. Common errors were taking the radius of the circular motion to be equal to the length of the string, and trying to resolve in the direction of the string instead of horizontally.

(iii) Very many candidates found the tension correctly, although a common error was to omit the component of the weight or to include it with the wrong sign.

(iv) It was not clear to all candidates that they should consider kinetic and potential energy here, and there were many unsuccessful attempts to derive the result from the radial equation of motion. Those who did consider energy were very often successful; the given answer enabled many candidates to correct errors which they had previously made.

(v) A lot of candidates assumed that the string would become slack when the velocity is zero. Those who did consider the tension were often successful, although here it was very common for the component of the weight to have the wrong sign, presumably because candidates did not appreciate that the term $mg\cos\theta$ would be negative when θ is obtuse.

3) *(Simple harmonic motion)*
The average mark on this question was about 10.

(i) Most candidates gave the tensions correctly.

(ii) This was generally done well, and the given answer was undoubtedly very helpful to many candidates who were able to adjust signs appropriately. The most common error was to omit the weight from the equation of motion.

(iii) This was usually answered correctly, but a common error was to assume that the tensions in the two strings would be equal.

(iv) The period was found correctly by most candidates.

(v) The methods for finding the amplitude and the maximum speed were well understood.

(vi) Although most candidates knew that they should use equations such as $v = v_{\max}\sin\omega t$, errors in signs or selecting the wrong solution to a trigonometric equation usually resulted in an incorrect answer.

4) *(Centres of mass)*
The average mark for this question was about 12.

(a)(i) The method for finding the centre of mass of a solid of revolution was well known, and most candidates noticed that the revolution was about the *y*-axis and adapted the formulae appropriately. However, a very common error was to have limits of integration (with respect to *y*) 0 to 2 instead of 0 to 8.

(a)(ii) The principles involved here were well understood, although a surprising number of candidates thought that the radius of the face in contact with the plane was 1 instead of 2.

(b) Finding the centre of mass of a lamina was done rather better than the solid of revolution in part (a)(i). Some candidates lost a factor ½ but the main errors were algebraic, such as expanding $\tfrac{1}{2}(8-2x^2)^2$ incorrectly.

4764 Mechanics 4

General Comments

Many candidates demonstrated a good understanding of Mechanics and high levels of algebraic competency. Approximately half of the candidates found the work on rotation difficult, and for these candidates question 3 was clearly their weakest question.

Comments on Individual Questions

1) (i) Most candidates realised that they had to consider momentum over a small increment in time (rather than simply write down Newton's second law). However, many had sign inconsistencies between the small change in mass and the rate of change of mass.

(ii) This was often done very well, although some candidates made errors in their integration. Some made errors when calculating the arbitrary constant and a few even omitted the constant.

(iii) There were many correct solutions.

2) (i) Most candidates were able to set up a correct differential equation, although some jumped to the printed version with insufficient working. Solving the differential equation presented some problems, but many candidates correctly used the standard result as given in the formula book.

(ii) Most candidates realised that at the terminal velocity the acceleration is zero and correctly substituted the value found into the expression for x.

3) (i) Finding the expression for the mass was usually done well, although a few candidates did not know how to set up the appropriate integral. Most candidates who found the mass went on to find the moment of inertia correctly.

(ii) There were many candidates who did not realise that they needed to use angular momentum. Of those who did, most equated the change in angular momentum to impulse, rather than moment of impulse.

(iii) Some candidates attempted to use energy, without success. Many did use the equation of motion, but some made errors when calculating the acceleration.

(iv) Some candidates produced good solutions for this part. Some were unable to solve their differential equation. Some attempted to set up and solve a second order equation for θ (which is beyond the requirements of the specification). A few could not even write down an appropriate equation of motion. Some unsuccessfully attempted to use energy.

(v) Of those who made some progress with the previous part, many realised that the model predicted that the angular velocity never reaches zero.

4) (i) Most candidates completed this correctly, but some seemed to be led by the printed answer and others mistakenly thought the given expression was the potential energy.

(ii) This was often correct, but sometimes lacked clarity, in particular not giving clear conclusions based on the relevant working.

(iii) Very few candidates appreciated what this part was asking. Most argued about the effect of friction on the whole system. The question specifically asked about the expression for *V*, and very few realised that the tension in the string would not be constant, hence invalidating the EPE term.

(iv) Many correct solutions were seen.

(v) The sketch was often done well.

(vi) There were many good answers seen, but many wrongly assumed that a negative θ meant that the string was slack.

4766 Statistics 1

General Comments

The standard this summer was variable. There were some excellent scripts seen by the examiners reflecting the hard work and dedication of teachers, lecturers and candidates. On the other hand there were a substantial number of candidates who seemed totally out of their depth who struggled to make any real progress.

Candidates should be reminded to work with total accuracy and not to round their answers severely as they progress through a calculation.

It was pleasing to see that a number of centres had acted on comments made in previous reports particularly with regard to the definition of p in the construction of hypotheses.

Comments on Individual Questions

1) The calculation of an estimate of the mean and standard deviation of grouped data presented unexpected problems for a sizeable number of candidates. Often 2 or more mid-points of the classes were incorrect thus throwing out any possibility of achieving the accuracy required. A common error even by the better candidates was to use mid-points of 1.5, 4.5, 6.5, 8.5 and 15. Some candidates had little idea how to obtain the mid-points and thought that the mean could be somehow calculated from multiplying the frequencies by the class widths or the frequencies by one of the boundary values. It was disturbing to see many candidates attempting to work out the standard deviation without using any frequencies. This is clearly a topic which deserves more attention to precision and process for the future.

