GCE

Mathematics

Advanced GCE **A2 7890 – 2**

Advanced Subsidiary GCE **AS 3890 – 2**

Report on the Units

June 2009

3890-2/7890-2/MS/R/09

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This report on the Examination provides information on the performance of candidates which it is hoped will be useful to teachers in their preparation of candidates for future examinations. It is intended to be constructive and informative and to promote better understanding of the syllabus content, of the operation of the scheme of assessment and of the application of assessment criteria.

Reports should be read in conjunction with the published question papers and mark schemes for the Examination.

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REPORTS FOR THE UNITS

GCE Mathematics and Further Mathematics Certification

OPTIMISING GRADES FOR GCE MATHEMATICS QUALIFICATIONS

Centres are reminded that when candidates certificate for a GCE qualification in Mathematics they are strongly advised to recertificate for any GCE Mathematics qualification for which they have previously certificated.
For example

- a candidate certificating for A level Mathematics is advised to recertificate for AS Mathematics if this has been certificated in a previous session.
- a candidate certificating for A level Further Mathematics is advised to recertificate (or certificate) for AS Mathematics, A level Mathematics and AS Further Mathematics.

The reason for this is to ensure that all units are made available to optimise the grade for each qualification.
Certification entries are free of charge.

MANUAL CERTIFICATION FOR FURTHER MATHEMATICS

It is permissible for candidates to enter for GCE Further Mathematics with the OCR specification if they have previously entered (or are simultaneously entering) for GCE Mathematics with another specification or Awarding Body. In this case OCR has to check that there is no overlap between the content of the units being used for the GCE Mathematics qualification and the GCE Further Mathematics qualification.
A Manual Certification Form must be completed for each candidate. A copy of the form is available on the GCE Mathematics pages on the OCR website. If you wish to have an electronic copy of the form email your request to fmathsmancert@ocr.org.uk

The table below summarises this.

Qualification	
7890	Candidates are **strongly advised** to apply for recertification for 3890 in the same series as certificating for 7890 if this has been certificated in a previous session.
3892	Candidates are **strongly advised** to apply for recertification (or certification) for 3890 (and 7890 if enough units have been sat) in the same session as certificating for 3892. If a candidate has certificated or is certificating for AS Mathematics or A-level Mathematics with a different specification or Awarding Body then a Manual Certification form* **must** be completed and returned to OCR.
7892	Candidates are **strongly advised** to apply for recertification (or certification) for 3890, 7890 and 3892 in the same series as certificating for 7892. If a candidate has certificated or is certificating for A-level Mathematics with a different specification or Awarding Body then a Manual Certification form* **must** be completed and returned to OCR.

*****A copy of the Manual Certification form is available on the GCE Mathematics pages on the OCR website. It may be photocopied as required, and should be returned to:
The Qualification Manager for Mathematics, OCR, 1 Hills Road, Cambridge, CB1 2EU; Fax: 01223 553242.
An electronic copy of the form may be requested by emailing fmathsmancert@ocr.org.uk When completed the form can be returned by email to the same address.

Chief Examiner's Report – Pure Mathematics

The section in the Specification booklet on synoptic assessment includes the following paragraph.

> Synoptic assessment in mathematics addresses candidates' understanding of the connections between different elements of the subject. It involves the explicit drawing together of knowledge, understanding and skills learned in different parts of the Advanced GCE course through using and applying methods developed at earlier stages of study in solving problems. Making and understanding connections in this way is intrinsic to learning mathematics.

The idea that effective study of mathematics involves an ever-growing body of knowledge and skills is one that eludes many candidates. The evidence seems to be that they regard each module as a self-contained collection of topics with limited connection to any mathematics that they might have met at an earlier stage of their mathematical education. In preparation for each unit, many candidates would benefit by considering and practising those skills and techniques from earlier which might occur in that unit.

It might seem that knowledge of basic algebra would be an exception and that candidates would readily use the appropriate skills of simplification and solution when required. However, the fact that in Core Mathematics 4 so many candidates had difficulty solving the quadratic equation $3 = 2t + t^2$ accurately suggests that even an assumption of basic algebraic competence is not always justified.

4721 Core Mathematics 1

General Comments

Candidates were generally well prepared for this paper and most worked through it in order, making good attempts at questions 1 to 10. The final question proved too demanding for many, although there were some excellent and well-written responses from the most able candidates.

As in previous sessions, the graph sketching and graph transformations questions were the least successfully answered. Candidates were often reluctant to use their graphical skills. However, as well as the requirement to produce a sketch in Question 6 and in Question 10(iii), a rough diagram may well have assisted candidates in Questions 9 and 11, and also in solving the quadratic inequality correctly in Question 8(ii).

Comments on Individual Questions

1) (i) Differentiation is a topic about which most candidates feel confident and this opening question was answered well by the vast majority of candidates. There was a small minority who differentiated $\frac{1}{x^2}$ as $\frac{1}{2x}$ or who started with $\frac{1}{x^2} = x^{-\frac{1}{2}}$, but these errors were fairly rare.

(ii) This notation for the second derivative of y was widely understood and nearly all candidates followed through correctly from part (i).

2) Although most candidates knew to multiply both numerator and denominator by the 'conjugate' of the denominator, there were many errors in the subsequent working and only the better candidates scored all 4 marks; in particular, dividing by -3 correctly to get the answer in the form required proved beyond many. Those who did not know the correct method usually multiplied by $\sqrt{7}$ or by $\frac{8-\sqrt{7}}{2-\sqrt{7}}$.

3) (i) The correct power of -2 was by far the most common answer although a few candidates thought that $n = \frac{1}{2}$ or left their answer as $\frac{1}{3^2}$.

(ii) This question was almost always answered correctly and, together with part (i), showed that candidates had a good understanding of the notation for powers of x.

(iii) However, this part of the question was very poorly done. A surprisingly large number of candidates of all abilities rewrote 9 as 3^3 and then expressed $(3^3)^{15}$ as 3^{45} and gave a final answer of 3^{55}. Another very commonly seen wrong answer was 3^{27} obtained from adding rather than multiplying the powers in their expression $(3^2)^{15}$. Other candidates thought that 9^{15} was equivalent to 3^{16} and a significant number just combined the given expression and gave the answer 27^{25}. Candidates should be encouraged to think about the meaning of powers in expressions such as these rather than trying (inappropriately) to simply apply rules.

4) Candidates usually perform well in questions requiring the solution of simultaneous equations but the fact that one of the given equations contained both x^2 and y^2 caused problems for many. In their attempts to eliminate one of the variables, some candidates 'square rooted' the first expression, writing $2x + y = \sqrt{10}$. Others 'squared' the linear expression term by term, resulting in either $4x^2 + y^2 = 16$ or $4x^2 - y^2 = 16$ or even $2x^2 \pm y^2 = 16$. They then added, subtracted or substituted but rarely obtained a 3-term quadratic of any kind. This meant that a disappointingly large proportion of candidates scored only the first mark for this question. Those who rearranged the linear expression and then substituted generally fared much better, as long as they were able to expand $(2x-4)^2$ correctly. As in previous sessions, some good candidates forgot to give the *y*-values.

5) (i) All but the very weakest candidates scored well on this question. The method for expanding three brackets was well known, although a small number of candidates combined the brackets in pairs and then added the resulting quadratic expressions together. Candidates who tried to combine all three brackets together in one go were sometimes successful, but often left out a term or made an error with a power.

(ii) Although a significant number of candidates expanded the given expression completely, many were able to spot the required terms immediately and combine them correctly. Some candidates used a grid method successfully.

6) (i) It was pleasing to note that only a few centres are still issuing graph paper and that most candidates realised that graph paper was not necessary for the sketch. Although some candidates mistakenly drew their graph in the second quadrant and others drew a graph in the correct quadrant but with the wrong curvature, many approximately correct graphs were seen. However, full marks were rarely awarded as inadequate care was taken with sketching. Very many otherwise well-drawn curves became horizontal or started to curve towards the *x*-axis at the end.

(ii) As in part (i), it was rare for candidates to score full marks. In this part it was because they failed to describe the transformation precisely, even though 'up 5' implied that they understood how the curve had been transformed. The word 'translation' was rarely seen, 'shift' or 'move' being the most common alternatives. Many candidates stated that the curve had been stretched by a scale factor of 5 or moved parallel to the *x*-axis.

(iii) Candidates proved even less successful at stating the equation of the curve after the given transformation. It was extremely rare to award both marks here. Candidates who understood that the change applied to the *x* variable but thought that the equation was $y = -\sqrt{2x}$ were able to gain one mark. There were, however, far more wrong equations such as $y = -2\sqrt{x}$, $y = \sqrt{x} - 2$ or $y = \dfrac{\sqrt{x}}{2}$ seen. Overall, question 6 proved to be one of the least successfully answered questions on the paper confirming that, for many candidates, this remains one of the most challenging parts of the specification.

7) (i) This was a straightforward question on completing the square. The vast majority of candidates correctly obtained $a = \frac{5}{2}$ but lost the final mark because they could not square this fraction correctly. There were many different wrong values for a^2 seen, most commonly $4{\cdot}25$, $5{\cdot}25$, $\frac{25}{2}$ and $\frac{10}{4}$.

(ii) Few candidates realised that their answer to part (i) could be used here and most attempted to complete the square again. The centre of the circle was generally stated correctly; the wrong answer $\left(-\frac{5}{2},\ 0\right)$ appearing occasionally. Far fewer candidates were able to find the correct radius and some who did obtain the constant 6 forgot to take its square root. There was a significant number of candidates who appeared to be very unsure of the equation of a circle and left this question unanswered.

8) (i) The responses to this linear inequality question showed much improvement on previous years and most candidates understood how to solve it. A small minority failed to change both constants when rearranging and there was much poor arithmetic in evidence from candidates of all abilities.

(ii) The method of solving a quadratic inequality still eludes most candidates. By far the most common final answer seen was $x > 4$, earning only 1 mark out of 3. Candidates who remembered the negative square root of 16 often simply wrote $x > \pm 4$. Candidates should be encouraged to do a sketch-graph in such questions.

9) (i) As in previous sessions, this coordinate geometry question was a good source of marks for candidates of all abilities. The vast majority of candidates used Pythagoras' theorem correctly to find the length of the line *AB*, although many did not recognise 169 as the square of 13.

(ii) Of those candidates who relied on memorising the midpoint formula, some found the difference between coordinates and halved while others paired an *x*- with a *y*-coordinate. Had this method been supported by a quick sketch showing the approximate positions of points *A* and *B*, candidates might have been able to spot their errors and correct their working.

