OCR
RECOGNISING ACHIEVEMENT

FREE-STANDING MATHEMATICS QUALIFICATION
ADVANCED LEVEL

Additional Mathematics 6993

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:
- Printed answer book 6993

Other materials required:
- Scientific or graphical calculator

Monday 13 June 2011
Morning

Duration: 2 hours

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given correct to three significant figures where appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **100**.
- The printed answer book consists of **20** pages. The question paper consists of **8** pages. Any blank pages are indicated.

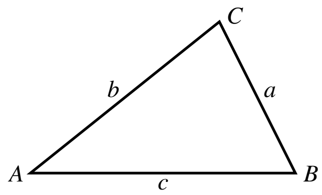
INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

Formulae Sheet: 6993 Additional Mathematics

In any triangle *ABC*

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$



Binomial expansion

When n is a positive integer

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Answer all questions on the Printed Answer Book provided.

Section A

1 Determine whether the point (5, 2) lies inside or outside the circle whose equation is $x^2 + y^2 = 30$. You must show your working. **[3]**

2 The equation of a curve is $y = x^3 - x^2 - 2x - 3$.

Find the equation of the tangent to this curve at the point (3, 9). **[5]**

3 In the triangle PQR, PQ = 8 cm, RQ = 9 cm and RP = 7 cm.

(i) Find the size of the largest angle. **[4]**

(ii) Calculate the area of the triangle. **[3]**

4 Solve the equation $5\sin 2x = 2\cos 2x$ in the interval $0° \leqslant x \leqslant 360°$.
Give your answers correct to 1 decimal place. **[5]**

5 The coordinates of the points A, B and C are (−2, 1), (5, 2) and (4, 9) respectively.

(a) Find the coordinates of the midpoint, M, of the line AC. **[1]**

(b) Show that BM is perpendicular to AC. **[3]**

(c) **(i)** Use the result of part **(b)** to state the mathematical name of the triangle ABC. **[1]**

(ii) Prove this by another method. **[2]**

6 Solve the inequality $x^2 - 12x + 35 \leqslant 0$. **[4]**

7 **(a)** Determine whether or not each of the following is a factor of the expression $x^3 - 7x + 6$.
You must show your working.

(i) $(x - 2)$ **[2]**

(ii) $(x + 1)$ **[1]**

(b) **(i)** Factorise the function $f(x) = x^3 - 7x + 6$. **[3]**

(ii) Solve the equation $f(x) = 0$. **[1]**

8 **(i)** On the axes given, indicate the region for which the following inequalities hold. You should shade the region which is **not** required.

$$\begin{aligned} 5x + 3y &\geqslant 30 \\ 3x + y &\geqslant 12 \\ y &\geqslant 0 \\ x &\geqslant 0 \end{aligned}$$

[5]

(ii) Find the minimum value of $6x + y$ subject to these conditions. **[2]**

9 The gradient function of a curve is given by $\frac{dy}{dx} = 3x^2 - 2x + 4$.

Find the equation of the curve, given that it passes through the point (2, 2). **[4]**

10 You are given that $\sin\theta = \frac{2}{5}$ with $0° \leqslant \theta \leqslant 90°$.

Using the identity $\sin^2\theta + \cos^2\theta = 1$, find an exact value for $\cos\theta$. **[3]**

Section B

11 Eggs are delivered to a supermarket in boxes of 6.
For each egg, the probability that it is cracked is 0.05 independently of other eggs.

Find the probability that

(i) in one box there are no cracked eggs, **[2]**

(ii) in one box there is exactly 1 cracked egg. **[4]**

The manager checks the eggs as follows.

- He takes a box at random from the delivery.
- He accepts the whole delivery if this box contains no cracked eggs.
- He rejects the whole delivery if the box contains 2 or more cracked eggs.
- If the box contains 1 cracked egg then he chooses another box at random.
- He accepts the delivery only if this second box contains no cracked eggs.

(iii) Find the probability that the delivery is rejected. **[6]**

12 Two cars, A and B, move from rest away from a point O on a straight road starting at the same time.

(a) Car A moves with constant acceleration of $2\,\mathrm{m\,s^{-2}}$.

Express the displacement of car A after time t seconds as a function of t. **[2]**

(b) Car B moves with acceleration given by $a = \frac{1}{2}t + 1$.

Express the displacement of car B after time t seconds as a function of t. **[4]**

(c) **(i)** Find the time at which the cars are the same distance from O. **[2]**

(ii) Find the distance they have travelled at that time. **[2]**

(d) Draw a sketch graph of the velocity of each car on the axes given. **[2]**

13 A pyramid has a square base, ABCD, with vertex E. E is directly above the centre of the base, O, as shown in Fig. 13.
The lengths of the sides of the base are each $2x$ metres and the height is h metres.
The lengths of the sloping edges, AE, BE, CE and DE, are each 5 metres.