   The concept of finding the upper boundary for any outliers was well known in terms of mean + 2 standard deviations but several tried to argue the case with $Q_3 + 1.5$ IQR (not that these data were available) or insisted using mean + 1.5 standard deviations. Candidates should be careful not to make rash statements such as 'there **are** outliers in the data' but instead be more circumspect and claim that 'there could be or may be some outliers in the final class'.

2) The work on testing for independent events was pleasing with a variety of methods used by candidates. Most went down the route of showing numerically that $P(W) \times P(C) \neq P(W \cap C)$ and hence the events were not independent. Some tried their luck with non numerical or qualitative attempts but to little avail.

   The Venn diagram was, unfortunately, often lacking in credibility. There are still too many candidates filling in the various regions with the incorrect probabilities. The region $W \cap C'$ was often given as 0.2 instead of the correct 0.14 and likewise the other region $C \cap W'$ was written as 0.17 instead of the correct 0.11. The region $W \cap C$ was invariably correct as 0.06. A curious number of candidates often labelled the region $W' \cap C'$ as 0.63 instead of the correct 0.69. Again, this is an area that deserves the attention of candidates for future examinations.

   The calculation of P (W/C) was well attempted and most scored 2 marks. The conclusion was usually sound but many did not choose their words carefully and quoted '**more** children speak Welsh' when really they meant 'the proportion of children speaking Welsh is higher.'

3) (i)(*A*) Many candidates had difficulty composing an equation which included p + q and a summation to 1.

(*B*) A little better, with some realising that the equation for E(X) must now include 2p + 3q.

(*C*) The solution of the resulting simultaneous equations seemed to be off the mathematical radar for many candidates with many struggling to find solutions for p and q.

(ii) The variance was usually calculated correctly bearing in mind that a generous follow through was applied for those candidates who did not find the exact values of p and q earlier. The only common error was the omission of 0.67^2 leaving an answer of 1.07.

4) This was a popular question which was well answered by many candidates. In (i) part (A) most gained the correct answer of 0.6634 but then did themselves no favours by unnecessarily rounding the answer to 0.66. Part (B) was well answered but there was some confusion about the meaning of P(X>1). Some believed it to be 1 – P(X=0) rather than the correct form of 1 – {P(X=0) + P(X=1)}. In the last part, most knew the E(X) = np formula and gained the marks, even on follow through.

5) Candidates need to be reminded that a hypothesis test on the binomial distribution requires an initial set up of the following conditions.

- The definition of the parameter p, in context
- The use of the correct notation for H_0 and H_1, namely in the case of this question that H_0: p = 0.35 and H_1: p > 0.35
- A clear explanation, in context, of why H_1 takes the form that it does.

Unfortunately, many omit the requirements of the first and last bullet points, thus losing 2 valuable marks. It is worth reminding centres again that sloppy or poor notation such as H_0: P(x = 0.35) and H_1: P(x > 0.35) is penalised by the examiners. Too many candidates are prone to this form of notation.
Many otherwise worthy initial set ups were spoilt by candidates using point probabilities or selecting the wrong tail. It was not uncommon to see $P(X \geq 8) = 0.0422$ when, in fact this was $P(X \geq 9)$. The correct solution required $P(X \geq 8) = 1 – P(X \leq 7) = 1 - 0.8868 = 0.1132$. Some candidates wrote ridiculous statements along the lines of 0.9578 > 5%. It must be emphasised, once again, that the tail probability must be compared with the significance level of the test. All further marks in the question are dependent on this important fact. The next stage is to accept or reject H_0 and then reach a valid conclusion in context.

6) There were many successful attempts to the first half of this question. Candidates were able to demonstrate a good understanding of probability calculations using their tree diagrams.

Part (A) was invariably correct as 0.04. Most were able to achieve 0.9559 in part (B) by adding the 5 separate probabilities but very few candidates realised the quick way to achieve the answer by $1 - 0.21^2 = 0.9559$. A common error in part (B) was the omission of the 0.79^2 term giving 0.3318 as an answer.

In part (C) most candidates preferred to list and add the 4 probability terms to gain 0.9801. Relatively few spotted the quick way of 0.99^2 would reach the same answer. Some candidates made the error of believing that neither of the people was born overseas could be calculated from $1 - 0.01^2 = 0.9999$. The conditional probability in part (ii) elicited some very good responses with most realising the correct method although some did write (0.04 x 0.9801)/0.9801with depressing regularity.

Only the better candidates made any progress in part (iii) with many finding $1 - 0.79^5$. Some candidates had become muddled by this stage and it was not uncommon to see $1 - 0.21^5$ or even $1 - 0.9559^5$. The latter two methods did, however, attract a partial award. Part (iii) (B) was often well attempted by the better candidates with equally as many opting for using logarithms as for using a trial and improvement method.

7) Part (i) was almost invariably correct with the response of positive skewness.

Part (ii) was well tackled with many achieving the answer of 950 000 but some candidates left their answer as 950 and lost a mark.

Many reached the required cumulative frequency of 2150 (thousands) via 1810 + 340 but there were instances of 1810 + 345 seen by the examiners. Almost all candidates were able to locate the position of the median as the 1385 or 1385½ value. Only the very talented candidates were then able to carry out the linear interpolation of $30 + \frac{145}{570} \times 10 = 32.54$, to achieve the median age.

It was pleasing to see many successful attempts at finding the frequency densities in part (iv). Without doubt, the frequency divided by class width was the most popular method but other strange but nevertheless correct methods were seen. The resulting histogram was well drawn but some candidates did make life difficult for themselves by choosing a bizarre scaling (e.g. 3cm = 10 units on the vertical axis).