(iii) While many candidates completed this question perfectly, in other scripts careless work led to a significant loss of marks. It was very common to see $\frac{9 - {}^{-}3}{{}^{-}1 - 4}$ become $\frac{12}{-3}$ which meant that the correct equation for the line could not be obtained. Even many of those who had a completely correct, unsimplified equation for the line were unable to rearrange it into the required form without making a slip. Some candidates found the equation of the line through (1, 3) perpendicular to *AB* rather than parallel to it.

10) (i) It was not altogether surprising that many candidates, including some of the most able, eschewed factorisation and instead attempted to complete the square or use the formula. However, it should be noted that those who attempted to factorise as their preferred method were usually successful while the others struggled to manipulate the numbers involved in their alternative methods.

10 (ii) This was one of the most successfully answered part questions. There was a variety of methods used. Differentiation was the most popular, although a small but significant proportion of those differentiating then substituted the values of x found in part (i) into their expression rather than setting it equal to zero and solving. Other candidates completed the square, often correctly, and a small number used the symmetry of the graph to find the mean of the roots obtained in part (i).

(iii) Although the vast majority of candidates knew that the graph was U-shaped, the quality of sketches was once again very disappointing. Many seemed intent on making the *y*-intercept the minimum point of the graph, which meant that, if they plotted their roots approximately correctly, the graph lost all symmetry. Other attempts had a very pointed vertex or straight sides. This sketch was considerably less demanding than some set recently and most candidates knew the general shape so it was dispiriting to see so many candidates lose marks here.

(iv) A pleasing proportion of candidates appeared to be familiar with this sort of question and realised that the answer required an inequality. There was a reasonable number of correct answers although $x \geq 0$, perhaps from use of a graph with its minimum point on the *y*-axis, and inequalities involving the roots, were common wrong answers.

11) (i) This last question proved too demanding for all but the best candidates. A large number of candidates made no attempt but it was not clear whether they had run out of time or ideas. Only a small minority realised that differentiation was required to relate the gradient of the curve to the gradient of the normal. Of those who correctly established that the gradient of the normal was $-\frac{2}{3}$, most equated either this gradient or its negative reciprocal to the equation of the curve and solved simultaneously. There were, however, concise and confident solutions from some candidates.

(ii) There was a follow-through mark available for candidates who substituted their value for k into the equation of the curve to find the *y*-coordinate of P and candidates who persevered with part (ii) were usually able to earn this mark. Unfortunately, many candidates then used $x = 0$ rather than $y = 0$ when attempting to find the coordinates of Q. Those who had drawn a rough diagram were often able to earn the final method mark for a correct method to find the area of the triangle. However, a surprising number of candidates assumed that the triangle was right-angled at P and did much unnecessary calculation to find the lengths OP and PQ.

4722 Core Mathematics 2

General Comments

This paper was accessible to the majority of candidates, and gave them an opportunity to demonstrate their knowledge. All but the very weakest were able to make an attempt at most questions, with questions 4 and 8 proving to be a good source of marks. There were also several parts that were sufficiently stretching for more able candidates.

It is important that candidates can express themselves clearly, and it was pleasing that the majority of scripts were well presented and showed clear detail of the methods used, though when an answer is given sufficient working must be shown. However, a number of candidates struggled when asked to provide an explanation in words.

It was disappointing that a significant number of marks were lost through an inability to manipulate algebraic expressions, even though candidates were obviously familiar with the topics being tested. There were a number of mistakes made when solving linear equations, dealing with simultaneous equations and rearranging an equation. Common errors included not dividing each term in an equation by a quantity, and squaring an equation term by term when a square root was involved. Candidates should also appreciate the need to make effective use of brackets, both to convey their intention to the examiner but also to ensure that subsequent work is accurately evaluated.

Candidates should ensure that they always use a method appropriate to the question posed. When a method is specified, such as questions 3 and 7(ii), they will gain no credit if an alternative method is used. On other questions, inappropriate methods are often less accurate and much more time consuming.

Comments on Individual Questions

1) (i) A surprising number of candidates struggled with this question, which builds upon prior GCSE knowledge. A minority assumed it was a right-angled triangle, but most realised that the cosine rule should be used. Whilst most could correctly quote from the formula book, a number could not rearrange the formula into the required form. The more astute realised that the largest angle must be opposite the largest side and gained all three marks easily. Many more found all three angles, often continuing even if the first angle found was obtuse. A number of candidates, having found one of the two acute angles, then used the sine rule to attempt the required angle without appreciating that it could be obtuse. Premature approximation led to a lack of accuracy in some answers and other candidates, having found the correct value for $\cos A$, decided to ignore the negative sign resulting in an angle of 65°.

(ii) Most candidates could attempt to use the correct formula for the area of a triangle, though a few failed to use the sides that were consistent with the angle that they were using.

2) (i) A number of elegant and concise solutions were seen, but many candidates struggled on this question. Most candidates gained at least the first method mark by producing a correct expression for an nth term, most typically the twentieth term as this had a numerical value. Many then found it difficult to link together the other two terms, often making an incorrect assumption such as the value of the tenth term was half of the value of the twentieth term. There were some more creative methods seen, equating the differences between the terms, and other candidates resorted to trial and improvement.

(ii) Nearly all candidates could correctly quote the relevant formula and substitute their values for *a* and *d*.

3) Most candidates gained the first three marks easily for introducing logarithms throughout and dropping the powers to obtain the correct linear equation, though a number of candidates omitted the bracket around $x + 1$. However, many then struggled to make any further progress as they were unable to either gather the like terms, or deal with the algebraic fraction that ensued. Candidates who moved into decimals were generally more successful, though there were a number of elegant algebraic solutions seen.

4) (i) This question proved to be very straightforward for the candidates, and was a good source of marks for many. Most attempted to use the binomial theorem, though there were a few attempts to expand the brackets. Whilst most candidates could state a correct unsimplified expansion, it was disappointing how many of them then struggled to deal with the indices involved. The most common error was for (×2)3 to become ×5, but other terms also caused problems. Some candidates failed to make effective use of brackets, which led to sign errors from powers of -5, and also led to each term being the sum rather than the product of its component parts.

(ii) Most candidates gained at least two marks in this question for correctly integrating their expansion, though a few failed to appreciate the link between the two parts of the question. There was an easy mark available for the constant of integration, and it was disappointing that a number of candidates either failed to include it, or spoiled their answer by still having an integral sign or a $\mathrm{d}x$ present.

5) (i) Responses to this question were very varied. Many candidates managed to obtain the first two marks for an angle of 15°. However, only the most able candidates could correctly find a second angle, with 165° being the most common error. A few candidates did find both the required angles, but spoiled this by including additional angles in the given range. However, a significant number of candidates failed to make any progress on this question. The most common error was to first divide by 2 and then take inverse sine, though a few were obviously not familiar with the conventions of trigonometric notation and treated $\sin 2$ as the coefficient of x.

(ii) This question was poorly done, with a number of candidates only getting the first method mark for correctly substituting for $\sin 2x$, and some didn't even achieve this. A surprising number could not even eliminate the 2s, and others used incorrect methods such as squaring the coefficients involved. However, the more able candidates obtained the correct quadratic and then attempted a sensible method to solve it. A common error was to divide by $\cos x$ thus losing one of the solutions, whereas those who used the quadratic formula tended to obtain both values for $\cos x$.

6) Most candidates appreciated the need to integrate and attempted to do so. Whilst the first term was usually correct, the second term caused more problems with ½ $a2$ being a common error. A number of candidates omitted the constant of integration, which meant that no further credit could be gained. However, a number of candidates attempted to make inappropriate use of the given gradient function, most typically substituting one, or both, of the given coordinates in an attempt to find a value for a, even if they then integrated. Other candidates felt that the equation of the chord joining the two points would be relevant, and another common error was to use the given algebraic gradient in an attempt at $y = mx + c$. There were a number of concise and accurate solutions seen, though some of these fell at the last hurdle by failing to give their expression as an equation

involving y.

7) (i) This part was very well done, with most candidates gaining full marks. The majority attempted to use the remainder theorem, though a lack of brackets sometimes resulted in an incorrect answer. Rather than use the most efficient method, a number of candidates decided to use long division or coefficient matching. This proved to be less accurate, and also took much longer.

(ii) In this part of the question the method was specified, though a significant minority ignored this instruction and thus gained no credit. Of those who attempted use of the factor theorem, the majority substituted the correct value of x though some lost a mark by not showing sufficient detail in their working.

(iii) Most candidates were able to make a good attempt at finding the quadratic factor, through a variety of methods. Despite the hint in part (ii), a few candidates could not identify the linear factor either using $(x + 2)$ or starting from scratch with the factor theorem. Relatively few candidates failed to gain the final mark by neglecting to give their final answer as a product.

(iv) In general this part was well answered with most candidates gaining at least one mark, though some seemed to confuse 'real' with 'negative'. Some candidates failed to gain the second mark as they simply stated that the discriminant was negative without actually demonstrating the working to support this conclusion. Others found the correct value for the discriminant and concluded that the quadratic had no real roots, but failed to consider the one real root from the linear factor.

8) (a) This entire question was done very well. In this part, most candidates were able to correctly quote the formula for the area of the sector and then attempt to find the radius. Most subsequently gained full marks for correctly finding the perimeter of the sector, though some simply found the length of the arc. A disappointing number of candidates lost marks through poor algebraic skills, with a common error being to divide by 2 rather than multiply when dealing with the factor of ½. A few candidates persist in converting the angle from radians to degrees. If appropriate methods are then used, this can be a successful, if rather long-winded, method but too many then attempt to use the angle in degrees in the radian formulae, thus gaining no credit. Some candidates still feel the need to append a π to any angle given in radians.

(b)(i) Most candidates gained both of the marks available, mostly through using the appropriate formula for the *n*th term of a GP though some employed less formal methods. A surprising number of candidates were unsure as to what to use as the first term, with a significant minority using 32, though 36 and 10 were also seen.

(ii) This question was also done very well with most candidates gaining both marks. Whilst most attempted to use the correct formula for the sum of a GP, a few used an index of $n - 1$ not n, and others listed and summed the relevant terms.

(iii) The sum to infinity was usually correctly calculated, but the explanations were often vague and lacked clarity. It was pleasing to see some candidates discuss the convergence of the sequence, or focus on the value of r, but many more candidates commented on the areas getting smaller. Some candidates discussed how the limit would be approached, but then neglected to state its value.

