

**Mathematics**

Advanced GCE

Unit **4724**: Core Mathematics 4

**Mark Scheme for January 2011**

---

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of pupils of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

© OCR 2011

Any enquiries about publications should be addressed to:

OCR Publications  
PO Box 5050  
Annesley  
NOTTINGHAM  
NG15 0DL

Telephone: 0870 770 6622  
Facsimile: 01223 552610  
E-mail: [publications@ocr.org.uk](mailto:publications@ocr.org.uk)

- 1 (i) First two terms are  $1 - \frac{1}{2}x$ ..... B1
- Third term =  $\frac{\frac{1}{2} \cdot -\frac{1}{2}}{2} [(-x)^2 \text{ or } x^2 \text{ or } -x^2]$  M1
- =  $-\frac{1}{8}x^2$  A1 3  $-\frac{1}{8}x^2$  without work  $\rightarrow$  M1 A1
- (ii) Attempt to replace  $x$  by  $2y - 4y^2$  or  $2y + 4y^2$  M1 or write as  $1 - (2y - 4y^2 \text{ or } 2y + 4y^2)$
- First two terms are  $1 - y$  B1
- Third term =  $+\frac{3}{2}y^2$  or  $\sqrt{(4b+2)}y^2$  A1√ 3 where  $b = cf(x^2)$  in part (i)
- 6**
- 2 (i)  $A(x-2) + B = 7 - 2x$  M1 or  $A(x-2)^2 + B(x-2) = (7-2x)(x-2)$
- $A = -2$  A1
- $B = 3$  A1 3
- (ii)  $\int \frac{A}{x-2} dx = \left( A \text{ or } \frac{1}{A} \right) \ln(x-2)$  B1 Accept  $\ln|x-2|, \ln|2-x|, \ln(2-x)$
- $\int \frac{B}{(x-2)^2} dx = -\left( B \text{ or } \frac{1}{B} \right) \cdot \frac{1}{x-2}$  B1 Negative sign is required
- Correct f.t. of A & B;  $A \ln(x-2) - \frac{B}{x-2}$  B1√ Still accept lns as before
- Using limits =  $-2 \ln 3 + 2 \ln 2 + \frac{1}{2}$  ISW B1 4 No indication of  $\ln(\text{negative})$
- 7**
- 3 (i) State/imply  $\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right)$  or  $\frac{d}{dx}(\cos x)^{-1}$  B1 Not just  $\sec x = \frac{1}{\cos x}$
- Attempt quotient rule or chain rule to power  $-1$  M1 Allow  $\frac{u dv - v du}{v^2}$  & wrong trig signs
- Obtain  $\frac{\sin x}{\cos^2 x}$  or  $-(\sin x)(\cos x)^{-2}$  A1 No inaccuracy allowed here
- Simplify with suff evid to **AG** e.g.  $\frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$  A1 4 Or vice versa. Not just =  $\sec x \tan x$
- (ii) Use  $\cos 2x = +/-1 +/- 2 \cos^2 x$  or  $+/-1 +/- 2 \sin^2 x$  M1 or  $\pm(\cos^2 x - \sin^2 x)$
- Correct denominator =  $\sqrt{2 \cos^2 x}$  A1  $\sqrt{2 - 2 \sin^2 x}$  needs simplifying
- Evidence that  $\frac{\tan x}{\cos x} = \sec x \tan x$  or  $\int \frac{\tan x}{\cos x} dx = \sec x$  B1 irrespective of any const multiples
- $\frac{1}{\sqrt{2}} \sec x$  (+ c) A1 4 Condone  $\theta$  for  $x$  except final line
- 8**