The comments in part (iv) were often not what the examiners were looking for. Many opted to compare numbers across the two histograms but it should have been evident that **all** the populations for **each** age group were higher in Outer London than Inner London. Some candidates did pick up on the salient points of the two histograms by comparing the different modal classes (20 – 30 for Inner London; 30 -40 for Outer London). In making comparisons it is advisable that candidates mention proportions rather than refer to 'more than' or 'less than' statements.

Part (vi) elicited some positive responses with many realising that the mean, midrange and standard deviation would all increase in the light of the new information. Some thought the standard deviation would decrease rather than increase but most knew the interquartile range would be unchanged.

4767 Statistics 2

General Comments

In keeping with recent sessions, the majority of candidates demonstrated good understanding and high marks were plentiful. No one question, or part of a question, stood out as being particularly difficult or particularly easy. This year saw a noticeable improvement in answers where some form of explanation or interpretation was required.

Comments on Individual Questions

Section A

1) (i) Well answered. Common mistakes involved premature rounding – rounding of $\bar{y}$ to 1.20, seen frequently. A few candidates omitted the square root in the denominator.

(ii) Well answered. Many obtained at least 5 marks out of the available 6; typically, the lost mark was for failure to define ρ as the population correlation coefficient, despite being asked to 'define any symbols' used. Some candidates lost a mark for failing to provide a conclusion in context. Those candidates providing nonsensical comparisons, for example comparing a negative p.m.c.c. with a positive critical value, were heavily penalised.

(iii) The requirement for an underlying 'bivariate Normal distribution' for the test to be valid was not widely known. Candidates were more familiar with the idea that the points in the scatter diagram should fall within an elliptical shape. Many candidates commented that the test would be valid if the scatter diagram showed negative correlation.

(iv) A good variety of comments were seen. Very few gained full marks. Many scored a mark for pointing out that there may be other factors involved. Comments relating to 'causation' were less common. Many candidates commented on dependence/independence without fully answering the question. A mark was available for those who pointed out that higher spending on education could be due to lower population growth rate, i.e. reversing the dependency.

(v) Reasonably well answered, but little credit was given to vague answers – e.g. simply stating that the a smaller sample size leads to a 'less accurate' result gained no credit, but pointing out that a smaller sample could be 'less representative of the population' gained a mark. Candidates are advised to use statistical words/reasons in their comments as much as possible.

2) (i) (*A*) Well answered.

(i) (*B*) Accuracy errors often prevented candidates achieving the provided answer. Few candidates showed awareness that, with the printed answer being correct to 4 decimal places, working should have included, ideally, values showing 5 decimal places. Candidates should be aware that when answers are given, sufficient working must be seen.

(ii) A significant minority failed to use the appropriate binomial model. For those who did, many simply found $P(X=1)$ then stopped. A common failure was omitting ${}^{30}C_1$. Those using Po(0.192) did so successfully in most cases.

(iii) Most candidates managed to use Po(3.7) tables to get the correct answer. The most common failure was saying $P(X > 8) = 1 - P(X \leq 7)$.

(iv) Most obtained N(74, 74) but other parameters were seen (e.g. $\sigma^2 = 200 \times 0.37 \times 0.63 = 46.62$). Knowledge of the need for, and how to apply, a continuity correction was lacking. Other frequent mistakes were use of $\sigma = 74$, not $\sqrt{74}$, and failure to use tables to a sufficient level of accuracy [i.e. $1 - \Phi(1.918)$ not $1 - \Phi(1.92)$].

(v) Many candidates recognised the correct method. A significant number simply thought that the answer was the same as that given in part (iv).

3) (i) Well answered with most obtaining $z = \pm 0.625$. A sizeable minority gave 0.266 as their answer (from using the wrong tail).

(ii) Nearly all showed an intention to use B(10, 0.734). [or B(10, their (i))] As in Q2, a common failure was omitting the ${}^{n}C_r$ term, here ${}^{10}C_7$. A few had just 10 instead of ${}^{10}C_7$. Sometimes the powers of p & q were reversed even though part (i) was correct.

(iii) Most found the correct $z = 2.326$ from the Inverse Normal table. Many proceeded to use this value of z and produced an incorrect upper tail value. A few bypassed Inverse Normal tables, using $(x - \mu) / \sigma = \pm 0.99$ or ± 0.01.

(iv) Hypotheses were usually correct. A precise definition of μ was lacking in most cases.

(v) Generally, well done. Those omitting $\sqrt{15}$ when standardising were heavily penalised. A number of candidates obtaining a test statistic of 1.094 went on to make a nonsensical comparison (e.g. 1.094 > 0.05 or 0.863 < 1.645). Though other approaches were seen, the majority of candidates kept to the approach in the published mark scheme.

4) (i) Well answered. A small number omitted the context from their hypotheses. Very few mentioned correlation or tried to use parameters in their hypotheses.

(ii) Well answered, although accuracy was an issue for a number of candidates. Obtaining the expected frequency of 9.975 was not a problem, but some rounded this to 9.98 which does not lead to the provided answer.

(iii) Well answered on the whole. Frequently seen mistakes included: use of a 2 ½ % critical value, using the wrong number of degrees of freedom, using $X^2 =$ 4.8773 instead of the value provided. For some reason, many candidates felt it necessary to calculate the test statistic themselves, despite it being provided in the question – there was no evidence that this prevented those candidates from finishing the paper.

(iv) Fully correct answers were seen, but despite the question stating clearly, "…compares with what would be expected…", many comments written did not address this. Many compared one expected frequency with another expected frequency, or one observed frequency with another. Some only made comments about the various contributions to the test statistic; of these, some recognised that the high contributions indicated some 'difference' between observed and expected, but did not explain whether there were more observed than expected or fewer. The candidates' attempts at this question were better than attempts at similar types seen in recent sessions.