9) (i) Whilst most candidates seemed to have a basic understanding of the shape of the required graph, a lack of care when drawing it often led to the mark being lost. Candidates were expected to show a reasonable amount of the graph in both quadrants, with enough detail to demonstrate its asymptotic nature in the second quadrant and having an increasing gradient throughout. Whilst the use of graph paper is discouraged, candidates are expected to use a ruler for the axes otherwise it is difficult to establish the significant features of the graph. All too often (0, 1) was given as the intercept on the *y*-axis.

(ii) This question was very poorly done, with most candidates gaining only the first mark for introducing logarithms. The majority of candidates then immediately dropped the powers, so that $\log 20k^2$ became $2 \log 20k$, and then attempted a division by $\log 4k$ rather than separating into two terms. Candidates who first divided by 4 tended to be more successful. Another common error was to look at the given answer and, rather than take logs to base k throughout, move the k from the terms to become the base of the logs. In questions where the answer is given it is important that each step of the proof is clearly shown.

(iii)(a) Candidates seemed familiar with the trapezium rule and most could make a good attempt to apply it. There were the usual errors of using incorrect *x* values, attempting integration first, or omitting the necessary brackets, but the most common errors involved an inability to correctly evaluate the *y*-coordinates. It was common for $4k0$ to become either 0 or 1, and also for $4k0.5$ to be used as $(4k)0.5$ or $40.5k$. Some assigned a particular value to k. However the majority of candidates could gain at least two marks on this question.

(b) Most candidates could gain the first mark for equating their expression to 16, but struggled to solve the disguised quadratic that ensued. The most common error was to square each term. Of those who did manage to employ an appropriate method, most arrived at the final answer of 9 but quite a few lost the final mark by failing to discard 25.

4723 Core Mathematics 3

General Comments

This was a paper with three particularly challenging requests – question 6 taking a possibly unfamiliar approach to differentiation, question 7(ii)(b) needing some insight and question 9(b) requiring candidates to decide on the appropriate mathematics to use and to provide a convincing explanation. It was therefore encouraging to note that a significant number of candidates recorded high marks, including some with full marks for the paper. These were candidates who displayed competent mathematical skills and presented their work with care.

Candidates of more modest ability found several questions where they could do well; questions 2, 4 and 5 were accessible to most and other questions had aspects which presented difficulties only to relatively few candidates. There was no indication that time pressure was a factor.

The level of algebraic competence shown by many candidates was disappointing – and not just among those candidates recording a low total for the paper. Care with signs, accuracy when solving simple equations, appropriate use of brackets and knowledge of logarithm properties are expected at this level as a matter of course but, too often, careless mistakes or basic misunderstanding meant that marks were lost. Mistakes equivalent to $\sqrt{a+b}=\sqrt{a}+\sqrt{b}$, $(a+b)^2=a^2+b^2$ and $\ln(a+b)=\ln a+\ln b$ were noted by examiners far too often.

Comments on Individual Questions

1) There was a mixed response to this question on standard graphs. Many candidates made the correct choices but it was clear that many other candidates were not familiar with these graphs at all. In particular, it was apparent that many candidates did not distinguish between the notations for inverse functions and reciprocal functions so that $y=\cos^{-1}x$ was a common wrong choice for Fig.1.

2) This question was generally very well done with most candidates aware of the formula for volume and able to integrate $(2x-3)^4$ without trouble. π was sometimes absent or disappeared partway through the solution and another error to occur was an upper limit of $\frac{2}{3}$ rather than $\frac{3}{2}$. Some candidates were not meticulous with the signs when applying the limits and had to explain away the negative sign in their answer of $-\frac{243}{10}\pi$. A number of candidates chose to expand $(2x-3)^4$ before integrating; sometimes this was concluded successfully but this inefficient method necessarily took extra time.

3) The identity $1+\tan^2\alpha\equiv\sec^2\alpha$ was not as well known as anticipated and, in part (i), many candidates could either make little progress or used an incorrect version of the identity. Unfortunate errors such as $(m+2)^2=m^2+4$ and $\tan^2\alpha-\tan^2\beta=16$ leading immediately to $\tan\alpha-\tan\beta=4$ were not seen infrequently. But the most significant cause of incorrect solutions involved the absence of necessary brackets. The equation involving m often appeared as $1+(m+2)^2-1+m^2=16$; sometimes the subsequent simplification assumed the absent brackets and no penalty was applied but, often, incorrect values of m were the inevitable outcome.
Many candidates answered part (ii) correctly and some credit was available for those candidates with an incorrect value of m. Attempts such as $\tan(\alpha+\beta)=\dfrac{\tan 5+\tan 3}{1-\tan 5\tan 3}$ were

not uncommon and revealed a fundamental lack of understanding. Other candidates resorted to finding the actual values of α and β.

4) This question was answered very well by many candidates. For others though, part (i) proved troublesome. Not all could integrate e^{3x} correctly and, when applying the limits, it was again the case that the absence of necessary brackets caused errors. It was also surprising that many candidates, having reached $e^{9a} + 2e^{3a} - 3e^{a} = 300$, did not spot that the next step should have been a rearrangement to $e^{9a} = \ldots$. The introduction of the logarithm was not always handled correctly and statements such as $\ln(e^{9a}) + \ln(2e^{3a}) - \ln(3e^{a}) = \ln 300$ appeared quite frequently. The fact that the answer to part (i) was given did prompt some candidates to go back over their solutions and make appropriate corrections; this was commendable but candidates are advised that such corrections must be applied to the whole of their solution and not just to the step before the conclusion.

The iteration process in part (ii) was carried out well. The only error to occur with any frequency was a final value of 0.6308, the result of truncating to 4 decimal places rather than rounding to 4 decimal places.

5) This question was another good source of marks for many candidates who were able to answer each of the three parts competently. In part (i), there were a few candidates who formed the composition the wrong way round or who considered the quadratic equation $(3x-2)(3x+7)=0$. Others substituted $x=0$ rather than $y=0$ and concluded with the value 19.

In part (ii), most candidates first found $g^{-1}(x)$ and then solved the equation $g(x) = g^{-1}(x)$. Most candidates knew how to find the inverse function although there were some who thought the inverse function was $\frac{1}{3}x - 7$. Some candidates immediately recalled that the two graphs would meet on the line $y = x$ and they opted for the easier task of solving $3x + 7 = x$.

Part (iii) was also answered well by many candidates, the method of squaring both sides of the equation being the more popular approach. In both this part and in part (ii), a significant number of candidates omitted to find the *y*-coordinate of the point of intersection. There was considerable uncertainty about the conclusion of part (iii). Some candidates offered two points, one with an *x*-coordinate of $-\frac{5}{6}$ and the other with an *x*-coordinate of $\frac{5}{6}$. Others decided that, in a context involving modulus functions, the value $-\frac{5}{6}$ had to be changed to $\frac{5}{6}$. A further error occurred not infrequently; the *x*-coordinate was substituted into either $(3x-2)^2$ or $(3x+7)^2$, resulting in a *y*-coordinate of $\frac{81}{4}$.

6) This question was not answered well. Most candidates recognised that the chain rule was needed in part (i) but the details were often incorrect. The most common error was the expression $\frac{1}{2} \times 10 \times (-4y)(37 + 10y - 2y^2)^{-\frac{1}{2}}$; perhaps candidates giving this had not previously met a case of the chain rule where differentiation of the main function gave an expression with two terms. The answer $5 - 2y(37 + 10y - 2y^2)^{-\frac{1}{2}}$ also occurred often; this did not earn the accuracy mark in part (i) even if the absent brackets were assumed by the candidate in subsequent work.

It was apparent from part (ii) that many candidates did not realise that there is a difference between $\frac{dy}{dx}$ and $\frac{dx}{dy}$. Their approach was to substitute 3 into their expression from part (i) and to use the resulting value as the gradient of the tangent. Others seemed to detect a difference but decided that substituting 7 into their expression would provide the required gradient. Many candidates did of course take the reciprocal and obtained the correct answer with ease. Impressive awareness of the notation involved was shown by those candidates who used the value $-\frac{1}{7}$ from their expression for $\frac{dx}{dy}$ and formed the equation of the tangent in the form $x-7=-\frac{1}{7}(y-3)$.

7) The first two parts of this question involved routine requests but the general response from candidates was somewhat disappointing. Many candidates did earn the first seven marks of the question without any difficulty but several errors were widespread on the scripts of many other candidates. The minus sign caused some problems with candidates deciding that $\tan\alpha=-\frac{6}{8}$ and going on to offer $10\sin(\theta+36.9°)$ as the equivalent expression. Others confused sine and cosine and offered either $10\sin(\theta-53.1°)$ or $10\sin(\theta+53.1°)$.

In part (ii)(a), two particular errors were common. Many candidates found the first angle, $101°$, without difficulty but then, sometimes with reference to a sketch of $y=\sin\theta$, offered $79°$ as a second answer. The second error occurred so often that it did not seem to be just the result of momentary carelessness. Having listed values of $\theta-36.9°$ correctly as $64.2°,\ 115.8°$, many candidates proceeded to calculate $64.2°-36.9°$ and $115.8°-36.9°$.

Part (ii)(b) was challenging and it was a small minority of candidates who appreciated what was involved and reached the answer 60. Many candidates did not seem clear about what was being asked and tried to solve some equation. Some merely assigned values of 1 to $\sin x$ and $\cos y$ and of 0 to $\cos x$ and $\sin y$, thereby claiming a maximum value of 44. Others did note the link with the earlier work but this often resulted in a final answer of 20. The fact that the smallest possible value of $\sin(y-36.9°)$ was -1 eluded most.

8) The vast majority of candidates recognised the two transformations needed in part (i) although an award of all three marks was denied to those who used inappropriate terminology such as ‘enlargement’, ‘move’ and ‘shift’. Almost all candidates recognised that the equation $2\ln(x-6)=\ln x$ had to be solved in part (ii) but the attempts at solution were often poor. Some candidates just stated the result 9 with no supporting evidence but a common first step was $2(x-6)=x$. This gave 12 for the value of a but there was no evidence of candidates using their calculators to check the validity of this value. Those candidates who correctly proceeded to $(x-6)^2=x$ usually had no further difficulty and often provided a sound reason for rejecting the value 4.

Many candidates failed to establish the correct lower limit for the shaded region in part (iii). Overlooking the evidence provided by the translation in part (i), they chose 6 and did not seem at all concerned at including $2\ln 0$ in their calculation of the approximate area. General knowledge of the process for using Simpson’s rule was sound but the various errors mentioned meant that not so many candidates reached the correct answer.