<p><b>4 (i)</b> Attempt to use <math>\frac{dy}{dx} = \frac{\frac{dy}{dt} \cdot dt}{dx}</math> or <math>\frac{dy}{dt} \cdot \frac{dt}{dx}</math></p> <p><math>\frac{4}{2t}</math> or <math>\frac{2}{t}</math></p> <p><b>(ii)</b> Subst <math>t = 4</math> into their (i), invert &amp; change sign Subst <math>t = 4</math> into <math>(x,y)</math> &amp; use num grad for tgt/normal <math>y = -2x + 52</math> AEF CAO (no f.t.)</p> <p><b>(iii)</b> Attempt to eliminate <math>t</math> from the 2 given equations <math>x = 2 + \frac{y^2}{16}</math> or <math>y^2 = 16(x - 2)</math> AEF ISW</p>	<p>M1 Not just quote formula</p> <p>A1 2</p> <p>M1</p> <p>M1</p> <p>A1 3 Only the eqn of normal accepted</p> <p>M1</p> <p>A1 2 Mark at earliest acceptable form.</p>
<b>7</b>	
<p><b>5 (i)</b> Attempt to connect <math>dx</math> and <math>du</math></p> <p><math>5 - x = 4 - u^2</math></p> <p>Show <math>\int \frac{4 - u^2}{2 + u} \cdot 2u \, du</math> reduced to <math>\int 4u - 2u^2 \, du</math> <b>AG</b></p> <p>Clear explanation of why limits change</p> <p><math>\frac{4}{3}</math></p> <p><b>(ii)(a)</b> <math>5 - x</math></p> <p><b>(b)</b> Show reduction to <math>2 - \sqrt{x - 1}</math></p> <p><math>\int \sqrt{x - 1} \, dx = \frac{2}{3}(x - 1)^{\frac{3}{2}}</math></p> <p><math>\left(10 - \frac{2}{3} \cdot 8\right) - \left(4 - \frac{2}{3}\right) = \frac{4}{3}</math> or <math>4\frac{2}{3} - 3\frac{1}{3} = \frac{4}{3}</math></p>	<p>M1 Including <math>\frac{du}{dx} =</math> or <math>du = \dots dx</math> ; not <math>dx = du</math></p> <p>B1 perhaps in conjunction with next line</p> <p>A1 In a fully satisfactory &amp; acceptable manner</p> <p>B1 e.g. when <math>x = 2, u = 1</math> <u>and</u> when <math>x = 5, u = 2</math></p> <p>B1 5 not dependent on any of first 4 marks</p> <p>*B1 1 Accept <math>4 - x - 1 = 5 - x</math> (this is not <b>AG</b>)</p> <p>dep*B1</p> <p>B1 Indep of other marks, seen anywhere in (b)</p> <p>B1 3 Working must be shown</p>
<b>9</b>	
<p><b>6 (i)</b> Work with correct pair of direction vectors</p> <p>Demonstrate correct <u>method</u> for finding scalar product</p> <p>Demonstrate correct <u>method</u> for finding modulus</p> <p>24, 24.0 (24.006363..) (degrees) 0.419 (0.41899..) (rad)</p> <p><b>(ii)</b> Attempt to set up 3 equations</p> <p>Find correct values of <math>(s, t) = (1, 0)</math> or <math>(1, 4)</math> or <math>(5, 12)</math></p> <p>Substitute their <math>(s, t)</math> into equation not used</p> <p><u>Correctly</u> demonstrate failure</p> <p><b>(iii)</b> Subst their <math>(s, t)</math> from first 2 eqns into new 3<sup>rd</sup> eqn</p> <p><math>a = 6</math></p>	<p>M1</p> <p>M1 Of <u>any</u> two 3x3 vectors rel to question</p> <p>M1 Of <u>any</u> vector relevant to question</p> <p>A1 4 Mark earliest value, allow trunc/rounding</p> <p>M1 Of type <math>3 + 2s = 5, 3s = 3 + t, -2 - 4s = 2 - 2t</math></p> <p>A1 Or 2 diff values of <math>s</math> (or of <math>t</math>)</p> <p>M1 and make a relevant deduction</p> <p>A1 4 dep on all 3 prev marks</p> <p>M1 New 3<sup>rd</sup> eqn of type <math>a - 4s = 2 - 2t</math></p> <p>A1 2</p>