(v) Well answered. Many were able to multiply the correct three fractions and score full marks. Putting the grand total on the bottom instead of the column total was, however, a fairly frequent error.

4768 Statistics 3

General Comments

There were 348 candidates from 72 centres (June 2007: 323 from 64) for this sitting of the paper. Once again the overall standard of the scripts seen was pleasing, on the whole, and many candidates appeared well versed in the content of this module. However Question 1 (b) (Sampling) was conspicuously badly answered. Also, candidates continue to display poor regard for clear and accurate notation in their work. Furthermore, following a gradual improvement in recent sessions, it was disappointing to see a marked deterioration in the quality of the language used in the conclusions to hypothesis tests.

Invariably all four questions were attempted. There was no evidence to suggest that candidates found themselves short of time at the end of the paper.

Comments on Individual Questions

1) **Continuous random variables; Sarah at the bus stop.**
Sampling; attitudes to pollution.

(a)(i) Almost all candidates started well by finding the value of k correctly. The sketch graphs that followed were usually correct too, with only occasional flaws. However the interpretation produced varied responses. In many cases it appeared that they had not read the context carefully enough, and so there were comments such as “the earlier Sarah arrives the longer she waits” with no mention of probability at all.

(ii) It was disappointing to see so many attempts to find the c.d.f. that were flawed by the absence of appropriate limits as part of the integral. Even when limits were present the notation was not, strictly speaking, correct. Perhaps more worrying was the number of instances where the c.d.f. was not used in the subsequent work. These candidates chose to repeat the integration (with limits this time).

(iii) There were many good answers to this part, though, as in the second half of part (ii), a noticeable number of candidates started off by doing yet another integration.

(b)(i) All three parts of part (b) were poorly answered.
Most candidates could not define the term “simple random sample”. Instead they wrote at length about sampling frames and random number generators.

(ii) There was considerable confusion between cluster sampling and stratified sampling; more often than not a description of the latter was given.

(iii) Once again answers were badly thought out. Many thought the population was the pupils at a particular school rather than secondary schools in general, and many believed that the issue was to do with trying to get a representative sample.

2) **Combinations of Normal distributions; confidence interval for a population mean from a large sample; resistances of resistors.**

(i) Most, if not all, candidates answered this part successfully.

(ii) This part was also well answered. There were only occasional problems with the variance of the sum.

(iii) This was another well-answered part.

(iv) Many candidates found difficulty with this part. Mostly they had problems in interpreting the requirement symbolically as a two-sided inequality. There was also the matter of finding the correct variance.

(v) There were many right answers to this part. Wrong answers resulted when candidates did not manage to locate and use the correct percentage point for the confidence interval. The final interpretation was usually appropriate.

3) **The *t* distribution: paired test for the population mean difference; Wilcoxon paired sample test; yields of tea plants and scorings of tea tasters.**

(a)(i) As in the past, the hypotheses were not well expressed in many cases. There seemed to be a reluctance to use the standard notation, μ, for a population mean difference. The necessary assumption often lacked the words "differences" and/or "population".

(ii) Usually the correct test statistic was found with little trouble. Most candidates identified the correct number of degrees of freedom, but not always the correct percentage point. As mentioned in the introductory comments to this report, the final conclusion to the test was less than satisfactory much of the time. Conclusions should be in context, contain a sense of "on average" and not be assertive. For example "The evidence suggests that there is no difference between the mean yields of the two types of tea plant."

(b) There were very many good solutions to this part of the question. However there was also evidence of confusion in the minds of some candidates. "$W_{test} > W_{crit}$ therefore the result is significant" was seen quite a few times. Also a small number of candidates wrote that ν (degrees of freedom) = $n - 1$, with the consequence that they looked at the wrong row in the tables of critical values. Final conclusions suffered the same faults as in part (a).

4) **Chi-squared test of goodness of fit; confidence interval for a population mean from a small sample; feeding habits of bees.**

(a)(i) It was extremely disappointing to see the relatively large number of candidates who were unable to calculate the variance of the data.

(ii) In contrast, much good work was seen in the calculation of the expected frequencies and the test statistic. However an appreciable number failed to find the frequency for $X \geq 8$ correctly and/or to combine cells appropriately.
No significance level for the test was given in the question, leaving it up to candidates to choose. It was hoped that they would notice both this and the fact that the test statistic turned out to be not significant whatever their choice, and that they would make a comment to that effect. Hardly any such comments were made.

(b) Many answers to this part were spoiled because a percentage point from the Normal distribution was used instead of one from t_9. Note that on this occasion the allocation of the marks for the confidence interval was adjusted to place more emphasis on the need to use the t distribution. As a result of this, the other details of the structure of the interval received less credit since the sample mean and standard deviation were given in the question.

4769 Statistics 4

General Comments

This is the third occasion on which the new-specification Statistics 4 module has been sat. There were 24 candidates from 10 centres. This rather small number is a disappointing reduction from the previous two years. There were several more candidates who had registered for the examination but in the event were absent.

The paper consists of four questions, each within a defined "option" area of the specification. The rubric requires that three be attempted. All four questions received many attempts, which is encouraging as it indicates that centres and candidates are spreading their work over all the options. The least popular of the questions was Q.1, on estimation theory, but even this received several attempts, some of which were highly successful. Overall, there was some extremely good work, but it has also to be reported that there was some work distinctly at the poorer end of the spectrum.

Comments on Individual Questions

1) This was on the "estimation" option. It was based on maximum likelihood estimation for a Poisson distribution.

   Most of the candidates who attempted this question did well. Unfortunately there were a few who simply did not know how to form the likelihood, though these were usually able to recover from part (ii) onwards, where another estimator was introduced for comparison. It was good that most of the candidates who dealt well with part (i) were aware that it needed to be checked that the turning point found was indeed a maximum.

