

**Mathematics (MEI)**

Advanced GCE 4756

Further Methods for Advanced Mathematics (FP2)

**Mark Scheme for June 2010**

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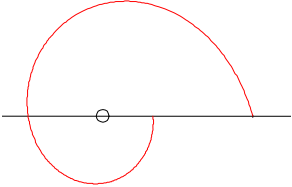
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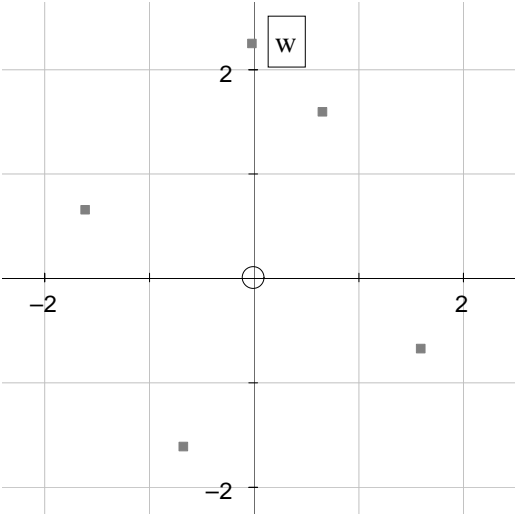
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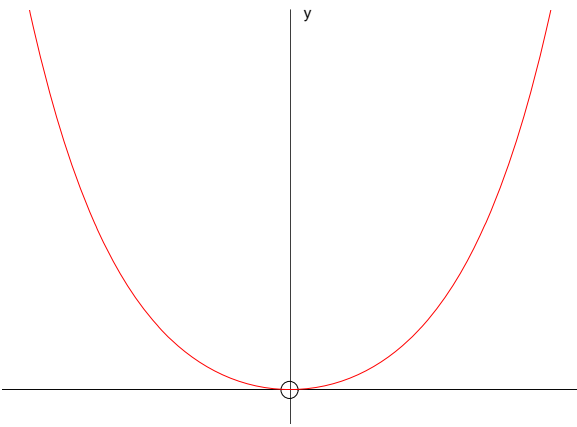
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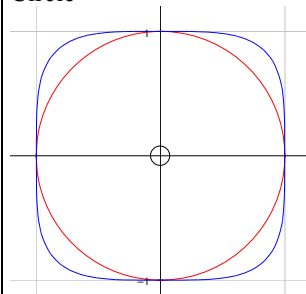
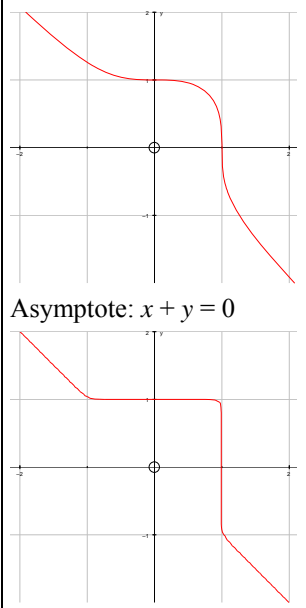
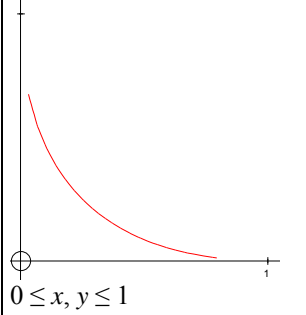
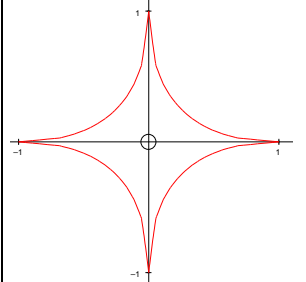
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1 (a)(i)	$f(t) = \arcsin t$ $\Rightarrow f'(t) = \frac{1}{\sqrt{1-t^2}} = (1-t^2)^{-\frac{1}{2}}$ $\Rightarrow f''(t) = -\frac{1}{2}(1-t^2)^{-\frac{3}{2}} \times -2t$ $= \frac{t}{(1-t^2)^{\frac{3}{2}}}$	B1 M1 A1 (ag)	Any form Using Chain Rule  <b>3</b>
(ii)	$f(x) = \arcsin(x + \frac{1}{2})$ $\Rightarrow f(0) = \arcsin(\frac{1}{2}) = \frac{\pi}{6}$ $f'(0) = \left(1 - \left(\frac{1}{2}\right)^2\right)^{-\frac{1}{2}} = \frac{2}{\sqrt{3}}$ $\text{and } f''(0) = \frac{\frac{1}{2}}{\left(1 - \left(\frac{1}{2}\right)^2\right)^{\frac{3}{2}}} = \frac{4\sqrt{3}}{9}$ $f(x) = f(0) + xf'(0) + \frac{x^2}{2}f''(0) + \dots$ $\Rightarrow \text{term in } x^2 \text{ is } \frac{2\sqrt{3}}{9}x^2$	B1 (ag) M1 A1 (ag)  M1 A1	$\frac{\pi}{6}$ obtained clearly from $f(0)$ www Clear substitution of $x = 0$ or $t = \frac{1}{2}$  Evaluating $f''(0)$ and dividing by 2  Accept $0.385x^2$ or better  <b>5</b>
(b)	 $\text{Area} = \int_0^{\pi} \frac{1}{2} r^2 d\theta$ $= \int_0^{\pi} \frac{\pi^2 a^2}{2(\pi + \theta)^2} d\theta = \frac{\pi^2 a^2}{2} \int_0^{\pi} \frac{1}{(\pi + \theta)^2} d\theta$ $= \frac{\pi^2 a^2}{2} \left[ \frac{-1}{\pi + \theta} \right]_0^{\pi}$ $= \frac{\pi^2 a^2}{2} \left( \frac{-1}{2\pi} + \frac{1}{\pi} \right)$ $= \frac{1}{4} \pi a^2$	G1 G1  M1 A1 M1 A1	Complete spiral with $r(2\pi) < r(0)$ $r(0) = a$ , $r(2\pi) = a/3$ indicated or $r(0) > r(\pi/2) > r(\pi) > r(3\pi/2) > r(2\pi)$ Dep. on G1 above Max. G1 if not fully correct  Integral expression involving $r^2$  Correct result of integration with correct limits Substituting limits into an expression of the form $\frac{k}{\pi + \theta}$ . Dep. on M1 above  <b>6</b>
(c)	$\int_0^{\frac{3}{2}} \frac{1}{9 + 4x^2} dx = \frac{1}{4} \int_0^{\frac{3}{2}} \frac{1}{\frac{9}{4} + x^2} dx = \frac{1}{4} \times \left[ \frac{2}{3} \arctan \frac{2x}{3} \right]_0^{\frac{3}{2}}$ $= \frac{1}{6} \arctan 1$ $= \frac{\pi}{24}$	M1 A1A1  M1 A1	arctan $\frac{1}{4} \times \frac{2}{3}$ and $\frac{2x}{3}$ Substituting limits. Dep. on M1 above Evaluated in terms of $\pi$  <b>5</b>