- 7 Attempt parts with  $u = x^2 + 5x + 7$ ,  $dv = \sin x$  M1 as far as  $f(x) + / - \int g(x) dx$
- 1<sup>st</sup> stage =  $-(x^2 + 5x + 7)\cos x + \int (2x + 5)\cos x dx$  A1 signs need not be amalgamated at this stage
- $\int (2x + 5)\cos x dx = (2x + 5)\sin x - \int 2 \sin x dx$  B1 indep of previous A1 being awarded
- =  $(2x + 5)\sin x + 2 \cos x$  B1
- $I = -(x^2 + 5x + 7)\cos x + (2x + 5)\sin x + 2 \cos x$  A1 WWW
- (Substitute  $x = \pi$ )  $-(\text{Substitute } x = 0)$  M1 An attempt at subst  $x = 0$  must be seen
- $\pi^2 + 5\pi + 10$  WWW AG A1 7
- 7**
- 8 (i)  $\frac{d}{dx}(y^2) = 2y \frac{dy}{dx}$  B1
- $\frac{d}{dx}(-5xy) = (-)(5)x \frac{dy}{dx} + (-)(5)y$  M1 i.e. reasonably clear use of product rule
- LHS completely correct  $4x - 5x \frac{dy}{dx} - 5y + 2y \frac{dy}{dx} (= 0)$  A1 Accept “ $\frac{dy}{dx} =$ ” provided it is not used
- Substitute  $\frac{dy}{dx} = \frac{3}{8}$  or solve for  $\frac{dy}{dx}$  & then equate to  $\frac{3}{8}$  M1 Accuracy not required for “solve for  $\frac{dy}{dx}$ ”
- Produce  $x = 2y$  WWW AG (Converse acceptable) A1 5 Expect  $17x = 34y$  and/or  $\frac{dy}{dx} = \frac{5y - 4x}{2y - 5x}$
- (ii) Substitute  $2y$  for  $x$  or  $\frac{1}{2}x$  for  $y$  in curve equation M1
- Produce either  $x^2 = 36$  or  $y^2 = 9$  A1
- AEF of  $(\pm 6, \pm 3)$  A1 3 ISW Any correct format acceptable
- 8**
- 9 (i) Attempt to sep variables in the form  $\int \frac{p}{(x-8)^{\frac{1}{3}}} dx = \int q dt$  M1 Or invert as  $\frac{dt}{dx} = \frac{r}{(x-8)^{\frac{1}{3}}}$ ;  $p, q, r$  const
- $\int \frac{1}{(x-8)^{\frac{1}{3}}} dx = k(x-8)^{\frac{2}{3}}$  A1  $k$  const
- All correct (+  $c$ ) A1
- For equation containing ‘ $c$ ’; substitute  $t = 0$ ,  $x = 72$  M1 M2 for  $\int_{72}^{35} = \int_0^t$  or  $\int_{35}^{72} = \int_0^t$
- Correct corresponding value of  $c$  from correct eqn A1
- Subst their  $c$  &  $x = 35$  back into eqn M1
- $t = \frac{21}{8}$  or 2.63 / 2.625 [C.A.O] A1 7 A2:  $t = \frac{21}{8}$  or 2.63 / 2.625 WWW
- (ii) State/imply in some way that  $x = 8$  when flow stops B1
- Substitute  $x = 8$  back into equation containing numeric ‘ $c$ ’ M1
- $t = 6$  A1 3

- 1 When an acceptable answer has been obtained, ignore subsequent working (ISW) unless stated otherwise.
- 2 Ignore working which has no relevance to question as set; e.g. in Qu.1, ignore all terms in  $x^3$  etc.
- 3 The 'M' marks are awarded if it is clear that candidate is attempting to do what he/she should be doing.
- 4 If an ans is given (**AG**), working must be checked minutely as answer shown will nearly always be 'correct'.  
More reasoning/explanation is generally required than when the answer is not given.

#### Comments or Alternative methods

##### Question 1(ii)

Beware: there are often double mistakes leading to the correct terms – errors invalidate marks.

##### Question 2(ii)

For the first 2 marks, we're really testing  $\int \frac{1}{x-2} dx$  and  $\int \frac{1}{(x-2)^2} dx$ ; this is why we accept  $\frac{1}{A}$  and/or  $-\frac{1}{B}$ .

For the 1<sup>st</sup> & 3<sup>rd</sup> marks, accept  $\ln(2-x)$  as these are the indef integ stages. At final, definite, stage, it will be penalised..  
'Exact value' is required; so 0.0945.... without equivalent log version  $\rightarrow B0$   $2\ln 2 - 3\ln 3$  need not be simplified.

##### Question 4

Allow marks for part (iii) to be awarded at any stage of question.

So, if the Cartesian equation is worked out first of all, then award marks in part (i) as follow:

if cart. eqn is found in the form  $x = f(y)$ , award M1 for finding  $\frac{dx}{dy}$ , inverting & subst  $y = 4t$  (in either order)

if cart. eqn is found in the form  $y = g(x)$ , award M1 for finding  $\frac{dy}{dx}$  and substituting  $x = 2 + t^2$

and, finally, A1 as in main scheme.

##### Question 5(i)

The problem here will centre on how the candidate manipulates the equation  $u = \sqrt{x-1}$  to get  $x$  in terms of  $u$ . He/she could get  $x = u^2 + 1$  (correct) or, perhaps,  $x = u^2 - 1$  or  $x = 1 - u^2$  (incorrect) or some other incorrect version.