   Moving onwards, the new estimator was usually dealt with successfully, and candidates knew how to find its relative efficiency with respect to the maximum likelihood estimator. Showing that this expression was always less than 1 and considering its limiting behaviour taxed the mathematical ingenuity of some candidates, but most had a reasonable idea of what to do.

2) This was on the "generating functions" option and was based on the geometric distribution.

   Many candidates proceeded thoroughly and carefully through the technical mathematical work, though this was one of the places where faking of answers was too common. The explanation in part (ii) was often somewhat sketchy, though usually sufficient to indicate that the candidate understood near enough what was happening. Most candidates knew the convolution theorem result at the end of part (ii) and could also straightway write down the mean and variance of the sum, but it was *very* disappointing, at Statistics 4 level, to find quite a few thinking that the variance of the sum had a factor of n^2 rather than n.

   In part (iii), not quite everybody realised that the approximation was simply a Normal distribution and, of those that did, some took it to be N(0, 1) even though they had explicitly obtained the mean and variance immediately before. Further difficulties arose in part (iv) where some candidates did not really know what to do and where use of a continuity correction was rare. It was however pleasing that many candidates gave a sensible reason why the model might not be appropriate, usually based on lack of independence for groups of passengers travelling together.

As some of the remarks above are somewhat critical of candidates' work, it is fitting to add that there were many very good attempts at this question.

3) This question was on the "inference" option, exploring ideas of Type I and Type II errors and the Operating Characteristic.

The opening explanations were usually correct, though perhaps it was inevitable that some candidates would get things the wrong way round. The technical work following in parts (ii) and (iii) was usually done well, though again it was perhaps inevitable that there would be a few errors in setting up the probabilities and/or in reading the Normal tables (the latter really should not occur in a Statistics 4 paper!). In part (iii), consideration of the upper limit 765 was sometimes forgotten; $\Phi(6)$ is indeed extremely near to 1, but it did need to be brought into account. Part (iv) was perhaps less successful; perhaps candidates were not too familiar with obtaining an algebraic expression for the Operating Characteristic. Nevertheless, there were some good solutions here.

4) This was on the "design and analysis of experiments" option.

It is extremely pleasing that there was some very good work here. Candidates seemed well prepared and to know the material thoroughly. Statements of the model and interpretation of its terms were often impeccable, though some candidates were still not careful enough in including words such as "population" and "independent". The analysis in part (ii) was also usually done well, and it is with much pleasure that I can report that nearly all candidates used the efficient "squared totals" method of calculation rather than the extremely cumbersome and error-prone "s_b^2/s_w^2" method. This is a real improvement from previous years and in particular from last year, when this had become much worse.

Part (iii) introduced a short discussion of "design" features for the experimentation. Most candidates appreciated that the design being introduced was that of randomised blocks and were able to give a good description of the layout.

4771 Decision Mathematics 1

General Comments

As in January the question on algorithms (Q2) caused most difficulty.
Again as in January the answer book worked well.

Comments on Individual Questions

1) **LP**
The question required candidates to identify the (obvious) scales in order to get started and to draw the third line. However, a surprisingly large number of candidates lost marks after adding the third line to the diagram. Problems included a failure to identify the points of intersection, a failure to identify them correctly (many gave (14,9) instead of (9,14)), and a failure to compare the values of the objective function at vertices of the feasible region (accepting that the alternative but rarely used profit line approach does not require this). Candidates needed to display their methodology for finding the optimal point.

2) **Algorithms**
This question was badly done. Even among those who successfully arrived at the answer (14 in both cases) few had fully correct applications of the algorithm.
Very few were able to explain the meaning of quadratic complexity.

3) **Graphs**
Most were able to collect the marks from parts (i) and (ii).
There were some good mathematical answers to part (iii) from candidates who could see how to do the count without making a listing.
Far too few candidates spotted that the answer to part (iv) should be half the answer to part (iii).

4) **Simulation**
This question was answered well.
Most candidates are now well drilled in producing simulation rules.
Applying the final step in this simulation was novel and not trivial, and it was pleasing to see most candidates being successful with it.
In their own simulation many candidates entered their simulated values incorrectly, listing 5 time intervals followed by 5 numbers of passengers.

5) **CPA**
(a) This was a different question to those more usually posed. Most managed part (i), but part (ii) was (intentionally) more difficult. This was intended to give good candidates a chance to show their mettle by realising that the answers depend on x.
(b) This was a mini version of the more usual CPA question. Candidates should have been able to deal with it easily, and most could.

6) **Networks**
Candidates are well schooled in the need to demonstrate that they are applying Dijkstra (part (iii) in this question) − they were less alert to the need to do so with respect to the tabular form of Prim's algorithm in part (i).
The second requirement in part (iii) − to list the arcs in Serena's connector − was intended to be differentiating, and it was.
Part (v) was even more differentiating. Few identified the cubic complexity.

4772 Discrete Mathematics 2

General Comments

Most candidates were able and well prepared, and gave good accounts of themselves. This report focuses on the difficulties encountered, and should not be allowed to detract from the overall success.

Comments on Individual Questions

1) **Logic**

(a) Most candidates were able to use their training in propositional logic to clarify this phrasing (even though a full analysis requires predicate calculus).

(b) There was some confusion here between switching circuits and logical circuits.

(c) Most candidates were able to produce the required four-line truth table correctly.

2) **Decision Analysis**

The essence and value of Decision Analysis is the modelling of the structure of the process − identifying the decisions and the chance outcomes, and getting them in the correct order.

In part (i) there were two independent decisions to be taken, and their order was irrelevant.

