Mark Schemes for the Units

June 2009
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Foundations of Advanced Mathematics FSMQ (6993)

### MARKSCHEME ON THE UNIT

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## Additional Mathematics – 6993

### Section A

**1**

- Pythagoras for third value:
  
  \[ c = \sqrt{5} \]
  
  \[ \Rightarrow \tan \theta = -\frac{\sqrt{5}}{2} \]

  Alt:
  
  \[ \sin^2 \theta = 1 - \cos^2 \theta = 1 - \frac{4}{9} = \frac{5}{9} \]
  
  \[ \Rightarrow \sin \theta = \frac{\sqrt{5}}{3} \]
  
  \[ \Rightarrow \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\sqrt{5}/3}{2/3} = -\sqrt{5}/2 \]

  - Includes negative sign.

  **M1**

  **A1**

**3**

- Using any means to find \( \sqrt{5} \)

**SC:** Allow B1 for \( \tan \theta = -1.12 \)

**2**

- \( \frac{dy}{dx} = 3x^2 + 5 \)
  
  \[ \Rightarrow \text{grad tangent} = 8 \]
  
  \[ \Rightarrow \text{grad normal} = -\frac{1}{8} \]
  
  \[ \Rightarrow y + 1 = -\frac{1}{8}(x - 1) \]
  
  \[ \Rightarrow 8y + 8 = -x + 1 \Rightarrow 8y + x + 7 = 0 \]

  - Attempt at differentiation with at least one term with correct power

  **M1**

  **A1**

  **F1**

  **M1**

  **A1**

  **5**

**3 (i)**

- \( 2x + 5y = 2 + 25 \)
  
  \[ \Rightarrow 2x + 5y = 27 \]

  **M1**

  **A1**

  **SC:** B2 from scale drawing only if absolutely correct

**2**

**3 (ii)**

- When \( x = 3, \ 6 + 5y = 27 \)
  
  \[ \Rightarrow 5y = 21 \Rightarrow y = \frac{21}{5} \]
  
  \[ \Rightarrow p = \frac{21}{5} = 4.2 \]

  - Substituting \( x = 3 \) into either their equation from (i) or the given equation in (i)

  **M1**

  **F1**

  **2**

  - Answer must specifically give \( p \)

  NB \( p = 0.2 \) comes from using original line. Give M1 A1 for this.

  - Dep on use of their normal gradient and correct point

  Any acceptable form. Acceptable means three terms only
4 (i) \[ AB = \sqrt{(5-1)^2 + (3-1)^2} \]
\[ = \sqrt{4^2 + 2^2} \]
\[ = \sqrt{20} = 2\sqrt{5} \]
M1
A1

NB M1 A0 for 4.47 with no sight of \( \sqrt{20} \)

(ii) \[ \left( \frac{1+5}{2}, \frac{1+3}{2} \right) = (3, 2) \]
B1

(iii) \[ (x \pm a)^2 + (y \pm b)^2 \] with \((a, b)\) from (ii)
\[ (x - a)^2 + (y - b)^2 = 5 \]
M1
Use of equation
F1
Their midpoint
A1
cau for 5
isw ie ignore any incorrect algebra following a correct equation

5 (i) \[ v^2 = u^2 + 2as \Rightarrow 0 = 4 - 2 \times 0.25s \]
\[ \Rightarrow s = 8 \]
Distance travelled = 8 m
M1
A1
A1
Use of right formula(e)
Substitution
Answer

If \( t \) is found first then M1 for any correct equations that lead to finding \( s \)
Careful also of \( 4 = 0 + \frac{1}{2} s \), this could be 3 if quoted formula is right.
Also of \( 0 = 4 + \frac{1}{2} s \Rightarrow s = -8 \)
Both of these M1 for formula only

(ii) \[ s = ut + \frac{1}{2}at^2 = s = 3 \times 4 - \frac{1}{2} \times 0.25 \times 16 \]
\[ = 12 - 2 = 10 \]
Length of ramp = 10 m
M1
A1
A1

NB Anything that uses \( v = 0 \) is M0

6 \[ \frac{dy}{dx} = 1 - 4x + 3x^2 \]
\[ \Rightarrow (y) = x - 2x^2 + x^3 (+c) \]
Through (2, 6)
\[ \Rightarrow 6 = 2 - 8 + 8 + c \Rightarrow c = 4 \]
\[ \Rightarrow y = x - 2x^2 + x^3 + 4 \]
M1
A1
A1