The 1<sup>st</sup>, 4<sup>th</sup> & 5<sup>th</sup> marks in part (i) are unaffected by the correctness or otherwise of this manipulation. However, any error seen must destroy the 2<sup>nd</sup> and 3<sup>rd</sup> marks – but candidates can still score 3 of the 5 marks.

For the A1, there must be some evidence of reduction to the given answer; the one main case that we are not accepting

is where  $\frac{8u-2u^3}{2+u}$  is said to be  $4u-2u^2$  without any supporting evidence; long division will suffice; or if  $8u-2u^3$  is said to be  $(2+u)(4u-2u^2)$ , then we will accept (as multiplication can easily be checked in the head whereas division is not reckoned to be). Note that '2' into '8u' gives '4u' and 'u' into ' $-2u^3$ ', gives ' $-2u^2$ '.

##### Question 5(ii)(a)

This is just a '1' mark part so we give 1 or 0 purely dependent on the answer and we ignore any sloppy working.

A candidate writing  $4-x-1=3-x$  will be awarded 0 marks; however, another candidate writing  $4-x-1=5-x$  will be awarded the B1 mark. This is not an **AG** so the candidate does not know the required answer.

Question 6(i)

For demonstrating correct method for finding scalar product, I expect to see at least  $2/3$  of the working correct.

Likewise for modulus: examine either vector,  $\sqrt{2^2 + 3^2 - 4^2}$  will score M1 { $\frac{2}{3}$  correct, prob  $\sqrt{29}$  will follow anyway}

Question 6(ii)

Occasionally candidates do not follow a 'sensible' method. However, the first M1 is always standard. The remaining 3 marks must be awarded for convincing arguments and/or accurate results.

Question 7

This is a question where signs are crucial and where the given answer may be obtained even with errors in the working; also the fact that the answer is **AG** means that many candidates will state it on the final line.

Using the standard method, 3 marks out of the 7 are fixed (the 2 @ M1 and the final A1) but the other 4 marks depend on the capability of the candidate to integrate  $\sin x$  and  $\cos x$ .

If he/she uses  $\cos x$  for the integral of  $\sin x$ , candidate should get  $-(\text{our version of 1st main stage})$ , so that's A0 but he/she still has to integrate  $(2x+5)\cos x$  for the 2<sup>nd</sup> stage. Admittedly he/she may then make a further mistake when integrating  $\cos x$  but the 2 @ B1 are available. These 2 marks are an independent pair and only depend on the integral of  $(2x+5)\cos x$  being attempted. Whether it's the integral of  $(2x+5)\cos x$  or of  $-(2x+5)\cos x$  is immaterial. This gives a maximum of 4 out of 7 if  $\sin x$  is incorrectly integrated.

Even though I have bracketed the 3 terms as  $(x^2 + 5x + 7)$ , we can expect some candidates to multiply out as 3 separate integrals.,  $\int x^2 \sin x \, dx$       and       $\int 5x \sin x \, dx$       and       $\int 7 \sin x \, dx$

Their equivalent 1<sup>st</sup> stages are:

$$-x^2 \cos x + \int 2x \cos x \, dx; \quad -5x \cos x + \int 5 \cos x \, dx; \quad -7 \cos x \quad \mathbf{M1 + A1}$$

Their equivalent 2<sup>nd</sup> stages are:

$$2x \sin x + 2 \cos x \quad \mathbf{B1} \quad 5 \sin x \quad \mathbf{B1}$$

To obtain the corresponding marks, all components must be correct.

**OCR (Oxford Cambridge and RSA Examinations)**  
**1 Hills Road**  
**Cambridge**  
**CB1 2EU**

**OCR Customer Contact Centre**

**14 – 19 Qualifications (General)**

Telephone: 01223 553998

Facsimile: 01223 552627

Email: [general.qualifications@ocr.org.uk](mailto:general.qualifications@ocr.org.uk)

**[www.ocr.org.uk](http://www.ocr.org.uk)**

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

**Oxford Cambridge and RSA Examinations**  
**is a Company Limited by Guarantee**  
**Registered in England**  
**Registered Office; 1 Hills Road, Cambridge, CB1 2EU**  
**Registered Company Number: 3484466**  
**OCR is an exempt Charity**



**OCR (Oxford Cambridge and RSA Examinations)**  
**Head office**  
**Telephone: 01223 552552**  
**Facsimile: 01223 552553**

© OCR 2011