In part (ii) one of those decisions was switched to a chance outcome and another decision was introduced. Most candidates failed to introduce an extra decision node to model this. They introduced an extra alternative to the existing decision node, thus failing to capture the logic.

With the changes there is only one correct order. First Jane has to decide whether or not to buy a car. Next comes the chance node for 2 trips or 4 trips. Thence there are the two costings in the event of having bought a car, or the decision between flying and driving if a car has not been purchased.

This is difficult modelling, and only very good candidates were successful with it.

3) **Networks**

(a) Again, candidates were very well prepared for Floyd's algorithm. Most scored all 6 marks.

The most common response to the instruction to draw the complete network of shortest distances was to ignore the instruction.

Nearest neighbour was done well by most, as was the construction of a lower bound. Surprisingly few candidates were able to put into words what a bound means.

(b) A surprising number of candidates ploughed on through this as if they were still being asked about the TSP. The question labelling indicates that this part is different.

Of those that were successful in constructing an inspection route of length 87, few were able to mount a convincing explanation of why it was optimal.

4) **LP**

Candidates were very competent both in the basic techniques of simplex, and, pleasingly, in its extension to two-phase or big-M.

Less pleasing was the almost universal failure of candidates to identify and define their variables. This is the first and essential step of LP and of much other modelling. Furthermore, it is essential that they remind themselves that they are dealing with numerical algebra here by starting their definitions with "Let ··· be the **number** of ···"

A similar point pertains to the interpretation of results. In interpreting a final tableau there are two steps. One is to identify the values of the variables, the second is to say what this means in the "real" world. Thus "b=5 and c=10" is only part way there. The remaining step may seem trivial but is vital in developing those modelling skills − "Make 5 tonnes of product B and 10 tonnes of product C."

4773 Decision Mathematics Computation

General Comments

Most candidates were quite well organised this year. It helps enormously if output is organised into a logical order, and if it is annotated. However, see comments below regarding excesses of output in some questions and lack of evidence in others.

Comments on Individual Questions

1) **LP modelling**

Modelling is always difficult, and this question tried to give as much help as possible to candidates in formulating this set covering problem. However, there were still many who failed to note that the variables were indicator variables.

In part (i) some did not include the “zero distance” village in their constraint. Those that succeeded with the modelling generally succeeded with the rest of the question, though not all dealt with part (iv) by introducing an extra constraint, or at least they did not show any evidence of having done so.

2) **Simulation**

There was a huge problem with responses to this question − the majority of candidates failed to capture the fact that birth and death rates are dependent on population size. By far the most common response was to model with a constant birth rate of 10×0.01 and a constant death rate of 10×0.04. This misses the point!

A subsidiary problem occurred with candidates who attempted to model both births and deaths by using just one random variable, instead of two independent random variables. This is OK provided that the probabilities are correct − they should be $(1-n\alpha)(1-n\beta)$, $n\alpha(1-n\beta)$, $(1-n\alpha)n\beta$ and $n\alpha n\beta$ − but it was not intended for candidates to show this level of skill in probability modelling.

There was a small problem with ensuring that a full minute was simulated. Many candidates started at t=1, and so had simulated only 59 seconds by time t=60.

In parts (ii), (iii) and (v) too many candidates failed to produce evidence of their work, whilst a number produced copious pages of repetitive output. There were still a few candidates who do not ensure that cells are wide enough to display all of the formulae.

3) **Networks**

This was a fairly straightforward two-part question. The question labelling and wording both indicated that, although specified by the same matrix, the problems were different.

In part (a) some candidates encountered difficulties by allowing back flows to the source and/or from the sink. This had the potential to derail the optimisation with, for instance, flows of AB=8, BA=8, AE=3 and EA=3 being preferred to the correct answer.

Some solutions included the redundant constraint AB+AE = CG+FG, whilst a number included all the variables in the objective function.

Part (b) presented no such difficulties. Some printouts lost the end of the objective function (this needed splitting on to two lines). There were inevitably some typing errors in variable names – these could have been identified if candidates had checked the list of variables in their solution output.

Candidates sometimes viewed "interpretation" as rewriting what was plain to see, rather than as interpreting back to the given scenario.

4) **Recurrence relations**

Candidates were largely well drilled in the algebraic solution. The spreadsheet was also well handled.

There were many alternative points made about the limitations of the spreadsheet solution, some valid but of marginal significance, and some false. Not many candidates gave the required answer that, using a spreadsheet, there is a (relatively small) limit to how far one can go into the sequence (accepting that Excel's programming capability could be used to circumvent this constraint).

4776 Numerical Methods (Written Examination)

General Comments

There were many good scripts seen, though as usual there were some candidates who appeared to be quite unprepared for this paper. The best candidates presented their work clearly and compactly, with due regard for the algorithmic nature of the subject. At the other extreme, some candidates presented their work as a jumble of figures, difficult to follow and frequently riddled with errors. It is worth saying yet again that candidates who adopt the latter approach put themselves at a considerable disadvantage.

Comments on individual questions

1) **False position method**

The single step of the false position method was surprisingly often done incorrectly. Even those who got the right answer were often unable to give the maximum possible error. Though this is elementary material the average mark scored was not high.

2) **Difference table**

This is elementary material too, but it defeated some. There were many errors of algebra with a common disregard of the rules for subtracting negative numbers – surely unforgivable at this level. Those who completed the difference table were sometimes unaware of the fact that, for a cubic, third differences will be constant. The more laborious methods used to find *k* were generally not successful.

3) **Central difference method**

The numerical values were generally found accurately, though some candidates worked with the forward difference formula. The differences and the ratio of differences then led in most cases to the right answer.