9) Most candidates managed to earn at least a few marks from this question but convincing and complete solutions were a rarity. The need for the quotient rule was noted in part (a) but the details were not always accurate. Differentiation of kx^2 occasionally produced $2k$ and, again, the absence of brackets sometimes caused difficulties. Even for those candidates who had differentiated correctly, an appropriate conclusion proved elusive. Faced with $4kx=0$, many candidates did not take the apparently obvious next step of stating $x=0$, the location of the one stationary point. Indeed the next step was sometimes $x=-4k$.

In part (b), the product rule was usually applied correctly and many candidates reached the equation $mx^2 + (m^2 + 2)x + m = 0$, although the disappearance of e^{mx} was seldom justified. At this point, some candidates merely commented that, because this was a quadratic equation, there must be two stationary points. Others applied the formula for the solution of a quadratic equation and mentioned the ± as justification. Some candidates gave m a particular value, usually 1, but doing so immediately meant that little further credit was available. Those candidates who did consider the discriminant of the quadratic equation and provided a convincing reason why it was positive were not very many but they did display pleasing mathematical skill.

4724 Core Mathematics 4

General Comments

The overall impression was that this paper was more difficult than some previous papers. There is no doubt that many candidates enter for this paper with little or no preparation; a large number are let down by poor algebra, incorrect cancelling of rational fractions and poor use of brackets leading to sign errors. It was very surprising to see so many candidates, in their final pure paper, unable to solve a quadratic equation. However, the better candidates coped well with the more difficult questions whilst the weaker ones tended to give up at early stages, often relying on drilled methods which were not always appropriate.

Comments on Individual Questions

1) This question was generally well answered with the methods used being fairly evenly split between long division and use of identities. Those attempting long division were generally successful, sign errors being the main problem. Those using identities usually realised that the remainder would be linear, but these candidates were, in general, more careless and not as successful as those using long division.

2) The majority knew they had to change the '$\mathrm{d}x$' term and most progressed to $\int \frac{1-\tan^2\theta}{1+\tan^2\theta}.\sec^2\theta\ \mathrm{d}\theta$ with fewer obtaining $\int 1-\tan^2\theta\ \mathrm{d}\theta$. This provoked the first main difficulty: whether to use $\int \tan\theta\ \mathrm{d}\theta = \ln\sec\theta$ leading to $\int \tan^2\theta\ \mathrm{d}\theta = \ln\sec^2\theta$ (this was often seen) or whether to use $\sec^2\theta = 1+\tan^2\theta$. Those using the second alternative frequently failed to convert the integrand to $2-\sec^2\theta$ because of carelessness with brackets and/or signs. Very few omitted an attempt to change the limits but a significant number used degrees, which were accepted at the initial stage but not, of course, in the answer.

3) (i) Few scored full marks in this part; the expansion of $\left(1+\frac{x}{a}\right)^{-2}$ was generally well accomplished but the exterior factor varied between a, a^2 and a^{-2}.

(ii) The correct process was usually attempted but the execution was poor with incorrect powers and/or signs appearing. A large number reached the stage of $\frac{3}{a^4}+\frac{2}{a^3}=0$ but were unable to solve this; many who correctly reduced the equation to $3+2a=0$ obtained a = $\frac{2}{3}$ or $\frac{3}{2}$ or $-\frac{2}{3}$.

4) (i) A few differentiated uv as $\mathrm{d}u.\mathrm{d}v$ but the vast majority were clear as to what they were expected to do. Sign errors in the derivatives of $\sin 2x$ and $\cos 2x$, coupled with the negative sign in $\sin 2x - 2\cos 2x$ and frequent omission of brackets all contributed to a failure to obtain $5\,\mathrm{e}^x\sin 2x$. A few converted $\sin 2x$ to $2\sin x\cos x$ and $\cos 2x$ to $\cos^2 x - \sin^2 x$ before differentiating but rarely did this make the problem any easier.

(ii) The 'Hence' was often ignored, even by those who had a correct answer to part (i). Again there were many sign errors after the limits had been used. For those who had a wrong answer to part (i), a lengthy double integration by parts was necessary and few correct solutions were seen.

5) (i) The expression for $\frac{dy}{dx}$ was usually found correctly. The association of $(3,-9)$ with $(2t+t^2, 2t^2+t^3)$ in order to find *t* was a difficult process and the standard solving of $3 = 2t + t^2$ proved too awkward for many. A few candidates noted (perhaps by reading part (ii) early) that something might emerge by considering $\frac{y}{x}$ and this was an excellent way of finding *t*. Those solving by the standard method who produced both -3 and 1 were generally careful in showing that *t* = 1 was not a valid solution to the second equation.

(ii) Most good candidates obtained $\frac{y}{x} = t$ but a very large number stated $\frac{2t^2+t^3}{2t+t^2} = \frac{2t^2}{2t} + \frac{t^3}{t^2} = t + t = 2t$. Unfortunately, many obtaining '*t*' were unsure what to do with it even though a clear specification item is conversion between parametric and cartesian forms.

6) (i) This was well done by the majority of candidates. Most used the identity $4x \equiv A(x-3)^2 + B(x-3)(x-5) + C(x-5)$; it has to be said that those using $x = 3$ and $x = 5$ in this identity were more often successful than those comparing coefficients.

(ii) The first two integrations almost always involved natural logarithms but whether the first became $\ln(x-5)$ or $\ln|x-5|$ was another matter. On many scripts, $\ln(-3)$ was seen and statements such as '$\ln|-3| = -\ln 3$' and '$\ln|-3|$ does not exist' indicated that the modulus notation was not always fully understood. Another problem that occurred was the question of whether $\int \frac{A}{x-5}\,dx = A\ln|x-5|$ or $\frac{1}{A}\ln|x-5|$. The third integration, that of $\frac{C}{(x-3)^2}$, was often thought to also involve a natural logarithm.
To accommodate the various problems arising in this part, the mark scheme included several follow-through marks; this ensured that candidates were not unduly penalised by a single error.

7) (i) The first part was well done on the whole although, once again, there were careless mistakes with signs when solving the two simple simultaneous equations. The second part, showing that **u** is a unit vector, was usually ignored; some candidates may have just missed this second request in part (i) but it seems more likely that the idea of a unit vector had not been introduced in many centres.

(ii) This was one of the most successful parts of the paper. A very minor issue that arose was that, although the answer was requested 'to the nearest degree', quite a number of candidates gave it in radian mode. Candidates ought to read questions more carefully to avoid such errors; on this occasion the error was not penalised.

8) (i) Implicit differentiation is now a well-understood topic and candidates rarely failed with the basic procedures. Just a few started with $\frac{\mathrm{d}y}{\mathrm{d}x} =$, but hardly anybody brought this term into action. Because the answer to this part of the question was given, the working was expected to be sufficiently detailed to indicate all the necessary steps. On this occasion, it was deemed insufficient to go directly from $28x - 7x\frac{\mathrm{d}y}{\mathrm{d}x} - 7y + 2y\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ to the given answer. Also it was expected that the given answer would be given as it is shown so, for example, a final answer of $\frac{7y - 28x}{2y - 7x}$ was not awarded the final mark unless it was then written in the required form.

(ii) There was a pleasing response to this part, especially since it was fairly unstructured. It was, however, purely a matter of simple thought and, provided the candidate sat and deliberated, it was relatively easy to get through. As in question 5, there were some poor attempts at solving the quadratic equation produced by substituting $x = 1$. Some candidates assumed that, because $\frac{\mathrm{d}y}{\mathrm{d}x}$ was involved, they had to find out when it was 0; others, having found the values of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the required points, then used $\frac{-1}{\frac{\mathrm{d}y}{\mathrm{d}x}}$ as the gradient for their tangents. Three very common errors were seen at the final stage when concentration lapsed: $4 = 7x - 4$ followed by $x = 0$; $7x = 8$ followed by $x = \frac{7}{8}$; forgetting that they had used $y = 4$ and managing to find a value of $y \neq 4$.

9) (i) Candidates should realise that, if the mark to be awarded is only 1, then there will be little work to do. The question stated that the temperature increased at a constant rate (k_1° per second) from 40° to 60°, which only required the candidate to say that the rise of 20° would take $\frac{20}{k_1}$ seconds. However, most candidates seemed to think a differential equation would be helpful at this early stage.

(ii) It was only at this stage that 'differential equation' was mentioned. The negative sign was often omitted; $\frac{\mathrm{d}\theta}{\mathrm{d}t}$ is the rate of increase and the temperature was decreasing. However, any absence of a negative sign was penalised only once. The left-hand side was often shown as $\frac{\mathrm{d}t}{\mathrm{d}\theta}$ or $\frac{\mathrm{d}k}{\mathrm{d}t}$; other variants were seen but usually $k_2(\theta - 20)$ did appear somewhere.

(iii) This question was a simple two-stage operation with the temperature going up and then going down. The separation of variables was usually recognised as the correct approach but some candidates seemed unaware that this only applied to the second stage, and that the first stage had already been covered in part (i). Of those who remembered to introduce a constant 'c', a considerable number thought that when *t* was 0, θ was 40 (instead of 60). As a result, many candidates were unable to conclude this question successfully.

4725 Further Pure Mathematics 1

General Comments

This paper proved to be a little more demanding than previous ones, with only a relatively small number of candidates scoring full marks on questions 5, 8 and 10. However, most candidates were able to provide good attempts at a reasonable number of the questions, showing a good understanding of most parts of the syllabus. There was little evidence that candidates were short of time and most answered the questions sequentially.

Comments on Individual Questions

1) Most candidates answered this question correctly. The most common error was to subtract S_{101} instead if S_{100} and many candidates did not give the full answer, but rounded to 3 significant figures. A small number used the formula for $\sum r^2$ and some evaluated S_{149}.

2) This was answered correctly by most candidates, some trying various values for a and b, rather than solving a pair of simultaneous equations. A common error was to attempt to multiply by $A - 1$ or $B - 1$, or both, which showed a rather serious gap in the understanding of scalar multiplication of a matrix and matrix addition.

3) (i) This was answered correct by most candidates.

(ii) This part was also well answered, the most common error was to use z rather than it's conjugate in the multiplication.

4) Most used the sum and product of the roots of the given equation correctly, while others found the quadratic equation with reciprocal roots correctly and usually went on to find the correct answer. A significant minority thought that $\frac{1}{p} + \frac{1}{q} = \frac{1}{p+q}$.

5) (i) This was found to be difficult by the majority of candidates. The given substitution was used, but then many candidates thought that each of the 3 terms could be squared to obtain the required cubic. Others simply squared their equation in u, which does not remove all the fractional indices.