For integrating - increase in power of one in at least two terms
Attempt to find \( c \)
Must be an equation
| 7  | (i) \[ AC^2 = 8^2 + 3^2 - 2.8.3 \cos 60 \]  
   | \[ = 73 - 24 = 49 \]  
   | \[ \Rightarrow AC = 7 \]  
   | \[ \Rightarrow \text{Total distance} = 18 \text{ km} \]  
   | M1 Use of formula  
   | A1  
   | A1 AC  
   | F1 Total distance  |
|----|--------------------------------------------------|
| (ii) | \[ \frac{\sin BCA}{8} = \frac{\sin 60}{9} \]  
   | \[ \Rightarrow \sin BCA = \frac{8}{9} \times \sin 60 \left( = 0.76989 \right) \]  
   | \[ \Rightarrow BCA = 50.3^0 \]  
   | M1 Alternative Scheme:  
   | A1 Use of cosine formula twice  
   | M1  
   | A1  
   | A1  |
| 8  | \[ 2x + 11 = x^2 - x + 5 \]  
   | \[ \Rightarrow x^2 - 3x - 6 \left( = 0 \right) \]  
   | \[ \Rightarrow x = \frac{3 \pm \sqrt{9 + 24}}{2} = \frac{3 \pm \sqrt{33}}{2} \]  
   | \[ = 4.37 \text{ or } -1.37 \]  
   | M1 Substitute  
   | A1 Quadratic  
   | M1 Solve  
   | A1 Correct substitution  
   | A1 Both answers  
   | 5 Ignore values for \( y \)  |
|    | Alternative Scheme 1 (relates to last 3 marks)  
   | Completion of square: \( (x - 1.5)^2 = k \)  
   | \[ x - 1.5 = \pm \sqrt{8.25} \]  
   | \[ \Rightarrow x = 4.37 \text{ or } -1.37 \]  
   | M1  
   | A1 Must contain \( \pm \)  
   | A1 Must be 2 dp  |
|    | Alternative Scheme 2: Only 2 marks from last 3  
   | Solving their quadratic by T&I  
   | Both roots  
   | M1  
   | A1  |
|    | Alternative Scheme 3: Only 4 marks  
   | Roots with no working: B2 each  
   | B2,2  |
|    | Alternative Scheme 4: Only 4 marks  
   | Finding a root from the original equations  
   | = one of them  
   | Finding the second root  
   | = the other  
   | M1  
   | A1  
   | M1  
   | A1  |
|    | Alternative scheme 5. Eliminate \( x \).  
   | Gives \( y^2 - 28y + 163 = 0 \)  
   | Gives \( y = 19.74 \) and \( 8.26 \) leading to \( x \) values  
   | M1 Eliminate \( x \)  
   | A1 Quadratic  
   | M1 Solve  
   | A1 Both \( y \) values  
<p>| A1 Both ( x ) values  |
| NB | Attempt to solve by graph - M0  |</p>
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<tbody>
<tr>
<td><strong>9</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i)</td>
<td>( a = 4 - 0.2t )</td>
<td>M1 Integrate (increase of power of one in at least one term)</td>
</tr>
<tr>
<td></td>
<td>( \Rightarrow v = 4t - 0.1t^2 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \Rightarrow v_5 = 20 - 2.5 = 17.5 )</td>
<td>A1 Ignore ( c )</td>
</tr>
<tr>
<td></td>
<td>Velocity is 17.5 m s(^{-1})</td>
<td>A1</td>
</tr>
<tr>
<td>(ii)</td>
<td>At ( t = 20 ), ( a = 0 )</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>ie Maximum velocity</td>
<td></td>
</tr>
<tr>
<td>(iii)</td>
<td>( v = 4t - 0.1t^2 )</td>
<td>M1 Integrate their ( v ) from (i)</td>
</tr>
<tr>
<td></td>
<td>( \Rightarrow s = \int_0^{20} (4t - 0.1t^2) , dt = \left[ \frac{2t^2}{3} - 0.1 \frac{t^3}{3} \right]_0^{20} )</td>
<td>A1 Ignore ( c )</td>
</tr>
<tr>
<td></td>
<td>( = 2 \times 400 - 0.1 \times \frac{8000}{3} = 533.3... = 533 )</td>
<td>A1 Allow exact answer or 3sf</td>
</tr>
<tr>
<td></td>
<td>Distance travelled = 533 m</td>
<td></td>
</tr>
<tr>
<td><strong>10</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i)</td>
<td></td>
<td>B2,1 Lines, (-1) each error</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B2,1 Shading, (-1) each error</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Correct side of line. ft if gradient is the same sign.</td>
</tr>
<tr>
<td>(ii)</td>
<td>( y = 2 )</td>
<td>E1 ft their graph</td>
</tr>
</tbody>
</table>
### Section B