4) **Newton-Raphson method**

The Newton-Raphson method was familiar to most, though a fair number were unable to completet the simple derivation of the formula. The numbers were generally processed correctly. The explanation of the fact that the method is faster than first order was usually poor (or non-existent). The point is that the *ratio of differences* is decreasing rapidly. Far too many candidates said that the *differences* are decreasing rapidly.

5) **Rounding errors**

Once again, the numbers were generally handled well. Some candidates, however, worked in radians and thereby lost the point of the question. The final part was often not done well and only a handful of answers gained both marks. There are two points to be made here. Firstly, there is the general observation that using mathematically equivalent expressions may result in rounding being performed in different ways. Secondly, there is the specific observation that one of the methods used involves the subtraction of nearly equal quantities, something that is well known for losing significant figures.

6) **Numerical integration**

Most candidates scored well on this question. The values of *M*, *T* and *S* were usually correct. Showing that the differences in *M* values reduce by a factor of 4 as *h* is halved was well done by most. Some then demonstrated rather than stated that a similar result holds for *T*. This got full marks, of course, though it took up candidates' time.

Candidates were required to use the differences and ratio of differences in the *S* values to justify their final answers, and many (though not all) did so.

7**)** **Finding the roots of an equation**

Again, the numbers were handled well in this question, though finding the third root without an iterative formula being given did defeat some. The only part of the question that caused trouble was the comment at the start of part (ii). The iterative formula given is in the form of a square root. The correct comment was that a square root is, by definition, positive so it cannot give a negative answer. A common incorrect comment was that it is impossible to take the square root of a negative quantity.

4777 Numerical Computation

General Comments

The candidature for this paper was, once again, small. Most of the candidates seemed well prepared for the paper and some scored very highly.

Comments on individual questions

1) **Solution of an equation; acceleration**

There was some fudging of the algebra in the first part and nobody scored full marks, but most candidates produced a respectable score.

2) **Romberg's method**

This was the least popular question, though it was well done by those who attempted it.

3) **Runge-Kutta method**

All the candidates who tackled this question new what to do, but some made slips in transferring the new value of *y* from one row to the next in the spreadsheet.

4) **Least squares curve fitting**

Again, all the candidates understood what to do, but there were some errors in solving the equations.

Coursework

Coursework report - Summer, 2008

Moderators were pleased to receive the MS1 and the sample of work from the vast majority of centres in good time. Additionally, it was rare to find the Authentication Form, CCS160, missing; when it had to be requested the majority of centres responded helpfully and efficiently.

There have been a number of cases where incorrect work is ticked and given credit (in C3 and in particular, NM). We do ask that if the work is not checked then assessors do not tick it, and preferably write "not checked".

As always, teachers will find that most of what is stated below has been said before. We feel the need to repeat what we have said before because we continue to experience the same difficulties with marking.
We would encourage Heads of Departments and Examination Officers to ensure that all those involved in the assessment have a copy of this report to inform them for future sessions.
We wish to stress that the vast amount of work we have seen displays a high level of commitment by candidates and assessors with appropriate marks being awarded.

Oral communication

It is a requirement of all three tasks that the assessor fulfils this criterion and writes a brief report on how it was done and the results. Assessors are reminded that it is not permissible to give credit for any of the other criteria as a result of this oral communication.

Methods for Advanced Mathematics (C3); Numerical solution of equations (4753)

We are still experiencing the use of an incorrect cover sheet, 7 sessions after centres have been requested to destroy them! The use of these sheets is not beneficial to candidates and Heads of Departments are urged to check their paperwork to ensure that none of these documents exist. This will have been noted on individual reports to centres, and if you receive such a comment please will you ensure that all incorrect sheets are destroyed.

Graphical work (in all domains) needs to relate to the equation being solved and needs to illustrate the method. The graph also needs to match the iterates of the method. Generic graphs are not credit-worthy.

Change of sign

In this domain the graph requires some annotation or successive "zooming in" to serve as the illustration of the method.
The failure of the method is sometimes carried out inappropriately. In particular, this includes situations where the search of a sign change actually locates the root or locates the discontinuity. Equations such as $\frac{1}{x-1}=0$ or $\left(x-2\right)^2=0$ are therefore deemed as trivial and should not be used. We also see on a number of occasions equations whose graph does not cross the axis, yet it is claimed that it does.

Newton-Raphson method

Assessors are still interpreting the second criterion as automatically having been satisfied for an equation with only one root once the first mark is awarded. This is not so. Page 62 of the

specification book makes it clear that an equation with more than one root should be used in this domain.
Error bounds need to be established. It is not enough to note that successive iterates agree to *n* decimal places and that therefore the root is found to *n* decimal places. Error bounds are typically established by a change of sign calculation.
Some candidates illustrate a “failure” of the method by simply moving the first value of x away from the root. The criterion states that they should be demonstrating a failure to locate the expected root “despite a starting value close to it”. Merely choosing an unrealistic value for x_0 does not satisfy this criterion. Candidate should choose a starting value at one of the end points of the integer range within which the root lies.
Particularly in this domain, candidates who use computer resources to do the work for them should give some indication that they understand the method by doing some of the work themselves, either using a spreadsheet or calculator. It is, of course, essential to see the formula applied to the equation being solved.

Rearrangement method

The main problem in this domain continues to be the description of why convergence or not was achieved. It is expected that candidates will make some reference to the fact that the gradient of the line $y = x$ is 1 and that convergence will therefore only be achieved if $|\mathrm{g}'(x)| < 1$. Merely stating that $\mathrm{g}'(x) < 1$ with no explanation does not fulfil the criterion.

Comparison

Candidates should comment on the resources (hardware and software) they have used and how effective it has been in aiding the coursework. There is no “right” answer to this and we expect candidates to comment constructively in the light of the resources they have available.

