

**ADVANCED GCE  
MATHEMATICS**

Further Pure Mathematics 2

**4726**

Candidates answer on the answer booklet.

**OCR supplied materials:**

- 8 page answer booklet (sent with general stationery)
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Monday 20 June 2011  
Morning**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a scientific or graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Express  $\frac{2x+3}{(x+3)(x^2+9)}$  in partial fractions. [5]

2 A curve has equation  $y = \frac{x^2 - 6x - 5}{x - 2}$ .

(i) Find the equations of the asymptotes. [3]

(ii) Show that  $y$  can take all real values. [4]

3 It is given that  $F(x) = 2 + \ln x$ . The iteration  $x_{n+1} = F(x_n)$  is to be used to find a root,  $\alpha$ , of the equation  $x = 2 + \ln x$ .

(i) Taking  $x_1 = 3.1$ , find  $x_2$  and  $x_3$ , giving your answers correct to 5 decimal places. [2]

(ii) The error  $e_n$  is defined by  $e_n = \alpha - x_n$ . Given that  $\alpha = 3.14619$ , correct to 5 decimal places, use the values of  $e_2$  and  $e_3$  to make an estimate of  $F'(\alpha)$  correct to 3 decimal places. State the true value of  $F'(\alpha)$  correct to 4 decimal places. [3]

(iii) Illustrate the iteration by drawing a sketch of  $y = x$  and  $y = F(x)$ , showing how the values of  $x_n$  approach  $\alpha$ . State whether the convergence is of the 'staircase' or 'cobweb' type. [3]

4 A curve  $C$  has the cartesian equation  $x^3 + y^3 = axy$ , where  $x \geq 0$ ,  $y \geq 0$  and  $a > 0$ .

(i) Express the polar equation of  $C$  in the form  $r = f(\theta)$  and state the limits between which  $\theta$  lies. [3]

The line  $\theta = \alpha$  is a line of symmetry of  $C$ .

(ii) Find and simplify an expression for  $f(\frac{1}{2}\pi - \theta)$  and hence explain why  $\alpha = \frac{1}{4}\pi$ . [3]

(iii) Find the value of  $r$  when  $\theta = \frac{1}{4}\pi$ . [1]

(iv) Sketch the curve  $C$ . [2]

5 (i) Prove that, if  $y = \sin^{-1} x$ , then  $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$ . [3]

(ii) Find the Maclaurin series for  $\sin^{-1} x$ , up to and including the term in  $x^3$ . [5]

(iii) Use the result of part (ii) and the Maclaurin series for  $\ln(1+x)$  to find the Maclaurin series for  $(\sin^{-1} x) \ln(1+x)$ , up to and including the term in  $x^4$ . [4]

6 It is given that  $I_n = \int_0^1 x^n (1-x)^{\frac{3}{2}} dx$ , for  $n \geq 0$ .

(i) Show that  $I_n = \frac{2n}{2n+5} I_{n-1}$ , for  $n \geq 1$ . [6]

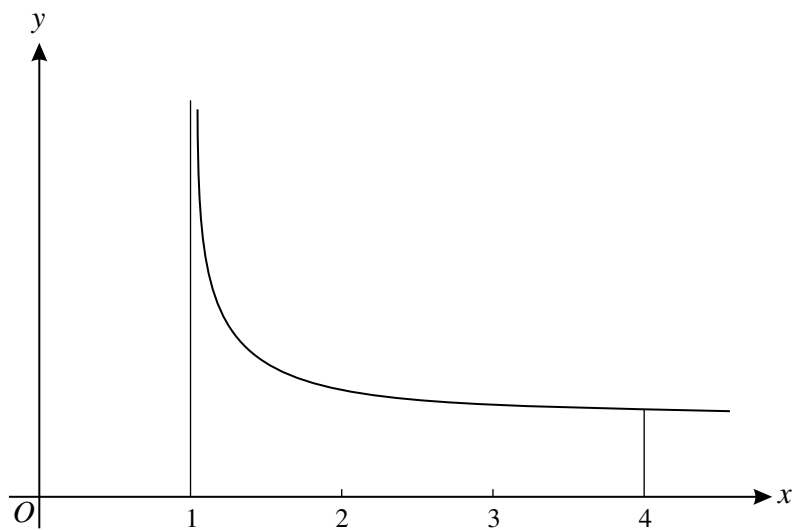
(ii) Hence find the exact value of  $I_3$ . [4]

- 7 (i) Sketch the graph of  $y = \tanh x$  and state the value of the gradient when  $x = 0$ . On the same axes, sketch the graph of  $y = \tanh^{-1} x$ . Label each curve and give the equations of the asymptotes. [4]

(ii) Find  $\int_0^k \tanh x \, dx$ , where  $k > 0$ . [2]

(iii) Deduce, or show otherwise, that  $\int_0^{\tanh k} \tanh^{-1} x \, dx = k \tanh k - \ln(\cosh k)$ . [4]

- 8 (i) Use the substitution  $x = \cosh^2 u$  to find  $\int \sqrt{\frac{x}{x-1}} \, dx$ , giving your answer in the form  $f(x) + \ln(g(x))$ . [7]



- (ii) Hence calculate the exact area of the region between the curve  $y = \sqrt{\frac{x}{x-1}}$ , the  $x$ -axis and the lines  $x = 1$  and  $x = 4$  (see diagram). [1]

- (iii) What can you say about the volume of the solid of revolution obtained when the region defined in part (ii) is rotated completely about the  $x$ -axis? Justify your answer. [3]

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