<p><b>2 (a)</b></p>	$z^n + \frac{1}{z^n} = 2 \cos n\theta, \quad z^n - \frac{1}{z^n} = 2j \sin n\theta$ $\left(z - \frac{1}{z}\right)^5 = z^5 - 5z^3 + 10z - \frac{10}{z} + \frac{5}{z^3} - \frac{1}{z^5}$ $= z^5 - \frac{1}{z^5} - 5\left(z^3 - \frac{1}{z^3}\right) + 10\left(z - \frac{1}{z}\right)$ $\Rightarrow 32j \sin^5 \theta = 2j \sin 5\theta - 10j \sin 3\theta + 20j \sin \theta$ $\Rightarrow \sin^5 \theta = \frac{1}{16} \sin 5\theta - \frac{5}{16} \sin 3\theta + \frac{5}{8} \sin \theta$ $A = \frac{5}{8}, B = -\frac{5}{16}, C = \frac{1}{16}$	<p>B1 M1 M1 A1 A1ft</p>	<p>Both Expanding <math>\left(z - \frac{1}{z}\right)^5</math> Introducing sines (and possibly cosines) of multiple angles RHS Division by 32(j)</p> <p style="text-align: right;"><b>5</b></p>
<p><b>(b)(i)</b></p>	<p>4<sup>th</sup> roots of <math>-9j = 9e^{\frac{3}{2}\pi j}</math> are <math>re^{j\theta}</math> where</p> $r = \sqrt{3}$ $\theta = \frac{3\pi}{8}$ $\Rightarrow \theta = \frac{3\pi}{8} + \frac{2k\pi}{4}$ $\Rightarrow \theta = \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$ 	<p>B1 B1 M1 A1 M1 A1</p>	<p>Accept <math>9^{\frac{1}{4}}</math> Implied by at least two correct (ft) further values Or stating <math>k = (0), 1, 2, 3</math> Allow arguments in range <math>-\pi \leq \theta \leq \pi</math> Points at vertices of a square centre O or 3 correct points (ft) or 1 point in each quadrant</p> <p style="text-align: right;"><b>6</b></p>
<p><b>(ii)</b></p>	<p>Mid-point of SP has argument <math>\frac{\pi}{8}</math> and modulus of <math>\sqrt{\frac{3}{2}}</math> Argument of <math>w = 4 \times \frac{\pi}{8} = \frac{\pi}{2}</math> and modulus = <math>\left(\sqrt{\frac{3}{2}}\right)^4 = \frac{9}{4}</math></p>	<p>B1 B1 M1 A1 G1</p>	<p>Multiplying argument by 4 and modulus raised to power of 4 Both correct <math>w</math> plotted on imag. axis above level of P</p> <p style="text-align: right;"><b>5</b></p>