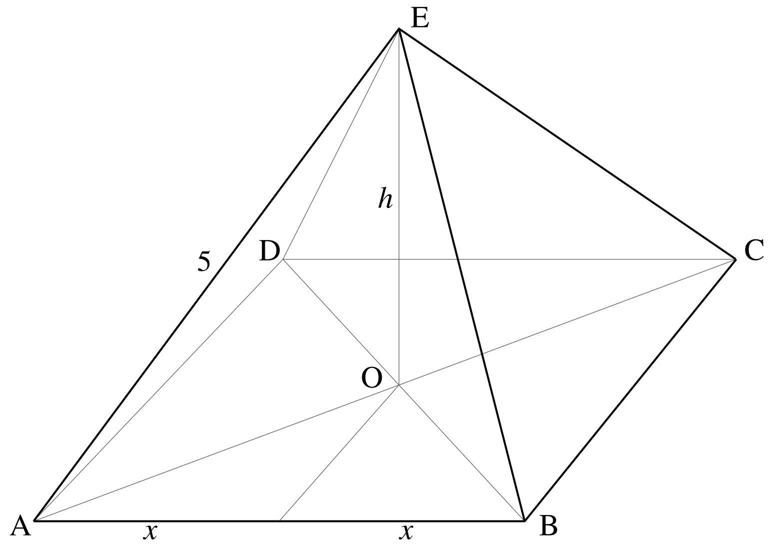


Fig. 13

(i) Show that $2x^2 = 25 - h^2$. **[2]**

(ii) Show that the volume of the pyramid, V m^3, is given by $V = \dfrac{50h - 2h^3}{3}$. **[2]**

(iii) As h varies, find the value of h for which V has a stationary value. **[4]**

(iv) Prove that this stationary value is a maximum. **[2]**

(v) Calculate the angle between the edge AE and the base when h takes this value. **[2]**

[Volume of a pyramid = $\frac{1}{3} \times$ base area $\times$ height.]

14 The cross-section of a speed hump is modelled by the region enclosed by the x-axis and the curve

$$y = \frac{1 - (x - 1)^4}{5}.$$

The graph is shown in Fig. 14.
Units are metres.

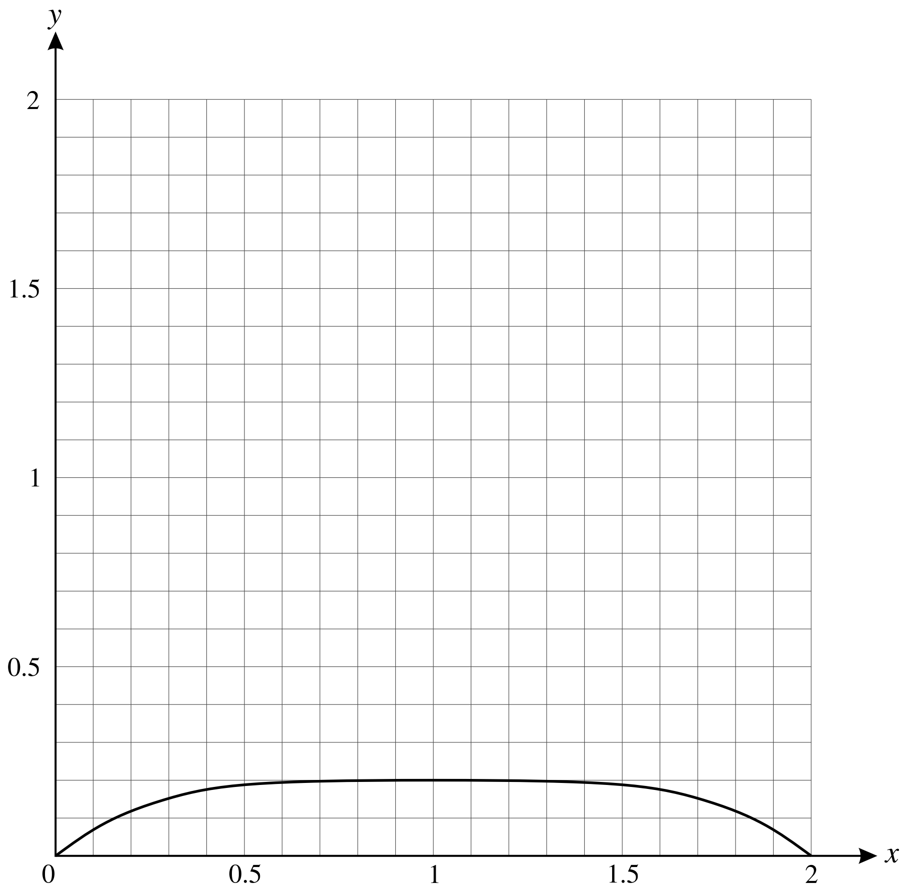


Fig. 14

(a) **(i)** Write down the maximum value of $1 - (x - 1)^4$. **[1]**

(ii) Hence write down the maximum height of the speed hump. **[1]**

(b) Show that $y = \frac{1}{5}(4x - 6x^2 + 4x^3 - x^4)$. **[3]**

(c) Find the area of the cross-section of the speed hump. **[7]**