#### 11 (i)

- $x^2 + 8x - 9 = x^2 - 6x + 11$
- $2x^2 - 14x + 20 = 0$
- $x^2 - 7x + 10 = 0$
- $(x - 5)(x - 2) = 0$
- $x = 2, 5$

Substitute: $x = 2 \Rightarrow y = 4 - 12 + 11 = 3$
$y = 25 - 30 + 11 = 6$

**M1** Equate

**A1** Quadratic

**M1** Solve: Factorisation needs 2 numbers to multiply to their constant

**A1** Or one pair, e.g. (2,3) or (5,6)

#### Alternative scheme:

Completion of square: $(x - 3.5)^2 = k$

$x - 3.5 = \pm \sqrt{2.25}$

$\Rightarrow x = 5$ or $2$

$\Rightarrow y = 6$ or $3$

**M1**

**A1**

**A1**

#### (ii)

\[
A = \frac{\int}{2} (y_1 - y_2) \, dx = \frac{\int}{2} (-2x^2 + 14x - 20) \, dx
\]

\[
= \left[ -\frac{2x^3}{3} + 7x^2 - 20x \right]_2^5
\]

\[
= \left( -\frac{2 \times 125}{3} + 7 \times 25 - 100 \right) - \left( -\frac{16}{3} + 28 - 40 \right)
\]

\[
= \left( -\frac{250}{3} + 75 \right) - \left( -\frac{16}{3} - 12 \right) = -\frac{234}{3} + 87 = 87 - 78 = 9
\]

**M1** Int between curves

**A1** Correct expression

Integrate their function (not if they divide by 2)

**M1** All terms, −1 for each error

**A1** Sub into integral Answer

**7**

**Alternative scheme:**

\[
A = \frac{\int}{2} (-x^2 + 8x - 9) \, dx - \frac{\int}{2} (x^2 - 6x + 11) \, dx
\]

\[
= \left[ -\frac{x^3}{3} + 4x^2 - 9x \right]_2^5 - \left[ \frac{x^3}{3} - 3x^2 + 11x \right]_2^5
\]

\[
= \left( -\frac{125}{3} + 100 - 45 \right) - \left( \frac{8}{3} + 16 - 18 \right)
\]

\[
- \left( \left( \frac{125}{3} - 75 + 55 \right) - \left( \frac{8}{3} - 12 + 22 \right) \right)
\]

\[
= 13 \frac{1}{3} - 4 \frac{2}{3} - 21 \frac{2}{3} = 18 - 9
\]

\[
= 9
\]

**M1** Subtracting 2 integrals

Integrate either

**A1** All terms of $y_1$

**A1** All terms of $y_2$

**M1** Substitute into either integral

**A1** For 18 or 9

**A1** Final answer

**SC** $A = \int (y_1 + y_2) \, dx$ M1 integrate and M1 sub only
<p>| | |</p>
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| 12 | (i)  

\[
\frac{100}{BE} = \sin 30
\]

\[\Rightarrow BE = \frac{100}{\sin 30} = 200 \text{ m} \]

M1 Fraction right way up  
A1 Correct expression for BE  
A1 3 Or B3 if the special triangle is noticed.

Alternative scheme:

\[
\frac{100}{BC} = \tan 30 \Rightarrow BC = \frac{100}{\tan 30} = 173.2
\]

\[BE = \sqrt{100^2 + 173.2^2} = 200 \]

M1 Ratio and Pythagoras  
A1 Allow not exact

(ii)  

AE by Pythagoras:

\[AE = \sqrt{500^2 + 200^2} = 100\sqrt{29} = 538.5... \]

\[\sin A = \frac{100}{538.5} \]

\[\Rightarrow A = 10.7^\circ \]

M1  
A1 soi

Alternative Scheme:

\[BC = \sqrt{30000} \approx 173.2 \Rightarrow AC = \frac{280000}{\sqrt{280000}} \approx 529.2 \]

\[\Rightarrow A = \tan^{-1} \frac{100}{\sqrt{280000}} = 10.7^\circ \]

\[NB A = 10.9^\circ \text{ comes from } \sin^{-1} \frac{100}{280000} \]

M1  
A1 4

(iii)  

Area = \(\frac{1}{2} \times 500 \times \text{ their } BE\)

\[= 50000 \]

Area = \(\frac{1}{2} \times BG \times \text{ their } AE\)

\[\Rightarrow BG = \frac{2 \times \text{ their area}}{\text{ their } AE} = 185.7..... \approx 186 \text{ m} \]

M1  
A1 5

Alternative Scheme:

Find angle A or E

Then \[\frac{BG}{500} = \sin A \Rightarrow BG = 186 \text{ m} \]

ie maximum 3 marks. The answer is found, but the question says “Hence” and this is “otherwise”.