<p><b>3 (a)(i)</b></p>	$2\lambda^3 + \lambda^2 - 13\lambda + 6 = 0 \Rightarrow (\lambda - 2)(2\lambda^2 + 5\lambda - 3) = 0$ $\Rightarrow \lambda = 2 \text{ or } 2\lambda^2 + 5\lambda - 3 = 0$ $\Rightarrow (2\lambda - 1)(\lambda + 3) = 0$ $\Rightarrow \lambda = \frac{1}{2}, \lambda = -3$	<p>B1 M1  A1A1</p>	<p>Substituting <math>\lambda = 2</math> or factorising Obtaining and solving a quadratic</p> <p style="text-align: right;"><b>4</b></p>
<p><b>(ii)</b></p>	$\mathbf{M} \begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix} = 2 \begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ -6 \\ 2 \end{pmatrix}$ $\mathbf{M}^2 \mathbf{v} = 2^2 \mathbf{v} = 4 \begin{pmatrix} 1 \\ -1 \\ \frac{1}{3} \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \\ \frac{4}{3} \end{pmatrix}$ $\mathbf{M} \begin{pmatrix} \frac{3}{2} \\ -\frac{3}{2} \\ \frac{1}{2} \end{pmatrix} = 2 \begin{pmatrix} \frac{3}{2} \\ -\frac{3}{2} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix}$ $\Rightarrow x = \frac{3}{2}, y = -\frac{3}{2}, z = \frac{1}{2}$	<p>B1  B2  M1  A1</p>	<p>Give B1 for one component with the wrong sign</p> <p>Recognising that the solution is a multiple of the given RHS</p> <p>Correct multiple</p> <p style="text-align: right;"><b>5</b></p>
<p><b>(iii)</b></p>	$2\lambda^3 + \lambda^2 - 13\lambda + 6 = 0$ $\Rightarrow 2\mathbf{M}^3 + \mathbf{M}^2 - 13\mathbf{M} + 6\mathbf{I} = \mathbf{0}$ $\Rightarrow \mathbf{M}^3 = -\frac{1}{2}\mathbf{M}^2 + \frac{13}{2}\mathbf{M} - 3\mathbf{I}$ $\Rightarrow \mathbf{M}^4 = -\frac{1}{2}\mathbf{M}^3 + \frac{13}{2}\mathbf{M}^2 - 3\mathbf{M}$ $\Rightarrow \mathbf{M}^4 = -\frac{1}{2}\left(-\frac{1}{2}\mathbf{M}^2 + \frac{13}{2}\mathbf{M} - 3\mathbf{I}\right) + \frac{13}{2}\mathbf{M}^2 - 3\mathbf{M}$ $\Rightarrow \mathbf{M}^4 = \frac{27}{4}\mathbf{M}^2 - \frac{25}{4}\mathbf{M} + \frac{3}{2}\mathbf{I}$ $A = \frac{27}{4}, B = -\frac{25}{4}, C = \frac{3}{2}$	<p>M1  M1 M1 A1</p>	<p>Using Cayley-Hamilton Theorem</p> <p>Multiplying by <math>\mathbf{M}</math></p> <p>Substituting for <math>\mathbf{M}^3</math></p> <p style="text-align: right;"><b>4</b></p>
<p><b>(b)</b> <math>\mathbf{N} = \mathbf{PDP}^{-1}</math> where <math>\mathbf{D} = \begin{pmatrix} -1 &amp; 0 \\ 0 &amp; 2 \end{pmatrix}</math> and <math>\mathbf{P} = \begin{pmatrix} 1 &amp; -1 \\ 2 &amp; 1 \end{pmatrix}</math></p>	$\Rightarrow \mathbf{P}^{-1} = \frac{1}{3} \begin{pmatrix} 1 & 1 \\ -2 & 1 \end{pmatrix}$ $\Rightarrow \mathbf{N} = \frac{1}{3} \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -2 & 1 \end{pmatrix}$ $= \frac{1}{3} \begin{pmatrix} -1 & -2 \\ -2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -2 & 1 \end{pmatrix}$ $= \frac{1}{3} \begin{pmatrix} 3 & -3 \\ -6 & 0 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -2 & 0 \end{pmatrix}$	<p>B1 B1 B1 B1ft  M1 A1</p>	<p>Order must be correct</p> <p>For B1B1, order must be consistent</p> <p>Ft their <math>\mathbf{P}</math></p> <p>Attempting matrix product</p>
<p>OR Let <math>\mathbf{N} = \begin{pmatrix} a &amp; c \\ b &amp; d \end{pmatrix}</math></p>	$\begin{pmatrix} a & c \\ b & d \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = -1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \text{B1}$ $\begin{pmatrix} a & c \\ b & d \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix} = 2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad \text{B1}$ $\Rightarrow a + 2c = -1, -a + c = -2 \quad \text{B1}$ $b + 2d = -2, -b + d = 2 \quad \text{B1}$ $\Rightarrow a = 1, c = -1; b = -2, d = 0 \quad \text{M1A1}$	<p style="text-align: right;"><b>6</b></p>	<p>Or <math>\begin{pmatrix} a+1 &amp; c \\ b &amp; d+1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}</math> Or <math>\begin{pmatrix} a-2 &amp; c \\ b &amp; d-2 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}</math></p> <p>Solving both pairs of equations</p> <p style="text-align: right;"><b>19</b></p>