Written communication

Candidates will continue to confuse equations with functions and even expressions.
A candidate who writes “I am going to solve the equation $y = x^3 + 2x - 8$” or even “I am going to solve the equation $x^3 + 2x - 8$” should not be credited with having written correct notation and terminology. The moderators continually find that a large number of candidates are awarded this mark with a positive comment given, yet the work is full of the errors described above.

Numerical Methods (4776)

Most candidates tackled appropriate integrals, and there were fewer instances of trivial tasks (e.g. find one root of an equation using Newton-Raphson method) being given inappropriately high marks.

Below are the main assessment issues which arose.

Problem Specification

The task should be specified using correct notation and terminology for the first mark. Referring to integrals as equations is not appropriate.
An explanation of why it is suitable should be made in clear English for the second mark.

Strategy

For the second mark, there is no need to replicate bookwork. A brief justification of the choice of algorithm is expected. (e.g. “I’m going to use the Midpoint Rule and the Trapezium Rule. By considering the shape of the graph, I know that the Midpoint Rule will over-estimate the value, and the Trapezium Rule will give an underestimate. Consequently I will be able to state an interval within which the value must lie.”)

Formula Application

Only going as far as 16 strips is not substantial. In many cases, going to 32 strips is not far enough, because “*r*” has not converged sufficiently for extrapolation to be reliable.

Use of Technology

The second mark is for explaining how the algorithm was implemented, not describing what software was used. An annotated printout of spreadsheet cell formulae works well.

Error Analysis

Comparison with the “true” value is seldom creditworthy.
Candidates should demonstrate convergence of r before using the theoretical value. They should not use the theoretical value at all if r converges to something else!
There is no credit for replicating bookwork. Analysis should be based on iterates produced.
Extrapolation should lead to a demonstrably better solution, which should be quoted in domain 6.

Interpretation

There should be a clear statement of the solution. The reader should not be expected to trawl through spreadsheet output.
Six significant figures is the minimum requirement. The stated level of precision should be justifiable from the iterates, and discussed in the commentary for the third mark in this domain.
Comments on the number of decimal places to which the spreadsheet / calculator works are seldom creditworthy. Rather, a discussion of problems encountered in achieving convergence should be discussed e.g. “ r seems to be tending to 0.353, but its theoretical value is 0.0625”.

Differential Equations (4758)

‘Cascades’ proved to be a very popular modelling assignment for this round of coursework. Generally speaking the standard of the work was good. However, it is expected that the first and second models (as far as assessment is concerned) are for the flow of water in the middle container; otherwise the level of sophistication of the differential equation is not sufficient at this level.
Some original pieces of work were submitted and some followed the modelling cycle well. Others did not, which was largely an inadequacy of the task itself. If novel tasks are set, it must be ensured that the modelling cycle can be followed and that the differential equations formed and solved, reflect the work covered in this module.
The assumptions are an important, if not vital, part of the process and a clear discussion of their relevance and relative importance is expected for full marks in Domain 1. There appears to be

an increasing amount of curve fitting rather than modelling where, instead of using the assumptions to create a model, the data are used.
In Domain 5, the modifying of the model should be based upon the assumptions and some sort of justification given for the new model. This was often absent, particularly in ‘Aeroplane Landing’ and ‘Cascades’.
Finally, wherever possible the comparison of the predicted and experimental data should be in both tabular and graphical form.

Grade Thresholds

Advanced GCE MEI Mathematics 3895 7895
June 2008 Examination Series

Unit Threshold Marks

Unit		Maximum Mark	A	B	C	D	E	U
All units	UMS	100	80	70	60	50	40	0
4751	Raw	72	61	53	45	37	30	0
4752	Raw	72	55	48	41	34	28	0
4753	Raw	72	59	52	46	40	33	0
4753/02	Raw	18	15	13	11	9	8	0
4754	Raw	90	75	67	59	51	43	0
4755	Raw	72	60	51	42	34	26	0
4756	Raw	72	57	51	45	39	33	0
4757	Raw	72	50	44	38	33	28	0
4758	Raw	72	58	50	42	34	26	0
4758/02	Raw	18	15	13	11	9	8	0
4761	Raw	72	57	48	39	30	22	0
4762	Raw	72	56	48	40	33	26	0
4763	Raw	72	53	45	37	29	21	0
4764	Raw	72	55	47	40	33	26	0
4766	Raw	72	53	45	38	31	24	0
4767	Raw	72	57	49	41	33	26	0
4768	Raw	72	56	49	42	35	28	0
4769	Raw	72	57	49	41	33	25	0
4771	Raw	72	58	51	44	37	31	0
4772	Raw	72	51	44	37	31	25	0
4773	Raw	72	51	44	37	30	24	0
4776	Raw	72	57	49	41	34	26	0
4776/02	Raw	18	14	12	10	8	7	0
4777	Raw	72	54	46	39	32	25	0

Specification Aggregation Results

Overall threshold marks in UMS (ie after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
7895-7898	600	480	420	360	300	240	0
3895-3898	300	240	210	180	150	120	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
7895	42.5	63.7	79.2	90.7	97.5	100	9600
7896	58.0	78.2	89.2	95.3	98.7	100	1539
7897	73.5	85.3	88.2	100	100	100	34
7898	27.8	52.8	61.1	77.8	91.7	100	36
3895	30.5	46.0	60.6	73.6	83.7	100	12767
3896	49.7	68.6	81.4	90.0	95.2	100	2039
3897	82.1	88.5	92.3	97.4	100	100	78
3898	47.8	52.2	69.6	87.0	95.7	100	23

For a description of how UMS marks are calculated see:
http://www.ocr.org.uk/learners/ums_results.html

Statistics are correct at the time of publication.

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