(ii) This part was often not attempted, with candidates being unable to connect the required value with the coefficients of their cubic equation. Some found, correctly, an identity connecting the symmetric functions for the given cubic and found the value -70 from this.

6) (i) Most found the modulus and argument correctly, the most common error was to give the argument as a positive angle.

(ii)(a) Most candidates realised that the locus was a circle, but a significant number failed to deduce that the circle passed through the origin.

(b) Most candidates realised that this locus was a straight line, but many failed to appreciate that it had a positive, rather than a negative, slope and their line often started at the origin instead of at the centre of the circle.

(iii) A good proportion of candidates found the required region for their Argand diagram; the most common error was to shade the region inside the circle, rather that the region outside it.

7) (i) This was usually answered correctly, but some candidates attempted to use Induction.

(ii) Most candidates established this result correctly.

(iii) Most understood how to use the two previous parts to establish the required result, the most common error being the use of $\sum 1$ as 1 rather than n. A number of candidates simply quoted the formula from the formula booklet, thus not appreciating that the first two parts had to be used.

8) (i) The images of the vertices of the unit square were generally clearly stated or scales were clearly indicated on the sketch, so most candidates scored well on this part

(ii) There were many incorrect answers to this part, with many simply writing down an incorrect matrix and not checking the image given in the question.

(iii) Most candidates found the inverse of the matrix in (ii), but then pre-multiplied instead of post-multiplying. Those who found the correct matrix for T often gave a description of a shear in rather ambiguous terms e.g. 'parallel to the *x*-axis', 'scale factor 2' instead of stating which axis is invariant and the image of one point.

9) (i) This part was answered correctly by the majority of candidates, sign errors being the common mistake.

(ii) Most equated the answer to (i) to zero and solved correctly, some candidates not giving the solution $a = 0$.

(iii) In both (a) and (b) many candidates simply stated that there were no solutions as $\det \mathrm{A} = 0$. A careful attempt to solve the particular equations in both cases is needed and a reasonable reason needs to be stated for each case.

10) (i) Nearly all candidates answered this part totally correctly.

(ii) A correct expression for u_n was found by those who realised that the last part of (i) could be used for u_3 or u_5. Many constructed difference tables, which did not help in finding u_n. A good proportion of candidates simply rearranged the given recurrence relation.

(iii) Candidates who had a correct expression generally completed the induction proof correctly. Those who had an explicit expression for u_n were usually able to verify the truth for $n = 1$ (or 2) and set up the induction step using the recurrence relation.

4726 Further Pure Mathematics 2

General Comments

Candidates generally answered the questions in the order set and completed the paper. A minority appeared to rush question 9 but largely as a result of a poor choice of methods and a tendency not to read the questions well enough. There were few poor scripts and candidates could gain some marks on each question, with no single question proving too difficult. There were more good scripts than usual, and it was pleasing that more candidates were able to produce thoughtful solutions. Some specification areas were not known well enough and it was surprising to see basic questions, for example on rational quadratics and/or partial fractions, done badly, often by whole centres. There was a general lack of precision, for example in integration where $\mathrm{d}x$ was often omitted, even when a substitution had been made, and where limits were often neglected. Candidates were not adept at using the earlier parts of a question to answer the later parts, usually viewing each part as a separate unit.
It is worth noting that graph paper is not usually called for or required and that it is the responsibility of centres to attach any extra sheets and not merely to insert them into answer booklets.

Comments on Individual Questions

1) This question was designed to test the candidates' appreciation that areas were being considered, albeit ones below the *x*-axis. A substantial number of candidates lost early marks by giving negative answers. Others used degrees to evaluate the areas. A majority of candidates gave positive answers and gained the marks. However, it was disappointing to see imprecise statements such as -1.313 = 1.313. Candidates used various signs in their work, from = to ≈ to > or <, all of which were allowed in this case. It has been highlighted before that candidates should take care in such questions, particularly by those who wrote A < 1.3131 and then rounded this so that A < 1.313.
Comments in part (iii) were usually sound, although often brief, leaving it to the examiners to interpret what was meant. 'More rectangles' did not give sufficient detail and accuracy. Candidates should be encouraged to give fuller statements.

2) Candidates using rational quadratic methods were often successful, although it was evident that many wrote down $b^2 \geq 4ac$ without knowing why they were using it. As long as it was used, marks were awarded. Others worked out $b^2 - 4ac$ and then decided it had to be positive or zero, presumably using the answer given. Again full marks were available.
Candidates using differentiation were less successful as the differentiation was not always accurate. Even when it was, candidates stopped after finding $x = -1$ and $y = ¼$. There were few attempts to prove it to be a minimum and only a handful of candidates thought of using their graphical calculators to sketch the curve and justify the inequality.
A substantial minority were unable to use either of the above methods, with some losing time by attempting partial fractions.

3) This question was answered well. It was pleasing that the majority of candidates were able to apply the chain and product rules accurately. Even the candidates who could not tended to pick up marks in part (ii) by using their values, although such candidates were penalised unless all terms were non-zero, as the question implied that there were three terms to be found. Candidates using $\ln y = \sin x$ often produced the required differentials quicker.

4) It was surprising how relatively few candidates were able to deal with a basic partial fractions question. Many candidates did not recognise the need to divide out in some way by considering the degrees of the numerator and denominator. The result was often a fudge to compensate for this or a method which clearly did not work. Such candidates gained a maximum of two marks.
It has been reported before that it would help candidates to use other methods as well as equating coefficients. In this case, candidates who merely equated coefficients in a basically incorrect method often showed that all their constants were zero, which should perhaps have led them to rethink their strategy.
Successful candidates either divided out or used $A + B/(x - 2) + (Cx + D)/(x^2 + 4)$.
Examiners accepted $A = 1$ quoted as obvious and $B = 1$ by the cover-up method.
Candidates who only equated coefficients spent some time in solving their equations. There was a general lack of precision, often leading to pages of work.

5) (i) Examiners were happy to see '$d\theta = 2\,dt/(1 + t^2)$' quoted rather than having to be derived. Most candidates were able to get some way into this question, which was answered better than similar questions in previous years. The only problem was the numerous basic algebraic errors.
Candidates should expect to deal with the limits at the same time as they change the variables and not introduce the limits at the end as an afterthought. It is also expected that the change from $d\theta$ to dt should be given in each integral seen.
Candidates used a variety of techniques to get from $(1 - t^2)/(1 + t^2)$ to the given answer. Dividing out and a form of 'partial fractions' were successful methods used, but the best solution used by many candidates was to write
$(1 - t^2)$ as $2 - (1 + t^2)$. It was perhaps surprising to see that very few candidates, who were unable to complete this part, did not use the answer given to get back to $(1 - t^2)/(1 + t^2)$.
Candidates who noted that $d\theta = (1 + \cos\theta)\,dt$, so that the integral became $\int\cos\theta\,dt$, should be commended.

(ii) Most candidates gained two marks, with only a small number believing the first integral to be $\ln(1 + t^2)$. Odd marks were lost unnecessarily when candidates took the 2 outside the integral sign, giving it as $2\int(1/(1 + t^2) - 1)\,dt$ for some reason.

6) The majority of the candidates gained the three method marks, but there was a general lack of care and accuracy. Methods varied between candidates who attempted an immediate logarithmic answer, usually not using the chain rule to get the correct fractions, candidates who attempted an immediate inverse hyperbolic answer, usually omitting the same fractions and/or not deriving the correct coefficients of x, and candidates who used substitution or simplification using $\sqrt{(ab)} = \sqrt{a}\sqrt{b}$ to get the integral in the form given in the Formulae Booklet. The latter two were most successful. Some candidates who completed the integrals then did not find a. It was felt that carelessness lost many candidates at least half marks.

7) (i) Most candidates could produce the graph, although a number, who might have used a graphical calculator, believed the x-axis to be an asymptote. Although the equations of the asymptotes were requested, some candidates only marked horizontal lines at
(y =) 1 and (y =) -1. These gained the mark if clear, although it was more important to clearly show an approach to the y-axis if the equation $x = 0$ was not given. Potentially two marks could be lost due to not reading and answering the question well enough. It would also be worthwhile for candidates to know the basic hyperbolic graphs rather than relying on exponential definitions and attempting them from scratch.

(ii) Most candidates produced the correct Newton-Raphson method and the accuracy of many answers was good.

(iii) The first part was answered well, although a number of candidates did not fully derive an answer given. In such cases it is important to show all relevant steps. Few candidates used the earlier parts of the question to answer the final part and this part was often omitted or a single answer was given. The instruction 'write down' enabled candidates to write various solutions down without explanation and some obtained the correct solutions somehow. Very few attempted to justify only two solutions.

8) (i)(a) This part was done well, with a majority of candidates able to write down $\frac{1}{2}(e^{\ln a} + e^{-\ln a})$ and simplify it to the given answer. However, because the answer was given the steps from $\frac{1}{2}(a + a^{-1})$ to $(a^2 + 1)/2a$ were required in full. A number of candidates believed the steps to be obvious.

(b) Again this part was done well with only a minority of candidates failing to expand $\cosh x \cosh y$ and $\sinh x \sinh y$ accurately. Minor errors occurred, such as $\frac{1}{2} \times \frac{1}{2} = \frac{1}{2}$, but errors such as $e^x e^y = e^{xy}$ were rarely seen.

(ii) Most candidates used $x = y$ in both sides of the identity, showing $\cosh^2 x - \sinh^2 x$ and $\cosh 0 = 1$. Other candidates lost the mark by deriving only one side of the identity. Candidates who resorted to the exponential definitions did not gain the mark.

(iii) There were various approaches to this part. Of all the correct approaches seen, the one using the earlier parts of the question was seen the least. In this method there was no need to deal with $\sinh(\ln a)$. Candidates who derived or wrote down $\sinh(\ln a) = (a^2 - 1)/2a$ were often successful, although it was disappointing to see how many candidates were unable to solve two equations in two unknowns. The third method of resorting to the exponential definitions also worked well.
A number of candidates who thought that they recognised a trigonometric link wrote $R = \sqrt{(13^2 + 5^2)}$ straight away, whilst others produced $\tanh(\ln a) = 5/13$ without an R being seen. Even those correctly deriving this equation often used a graphical calculator to find a. Candidates who wrote $\tanh^{-1}(5/13)$ in logarithmic form should be commended, although both methods gained the marks if accurate.

(iv) Many candidates did not read 'write down' and produced a page of work, usually involving differentiation, to get x and/or y. Although these candidates did not prove their point was a minimum, marks were awarded although time was lost.