<p><b>4 (i)</b></p>	$2 \sinh x \cosh x$ $= 2 \times \frac{e^x + e^{-x}}{2} \times \frac{e^x - e^{-x}}{2}$ $= \frac{e^{2x} - e^{-2x}}{2}$ $= \sinh 2x$ <p>Differentiating,</p> $2 \cosh 2x = 2 \cosh^2 x + 2 \sinh^2 x$ $\Rightarrow \cosh 2x = \cosh^2 x + \sinh^2 x$	<p>M1 A1 (ag) B1 B1</p>	<p>Using exponential definitions and multiplying or factorising</p> <p>One side correct Correct completion</p>
<p><b>(ii)</b></p>	 $\text{Volume} = \pi \int_0^2 (\cosh x - 1)^2 dx$ $= \pi \int_0^2 \cosh^2 x - 2 \cosh x + 1 dx$ $= \pi \int_0^2 \frac{1}{2} \cosh 2x - 2 \cosh x + \frac{3}{2} dx$ $= \pi \left[ \frac{1}{4} \sinh 2x - 2 \sinh x + \frac{3}{2} x \right]_0^2$ $= \pi \left[ \frac{1}{4} \sinh 4 - 2 \sinh 2 + 3 \right]$ $= 8.070$	<p>G1 M1 A1 M1 A2 A1</p>	<p>Correct shape and through origin</p> $\int (\cosh x - 1)^2 dx$ <p>A correct expanded integral expression including limits 0, 2 (may be implied by later work)</p> <p>Attempting to obtain an integrable form Dep. on M1 above</p> <p>Give A1 for two terms correct</p> <p>3 d.p. required. Condone 8.07</p>
<p><b>(iii)</b></p>	$y = \cosh 2x + \sinh x$ $\Rightarrow \frac{dy}{dx} = 2 \sinh 2x + \cosh x$ <p>At S.P. <math>2 \sinh 2x + \cosh x = 0</math></p> $\Rightarrow 4 \sinh x \cosh x + \cosh x = 0$ $\Rightarrow \cosh x(4 \sinh x + 1) = 0$ $\Rightarrow \cosh x = 0 \text{ (rejected)}$ $\Rightarrow \sinh x = -\frac{1}{4}$ $\Rightarrow x = \ln \left( -\frac{1}{4} + \frac{\sqrt{17}}{4} \right)$	<p>B1 M1 M1 A1 A1 M1 A1</p>	<p>Any correct form</p> <p>Setting derivative equal to zero and using identity</p> <p>Solving <math>\frac{dy}{dx} = 0</math> to obtain value of <math>\sinh x</math></p> <p>Repudiating <math>\cosh x = 0</math></p> <p>Using log form of arsinh, or setting up and solving quadratic in <math>e^x</math> A0 if extra "roots" quoted</p>