NB If area is attempted but not used then give M1 A1. If area is found after BG is found then do not mark it.
### Question 13

**In all parts of this question allow answers to 3sf or 4 dp**

<p>| | | | | |</p>
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</table>
| **13 (a)** | The selection is random.  
*Allow anything that implies equal chance of selection* | B1 | 1 |
| **(b)(i)** | \( P(\text{all are female}) = 0.6^6 \) (= 0.046656)  
\[ = 0.0467 \] | M1 | A1 | Sight of 0.6^6  
Must be 3 sf |
| **(ii)** | \( P(3 \text{ of each}) = \text{Bin coeff} \times 0.6^3 \times 0.4^3 \)  
\[ = 20 \times 0.6^3 \times 0.4^3 \]  
\[ = 0.2765 \text{ or } 0.276 \] | M1 | A1 | One term with binomial coeff  
20 (may be implied)  
Powers (may be implied) |
| **(iii)** | \( P(\text{more females than males}) = 6, 0 \text{ or } 5,1 \text{ or } 4,2 \)  
\[ = 0.6^6 + 6 \times 0.6^5 \times 0.4 + 15 \times 0.6^4 \times 0.4^2 \]  
\[ = 0.04666 + 0.1866 + 0.3110 \]  
\[ = 0.5443 \]  
Allow 0.544, 0.545, 0.5444 | M1 | B1 | Add 3 terms  
Binomial coefficients correct in at least two terms  
Powers correct in at least two terms  
At least 2 terms correct. |
| Alternative scheme:  
\( P(\text{more females than males}) \)  
\[ = 1 \text{ – } P(\text{more males than females or equal numbers}) \]  
\[ = 1 \text{ – } (0.4^6+6 \times 0.4^5 \times 0.6+15 \times 0.4^4 \times 0.6^2+20 \times 0.4^3 \times 0.6^3) \]  
\[ = 1 \text{ – } (0.0041 + 0.0369 + 0.1382 + 0.2765) \]  
\[ = 0.5443 \] | M1 | B1 | Take 4 terms from  
Binomial coeffs  
Powers  
At least 2 terms correct |
|   | The terms are:  
0.0467, 0.1866, 0.3110, 0.2765, 0.1382, 0.0369, 0.0041 |   |   |
|   | If \( P(\text{more males than females}) \), treat as MR and –2  
If \( p = 0.4 \text{ and } q = 0.6 \) then MR –2 (but also 0 for (b)(i) where answer is given!) |   |   |
(a)(i) Line with +ve intercepts and −ve gradient
Curved Condone +ve gradient for cubic at origin. Must pass through the origin

(ii) Can only intersect in one point.
NB Do not allow if the curve implies that there could be more than one root but the line has not been drawn long enough - eg if curve is quadratic

(b)(i) \[ \frac{dy}{dx} = 3x^2 + 3 \]
Greater than 0 for all x or attempt to solve their \( \frac{dy}{dx} = 0 \)
so no solution to \( 3x^2 + 3 = 0 \)
Correct two terms
M1 = 0
A1 No solution

(ii) Because the curve is always increasing can only cross the x axis in one point which is the root
There must be some reference to (b)(i)

(c)(i) By trial \( f(2) = 0 \)
Condone \( (x - 2) \) is a factor

(ii) \( (x - 2)(x^2 + 2x + 5) = 0 \)
In long division at least \( x^2 \) must be seen

(iii) Discriminant “\( b^2 - 4ac \)” = -16 < 0
So no roots.
This means that \( x = 2 \) is the only root
NB “Quad does not factorise” is not good enough

(d) The equation will only have one root (for all \( r \) and \( s \))
Ignore extra comments even if wrong
Grade Thresholds

Additional Mathematics (6993)
June 2009 Assessment Series

Unit Threshold Marks

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<tr>
<th>Unit</th>
<th>Maximum Mark</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>U</th>
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<tr>
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<td>100</td>
<td>73</td>
<td>63</td>
<td>53</td>
<td>44</td>
<td>35</td>
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The cumulative percentage of candidates awarded each grade was as follows:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>U</th>
<th>Total Number of Candidates</th>
</tr>
</thead>
<tbody>
<tr>
<td>27.7</td>
<td>39.7</td>
<td>48.7</td>
<td>56.9</td>
<td>66.0</td>
<td>100</td>
<td>9859</td>
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