9) (i) As a fairly standard reduction formula, it was surprising how many candidates did not start by using '$\sin^n\theta = \sin^{n-1}\theta.\sin\theta$'. Candidates who did were generally successful, although in some cases limits went missing and terms mysteriously disappeared. For example, $[-\cos\theta.\sin^{n-1}\theta]$ given without limits was then omitted on the next line without explanation. Again it was expected that because the answer was given it should be derived carefully. In the above case, the same bracket given with limits was allowed to disappear on the next line. It would have been better (and safer) for candidates to show clearly that the use of the limits produced 0.

A number of candidates who started
'$\int \sin^{n-2}\theta.\sin^2\theta \, d\theta = \int \sin^{n-2}\theta - \sin^{n-2}\theta.\cos^2\theta \, d\theta$
$= I_{n-2} - \int (\sin^{n-2}\theta \cos\theta)\cos\theta \, d\theta$'
and who accurately used parts on the last integral, should be commended. However, a significant minority of candidates gained no marks on this part.

(ii)(a) The equations of the tangents at the pole were usually found, although a number of candidates were unable to solve $\sin^3\theta = 0$ without resorting to $\sin\theta(1 - \cos^2\theta) = 0$ or double-angle formulae and again wasted time. Sketches were generally sound, although the tangents found earlier were often not clearly shown. Although not penalised in this case, candidates should take care and attempt to show any symmetry.

(b) Again many candidates did not appear to realise the significance of the earlier parts of the question, with candidates using double-angle or complex number equivalents, not always accurately. Even those candidates who saw the connections were very vague about limits, often applying without proof the limits 0 and π to the reduction formula in part (i). Such candidates often picked up the four method marks. The best candidates showed carefully that the area equalled I_6.

4727 Further Pure Mathematics 3

General Comments

This paper was done well by many candidates, who are to be commended for their knowledge of the demanding aspects of the specification and for their ability to apply them. The questions which were answered best were 3, 4, 5 and 6. The last two questions were more difficult and only the better candidates scored high marks on them. A few candidates appeared to have run out of time towards the end of question 8: as is often the case, some unnecessary work may have been done earlier, particularly in questions 7 and 8. Presentation was, in most cases, better than last year, but careless arithmetic and miscopying of figures were seen quite often, especially in the vector calculations.

Comments on Individual Questions

1) This was generally attempted well. The method for changing into polar coordinates was well understood; almost all answers had the correct modulus, but an incorrect argument was seen fairly often, usually $\frac{1}{3}\pi$ Common errors were to give only one of the three roots, and to give the third root outside the required range. A basic mistake, which occurred fairly often, was to add multiples of $\frac{2}{3}\pi$ to the argument, without having divided it by 3. Answers in exponential form were accepted, but some who gave only that form omitted the i.

2) (i) The majority of candidates gave the correct inverse. The most common error was to give the modulus as 5, but other errors included gibing an incorrect argument or omitting the i. Throughout this question only the exponential form was allowed.

(ii) All that was necessary here for the two marks was to show two distinct elements and to multiply them together correctly. Many answers, commendably, included reducing the argument mod 2π, but as this aspect was to be tested in part (iii) it was not essential here. There were still candidates who used the same modulus and/or argument for their two elements, and a small minority who used numerical values.

(iii) The first mark in this part was usually obtained, by writing $e^{2i\gamma}$, but very few realised that more was needed. Quite a number tried to do something with the range of values of θ, instead of reducing the argument by 2π. Of the small minority who tried to do the right thing, some reversed the sign of the argument, omitted the i, or bracketed wrongly. The letter γ seemed unfamiliar to many, but reasonable alternatives were allowed, apart from θ.

3) Vector questions are generally well done, and this was no exception. About half the answers scored full marks.

(i) The majority of candidates answered this part correctly, using the parametric approach shown in the mark scheme. A few tried solving the cartesian equations directly by elimination, and this was sometimes successful, but more often algebraic errors were made, and in any case the method took much longer.

(ii) Again, this part was often done well without any difficulty, using the first method shown in the mark scheme. Arithmetical errors were more common here, particularly in the signs in the vector product and in miscopying. Other correct methods were sometimes seen, but some weaker candidates did not know how to start.

4) (i) The majority of candidates answered this part correctly, using the List of Formulae appropriately. Occasionally only the integral was worked out, omitting the essential exponential part.

(ii) The subsequent solving of the equation was done very well in many cases, and full marks were often obtained. The most serious fault, in a small number of answers, was the to failure to multiply the right-hand side of the equation by the integrating factor; in some cases this might have been due to careless cancelling, but if the evidence of multiplication was missing and the term on the right-hand side was unaltered, then no credit could be given. Otherwise, the integration was usually done correctly and the constant inserted. Some then failed to divide through properly by the integrating factor, leaving *c* unchanged or making algebraic errors such as having $\left(\frac{1-x}{1+x}\right)^{\frac{1}{2}}$ upside down. Occasionally the step of giving x and y numerical values was not done.

5) (i) Most candidates answered this part correctly. Occasionally the solutions of the auxiliary equation or the form of the complementary function were wrong, but these were rare mistakes.

(ii) The mark for this part was frequently not obtained, in an otherwise correct answer. A simple statement that the given forms of the particular integral were in the complementary function was all that was required (and the mark was still awarded to the considerable number of candidates who wrote 'complimentary', when the correct spelling was on the question paper and when 'CF' would have been acceptable!). Some candidates wrote that 'all terms cancel' or that there was a repeated root, or mentioned only one of the suggested forms of the particular integral (PI); none of these were accepted. In a few cases a lot of working was done to prove that the given forms did not work, which was accepted if correct, but for one mark much time was wasted.

(iii) The majority of answers found the value of k correctly, with no errors in differentiation or substitution. The small number who differentiated wrongly then found that terms did not cancel out as expected and a numerical value of k could not be obtained.

6) (i) This vector problem was also done well. Most attempted this part by using the vector product and obtained the correct equation. It was pleasing to find that most candidates simplified the vector product, since only the direction was needed, although examiners had to overlook the misuse of equal signs. Common mistakes were in the arithmetic of the calculation and miscopying signs.

(ii) All of the methods shown in the mark scheme were seen at some time, with the first two being the most popular. Those who used the vector product and then found a point on the line of intersection were usually successful. Solving the two cartesian equations by elimination and obtaining parametric solutions was often done correctly, but some were unable to translate the parametric form into the vector equation. It was sometimes claimed, without any working, that the point (2, 2, 1) (on Π_1) was on the line of intersection. This was incorrect and resulted in the loss of the last 3 marks. A few answers omitted the left-hand side of the final equation. Many candidates were helped in this part by some generous follow-through marks when an early error would otherwise have meant the loss of some of the subsequent marks.

7) (i) The majority of candidates were able to establish this standard result correctly. In a number of cases reference to de Moivre was minimal, and the fractional expression for $\tan 3\theta$ in terms of c and s appeared early in the working. This was accepted, but those who quoted the results without any indication that the theorem had been used lost the first two marks.

(ii) The question then became more demanding. It should have been straightforward to realise that $\tan 3\theta = 1$ and to rearrange the result of part (i), but many were unable to do this. As far as showing that $\tan \frac{1}{12}\pi = 2 - \sqrt{3}$ was concerned, it was very common for the given value to be substituted into the cubic equation and worked out to obtain $0 = 0$ or equivalent; this simply verifies that the value is one of those which satisfy the equation and it earned no marks. Those who attempted factorisation were almost always successful in obtaining at least 3 of the 4 marks. The final mark was for showing that it was $\tan \frac{1}{12}\pi$, rather than $\tan \frac{5}{12}\pi$, which was $2 - \sqrt{3}$ and only the better candidates obtained this mark.

(iii) When an instruction to use a particular method is given, candidates should be aware that they may not obtain any marks if they use another method. In this case attempted solutions by partial fractions almost always came into this category. Furthermore, there was a considerable amount of work to be done before the integration started. The integral was designed to follow on from the previous parts of the question, but the method of substitution seemed to have been only half-recalled in many cases. This was partly on account of slackness in writing and dealing with $\mathrm{d}t$ and $\mathrm{d}\theta$ properly. It was quite common for candidates to not realise that the integrand, when $(1 + \tan^2 \theta)$ in the denominator had been cancelled, was just the expression for $\tan 3\theta$. Those who reached this point correctly sometimes omitted to change the limits or else they integrated to $\ln(\sec\theta)$ or omitted the factor of $\frac{1}{3}$. There was much scope for error, but it was pleasing that a reasonable number of completely correct answers were seen.

8) Throughout this question many candidates assumed that all products of the elements commuted. It is not expected that the structure of groups of order 8 and higher is known, but candidates should be aware that some groups are non-commutative, including one of those of order 6 whose structure is within the specification. Some answers began by writing out the complete table for the group; this was not asked for and most such attempts were wrong because they assumed commutativity.

(i) The better candidates used pre- and post-multiplication in $a^2 = apap$ and $p^2 = apap$ to obtain the results quickly. Some assumed what was to be shown and worked backwards; this is to be discouraged, but was allowed provided the steps were reversible. A few claimed incorrectly that $a^2 = (pap)^2 \Rightarrow a = pap$.

(ii) Even those who assumed commutativity often gained the first 3 marks, for the orders of p^2, a and ap, but for ap^2 it was necessary to show the working correctly. Those who used $ap^2 = a^3$ were often successful, but the incorrect use of commutativity was apparent in many answers.

(iii) The majority of answers used Method 3 of the mark scheme, but in many cases candidates thought that only closure was required. They gave no evidence that there was an identity and that all elements had inverses, both of which are essential for a subgroup; these had to be clearly identified. Surprisingly, a few attempts were made to justify closure by listing the possible products instead of making a table; all 16 products were required, but were hardly ever seen. It was good to see occasional proofs which showed that a was a gener ator, as in Method 1.

(iv) Only the better candidates scored any marks in this final part. The majority thought that Q was commutative, either 'because it was multiplication' or because they had constructed a complete table which was symmetric about the leading diagonal. Many answers just used two of the elements from part (iii) and claimed that their commutativity meant that Q was commutative. It is possible that some misread the question, and thought it was asking if the subgroup in part (iii) was commutative. For those who knew what to do, there were some very good, neat, answers, either by finding a pair of elements which did not commute, or by contradiction, perhaps using one of the results of part (i). Some forfeited the last mark, if their argument was not sufficiently clear.