<p>5(i)(A) (B)</p>	 <p>(C) Square (D) <math>-1 \leq x \leq 1</math> <math>-1 \leq y \leq 1</math></p>	<p>B1  G1 G1 B1 B1 B1</p>	<p>Sketch of circle, centre (0, 0) Sketch of “squarer” circle on same axes  Give B1B0 for not all non-strict or unclear</p>
<p>(ii)(A) (B) (C)  (D)</p>	<p>Odd roots exist for all real numbers Line</p>  <p>Asymptote: <math>x + y = 0</math></p>	<p>B1 B1  G1 B1  G1 G1</p>	<p>Any equivalent explanation Sketch insufficient          Line <math>x + y = 0</math> outside unit square Lines <math>y = 1</math> and <math>x = 1</math> on unit square</p>
<p>(iii)</p>	 <p><math>0 \leq x, y \leq 1</math></p>	<p>G1 B1</p>	<p>G0 if curve beyond (1, 0) or (0, 1) Accept strict, or indication on graph</p>
<p>(iv)(A)  (B)</p>	 <p>Limit is a “plus sign” where <math>x \rightarrow 0</math> for <math>-1 \leq y \leq 1</math> and vice versa</p>	<p>G2ft B1 B1</p>	<p>Give G1 for a partial attempt. Ft from (iii) on shape</p>

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