Chief Examiner Report - Mechanics

The standard of work which candidates produced was high, and only a small minority showed weakness on all aspects of their particular syllabus. A feature of responses on all four papers was the needless loss of marks, which can have as significant an impact on achievement as ignorance of a topic.

Calculator errors can easily be made, such as striking the minus key rather than the divide, and can only be noticed by candidates if they routinely reflect on the plausibility of an answer. A second problem when dividing is that in written work a division sign implies bracketing of the numerator and denominator, but calculators do not have this feature.

A second source of error is poor notation. Does “R” mean resultant or reaction; which of (at least) four meanings of “F” is intended? Is a particular letter used to represent a “before” or “after” quantity, and to which of several objects does it refer? When a candidate’s solution is entirely correct, its internal coherence supplies the answer. However, an incorrect solution (often prompted by a candidate’s own uncertainty about what is intended) gives no clue unless supported by a clear diagram. It may be that candidates imagine that examiners are required to make the most generous interpretation of ambiguous work. This is not the case.

A third way in which some candidates could achieve higher marks is to use given answers more judiciously. It is essential when correcting a solution which has initially given a wrong result, that changes be made clearly and (if appropriate) tracked back to the very first line of their solution. However, answers are given in papers usually with the intention of ensuring a subsequent part of a question can be answered correctly, and candidates should be warned that persisting with their own wrong value can only lead to a further loss of marks.

4728 Mechanics 1

General Comments

Many excellent scripts were seen, and the majority of candidates had been well prepared for the examination. The two questions involving friction (3 and 6) were found to be conceptually the most difficult. However, many candidates lost marks needlessly, as indicated below.

Comments on Individual Questions

1) (i) Though many correct answers were seen, a common error was to omit brackets when squaring the 3x term. The equation $4 \times 2 = 36$ (leading to $x = 3$) was unexpectedly frequent. Almost no candidate reflected on the impossibility of having a triangle with sides 3, 6, and 9. No penalty was levied when candidates gave the answer 1.9, rather than 1.90.

(ii) Candidates nearly always attempted to find the correct angle. Using 1.9 in conjunction with sine or cosine gave an answer which was incorrect to 3 significant figures, and gained only 2 marks out of 3.

2) (i) This part of the question was answered well.

(ii) The most common approach was to use constant acceleration formulae. As the situation describes both acceleration and deceleration, the method gained no marks except when a candidate distinguished between t s of acceleration, and $(3 – t)$ s of deceleration.

(iii) The correct solution to this part of the question was often seen, candidates sometimes losing the final mark through stopping at $a = -3$ ms^{-2} and not giving the positive answer needed.

3) (i) Correct justifications of the printed answer were frequently given, but simple subtraction of 0.8×0.2 from $0.8 \times 9.8\sin30$ without the underpinning of Newton's Second Law was inadequate.

(ii) This half of the question logically involves only the 3 kg block, and the application of $F = ma$. One frequent error, perhaps a consequence of this notation, was to treat the product of 3 and 0.2 (0.6) as showing the frictional force. A second error was to use the tension in the string as friction, while some candidates using both terms added them, as though friction were creating the acceleration.

4) (i) Nearly all candidates used the appropriate constant acceleration correctly.

(ii) This part of the question was answered well.

(iii) Though this part of the question was tackled successfully, a significant minority of candidates found only the time to descend from the highest point. Indeed the most common approach was to find separately the rising and falling times. Few candidates employed $-5.7 = 7 - 9.8t$.

5) (i) This part was well answered.

(ii) Most candidates achieved the correct result, setting up and simplifying the conservation of momentum equation confidently.

(iii) Candidates were able to solve their simultaneous equations successfully. However, the minority who did not obtain 0.9 as the value for *m* usually persisted with their incorrect value. It had been anticipated that the final 2 marks would be awarded on nearly all scripts as a result of substitution of 0.9 in the equation given in part (i).

6 (i) The first 4 marks were commonly obtained, but some candidates incorporated *g* in their expressions. Some scripts showed the diagram specific to (ii) in part (i).

(ii) Only a minority of candidates successfully created expressions for the normal reaction and the frictional force involving both *T* and 10. One common assumption was that the value of the frictional force would be unchanged from part (i). Candidates who did obtain these expressions sometimes made no further progress through not using $F = \mu R$ to create an equation with *T* as the only unknown.

7) (i) Very rarely did any candidate divide by *t*, and a failure to gain both marks was rare.

(ii) There were few scripts in which attempts to use constant acceleration were seen. The main error was not justifying the value of the constant of integration to be zero.

(iii) Few candidates made any error in this part.

(iv) Nearly all candidates correctly based their solution on discovering how long it took the decelerating sprinter to cover 11 metres. A few scripts used 0.6 as the acceleration. The most common method was to find the velocity after running 11 metres, and then calculating the time by employing $v = u + at$. However, the correct solution, by way of the quadratic equation $11 = 9t - 0.3t^2$, was also seen. The candidates who used calculus to obtain expressions for *v* and then *s* were seldom successful.

4729 Mechanics 2

General Comments

There was a large number of excellent candidates, a significant number of whom scored full marks. However, there was also a fair number of candidates who scored very few marks and were not prepared for the examination. Questions 3 and 4 posed the greatest difficulty and it was evident that some candidates did not know the formula relating impulse to change of momentum (question 6). It was also fairly common for a kinematic formula to be misquoted (e.g. missing half or additional half with $s = ut + ½ at^2$ / $v = u + at$) and even when formulae were quoted correctly it was not uncommon for a square to be omitted from kinetic energy or other formulae requiring the squaring of time or velocity. As a general guideline, it is helpful if when moments are taken, the point about which they are being taken is stated. Similarly for the direction in which forces are being resolved.

Comments on Individual Questions

1) Most scored full marks. However, marks were lost for wrongly balanced energy equations or the repetition of expressions attempting to calculate potential energy. This calculation sometimes involved an unnecessary use of trigonometry. A few lost a mark by not following the instruction to consider the change in energy (an alternative method used was via acceleration and '$F = ma$') and some defined the change in kinetic energy as ½m(12 − 3)2.

2) (i) The easiest two marks on the paper. The vast majority gained these.

(ii) Generally well done although some omitted 'g' or the resistance or sign errors were made.

(iii) Less well done. Some failed to halve the power and other errors were of the same nature as those in (ii).

3) (i) To isolate the force acting on the beam at A, it was necessary to take moments about A. Many failed to do this. It was also common to only find the vertical component of the required force. The beam's weight (not shown on the diagram) was also sometimes missed, although the mark scheme took this into account.

(ii) It is simplest to find the two components at A and then combine. Many just found one of the two components and thought that that was the final answer.

4) (i) The question came as a surprise as many clearly expected a conical pendulum question and hence drew inappropriate diagrams and produced inappropriate equations. However, many correctly calculated the tension in AB.

(ii) This was more challenging. A correct diagram with separate tensions in a horizontal string would have helped.

(iii) Most recognised the need to calculate two separate speeds but many made things difficult by comparing accelerations and using $r\omega^2$ and v^2/r rather than simply using $v = r\omega$. Some candidates did not attempt to find separate speeds and simply added the masses together and multiplied by a single speed squared.

5) (i) Generally well done with most recognising the need for $\Pi/4$ and only a few fiddling their working to achieve the given result.

(ii) The two separate masses were given but sadly many treated the lamina as uniform and calculated areas. It was also common for candidates to use the (x, y) distances to the centre of mass of the quarter circle as (3.60, 3.60) rather than (3.60cos45°, 3.60sin45°).

(iii) The geometry of the situation was generally well understood and follow through marks were awarded for incorrect coordinates from (ii).

6) (i) Many were tied in knots because they failed to recognise the need to consider the change in momentum of *B*.

(ii) Most candidates recognised the need to find and compare the final velocities of *A* and *B* and many were successful. For this second collision a significant number of candidates thought that the speed of *B* was 1.5 ms^{-1}, rather than 6 ms^{-1} immediately before the second collision. In many cases, both the candidate and the examiner would have been helped greatly if the new velocities of the spheres had been made clear with the use of diagrams.

7) (i) The unusual projection direction in this question confused many candidates and led to sign errors in the use of kinematic formulae. Some quoted the trajectory equation but substituted θ = 25° instead of -25°. A remarkably common error in calculating the time to hit the fence was to not work out 9/(17cos25°) (0.584) but 9/17 × cos 25° (0.480) – simple use of the calculator! A few incorrectly assumed that v = 0 on hitting the fence. The quoting of standard formulae for range or maximum height also led to errors.

(ii) Generally well done although some did not make it clear whether their angle was to the horizontal or vertical. Some inappropriately used displacements to find the direction of motion.

(iii) Also good but it was not uncommon to calculate the final speed having lost 70% rather than 30% of the kinetic energy.

4730 Mechanics 3

General Comments

The work of candidates was generally of a good standard and well presented. Questions 6 and 7 were found to be the most difficult, reflecting the intended incline of difficulty. Although a fair number of candidates scored full marks in each of these questions, a significant minority scored no marks in each case.

Questions 3 and 4 proved to be the most straightforward for candidates.

Comments on Individual Questions

1) Most candidates found this question straightforward. However some candidates thought it necessary to calculate the speed of the sphere before impact, using the result in erroneous calculations of the coefficient of restitution and of the magnitude of the impulse.

   The incorrect answer 0.9 N for the magnitude of the impulse, from 0.3(6 – 3), was common.

2) (i) Many candidates made the calculations unnecessarily complicated by using moment distances such as 3 m in the form 3.35cos26.6°.

   (ii) This part was less well attempted than part (i), a significant number of candidates being unable to determine a suitable strategy.

   Many candidates showed by their sketches that they had assumed the force on *BC* to act in the direction from *B* to *C*. This did not prevent candidates from producing a correct solution in most such cases, but candidates who also used the erroneous assumption that the horizontal and vertical distances between *B* and *C* are 1.5 m and 3 m, instead of making use of the horizontal component of the force at *C*, scored no marks.

3) This question was very well attempted with many candidates scoring full marks. Problems arose, however, when some candidates used symbols without making it clear what they represented. Does *x* represent the component of velocity of *A* or of *B*, before or after the collision, to the left or to the right, or to the north or south? The answer is most clearly communicated in simple diagrams, one representing 'before' and one 'after'.

   Another problem is that some candidates found the components of the velocity of *B* to be unchanged in magnitude, but failed to deduce from this that the speed of *B* is therefore unchanged.

4) This question was very well attempted and errors that arose were usually manipulative rather than misunderstandings. A fairly common error was to use 'resultant force = -120v dv/dx' at the outset, presumably anticipating the minus sign in the given answer to part (i).

5) (i) Some candidates found the tension when *P* is at its lowest point by resolving forces vertically, ignoring the acceleration of *P*.

(ii) The most common error in this part was to consider only one of the strings when finding the elastic energy. In part (b) a significant minority of candidates omitted the term representing elastic energy when applying the principle of conservation of energy.

6) There was a fairly large number of confused attempts in part (i), with equations that are dimensionally unbalanced appearing. Most of the confusion relates to the failure to distinguish clearly between linear displacement, velocity and acceleration and angular displacement, velocity and acceleration. This failure was often repeated in parts (iv) and (v). The clearest correct answers in part (i) were usually accompanied by simple sketches.

7) Part (i) was well attempted but many candidates thought that $v = 0$ should be used in part (ii), despite the requirement to show that v^2 is equal to ag.

Many candidates who correctly used $R = 0$ in part (ii) erroneously carried this feature into part (iii).

4731 Mechanics 4

General Comments

The candidates sitting this paper generally demonstrated a good understanding of most of the topics being examined, and there was a lot of confident and correct work. The topics which caused the most difficulty this year were the compound pendulum (question 3) and force at the axis of rotation (question 7). There was a good spread of marks, with about one third of the candidates scoring 60 marks or more (out of 72), and about 20% scoring fewer than half marks.

Comments on Individual Questions

1) This question, on constant angular deceleration, was answered well, with about three quarters of the candidates scoring full marks. In part (ii), the direct approach using $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$ led to two values of *t* (5 and $133\frac{1}{3}$). Some candidates gave both values as possible answers to the question, and a few selected the larger value.

2) Finding the centre of mass of a solid of revolution was very well understood and generally carried out accurately; about 70% of the candidates scored full marks on this question. The most common errors were slips in evaluating the definite integrals, and a few candidates appeared to be finding the centre of mass of the lamina *R* rather than the solid of revolution.

3) This question, on moments of inertia and a compound pendulum, caused quite a lot of confusion and only about a quarter of the candidates scored full marks.
In part (i) most candidates applied the parallel axes rule appropriately, although some took the mass of the disc to be *m* instead of 4*m*, and the particle at *B* was quite often omitted from the calculation.
In part (ii) if the formula $2\pi\sqrt{\frac{I}{Mgh}}$ for the period of a compound pendulum was used, it was necessary to find the centre of mass (not usually a problem for those who realised that they needed to do so, but many just assumed that $h = a$), and a common error was to put $M = m$ instead of $M = 5m$. Candidates who formed an equation of rotational motion, considering the weights of the disc and the particle separately, were perhaps more likely to obtain the correct answer. A fair number of candidates completely ignored the particle at *B* in this part as well.

4) Relative velocity remains a difficult topic for some candidates with about 10% scoring no marks (sometimes omitting the question altogether). However, the standard of work generally is improving, and there were very many who answered this question efficiently (using a velocity triangle) and confidently. Half the candidates scored 7 marks or more (out of 9), and about a third scored full marks. When one or two marks were lost, it was usually through not giving the bearings correctly in part (i).

5) In part (i) some candidates made no progress beyond finding the mass per unit area, but most knew how to find the moment of inertia of a lamina. It was possible to use strips parallel to the *y*-axis or parallel to the *x*-axis (for both cases the work required was of similar difficulty) and both methods were common; the given result was very often obtained correctly.
In part (ii) many candidates lost a mark for not giving sufficient explanation to establish a given result. A statement such as '800 is greater than 63 × 9.8 × $\frac{4}{5}$, so it rotates anticlockwise' is not enough; there should be some reference to anticlockwise and clockwise moments.
In part (iii) the moment of inertia was almost always found correctly using the perpendicular axes rule, and most candidates realised that energy considerations should be used to answer this part; only a few tried to use the constant acceleration formulae here. Sign errors and mistakes in the potential energy term were quite common, and the work done by the couple was often omitted altogether.

6) About 40% of the candidates scored full marks on this question about the energy approach to equilibrium.
In part (i) most candidates obtained a correct expression for the total potential energy, although some could not use the double angle formulae to obtain the given form convincingly.
Using the given result to investigate the positions of equilibrium in parts (ii) and (iii) was very well understood and often completed accurately. The most common error was to give the second position of equilibrium as $\theta = \frac{7}{12}\pi$ instead of $\theta = -\frac{5}{12}\pi$.

7) About 15% of the candidates scored no marks on this question (about the forces at the axis of rotation). Several did not attempt the question at all, and in some cases this appeared to be through lack of time.
Parts (i), (ii) and (iii) were reasonably well answered, although sign errors were quite common in the equations of motion.
Few candidates were able to answer part (iv) correctly. The anticipated method was to assume that the rod does not slip, use the expressions from part (iii) to calculate *F* and *R* at the moment when *B* hits the ground, and show that these values are impossible; this was completed successfully by some. However, a much more common approach was to set *F* = 0.9*R* and solve the resulting equation to find the point at which the rod slips. Unfortunately the frictional force changes direction during the motion, so it is actually necessary to start with *F* = -0.9*R*.

Chief Examiner's Report - Statistics

Some Centres have acted on the notice given in previous reports concerning statements of hypotheses and that over-assertive conclusions to hypothesis tests (for instance, 'the time taken has changed') would be penalised. (Preferable is 'there is insufficient evidence that the time taken has changed'.)

In all statistics units, the incorrect use of formulae given in MF1 continues to be an issue. Centres would be well advised to spend time teaching the correct use of MF1 explicitly. With the increase in statistical functions available on many calculators, it needs to be emphasised that answers obtained by a calculator with no justifying working risk scoring no marks.

Questions on the assumptions needed for standard mathematical models such as the binomial distribution have been asked for many years now and it is disappointing that in this area there has been no real improvement. Many candidates seem to be able to go no further than quoting standard phrases such as 'they must be independent' without saying what 'they' are or giving any indication that they understand what the phrase actually means. Answers such as this receive little or no credit.

4732 Probability & Statistics 1

General Comments

Many candidates showed a reasonable understanding of a good proportion of the mathematics in this paper. There were some very good scripts, although very few candidates gained full marks. There were several questions that required an interpretation to be given in words, and these were not generally answered well. Some answers were just wrong, while others suggested an inkling of something correct but expressing this understanding in words was beyond many candidates. Even some of the most able candidates did not score full marks on these questions.

It was noteworthy that many candidates seemed uncomfortable with manipulation of fractions, often resorting to the calculator to evaluate simple expressions. In some cases they consequently gave approximate answers where exact answers could easily have been obtained.

This year again it was pleasing to note that very few candidates ignored the instruction on page 1 and rounded their answers to fewer than three significant figures, thereby losing marks. However, in a few cases a mark was lost through an incorrectly rounded answer without any previous answer having been shown.

Responses to question 5 suggested that many candidates had not revised GCSE work recently. Also, many candidates appeared not to be aware of how to handle a discrete variable when drawing a histogram or cumulative frequency graph. Perhaps some centres have not appreciated that this is possibly just beyond what candidates will have met at GCSE and so have not covered this topic.

One possible solution to question 9(i) involved logarithms although trial and improvement was also acceptable, but no other questions made a significant call upon candidates' knowledge of Pure Mathematics.

Few candidates appeared to run out of time.

Most candidates failed to fill in the question numbers on the front page of their answer booklet.

In order to understand more thoroughly the kinds of answers which are acceptable in the examination context, centres should refer to the published mark scheme.

Use of statistical formulae and functions

The formula booklet, MF1, was useful in questions 1 (for binomial tables), 2 and 3(iii). However, as usual a few candidates appeared to be unaware of the existence of MF1. Other candidates tried to use the given formulae, but clearly did not understand how to do so properly (e.g. Σd^2 was sometimes misinterpreted as $(\Sigma d)^2$ in question 2). In question 3(iii) a few candidates quoted their own (usually incorrect) formulae for r_s, rather than using the one in MF1. Some thought that $S_{xy} = \Sigma xy$. Some candidates used the less convenient version, $r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2\Sigma(y-\bar{y})^2}}$ from MF1, but many of these completely misunderstood this formula, interpreting it as, for example, $\frac{(\Sigma x-\bar{x})(\Sigma y-\bar{y})}{\sqrt{(\Sigma x-\bar{x})^2(\Sigma y-\bar{y})^2}}$. Some candidates' use of the binomial tables showed that they understood the entries to be individual, rather than cumulative, probabilities. Others did not know how to use the tables to handle, for example, $P(X > 3)$.

It is worth noting, yet again, that candidates would benefit from direct teaching on the proper use of the formula booklet, particularly in view of the fact that text books give statistical formulae in a huge variety of versions. Much confusion could be avoided if candidates were taught to use exclusively the versions given in MF1 (except in the case of *b*, the regression coefficient). They need to understand which formulae are the simplest to use, where they can be found in MF1 and also how to use them.

A good number of candidates successfully used their calculator functions for standard deviation and for the correlation coefficient. Some gave fewer than three significant figures in their answer. A few obtained the wrong answer and could not be awarded even any method marks as no method was seen. Candidates who wish to use these functions should be advised to work through each calculation twice.

Comments on Individual Questions

1) Many candidates answered this question well.

(i) Some candidates interpreted this as a question on the geometric distribution. Others omitted the binomial coefficient. Some just looked up 3 in the binomial table for $n = 8$.

(ii) Some candidates found $P(X = 2)$ (presumably not appreciating that the table probabilities are cumulative). Others found $P(X < 3)$ or $1 – P(X < 3)$. Many candidates used the formula rather than tables. Ths is a much more protracted process and candidates frequently included an extra term, i.e. found $P(X > 3)$. Some candidates just found 1 – their answer to part (i).

(iii) Many candidates started from scratch, attempting to find the probability distribution of X and then to use Σxp. Others used the formula for the mean of a geometric distribution.

2) Many candidates answered this question well. Most candidates found no difficulty in interpreting the slightly unusual way of presenting the information, but a large number found $(\Sigma d)^2$ instead of Σd^2. Some used an incorrect value of n. Others misquoted the formula (e.g. by omitting '1 – ' or by using $1 - \frac{6 \times 2}{7(7-1)}$). As usual some placed the '1 – ' in the denominator.

3) (i) Many candidates appeared not to appreciate the fact that two regression lines are possible and that there are reasons for choosing one over the other. Their answers bore no relation to the question. Others saw the point but gave answers without reference to the context.

(ii) The question specifically asked for the significance 'for the regression line' and therefore required an answer which referred to the minimum sum of squares. Many answers referred to the measure of correlation and how the dotted lines illustrate this. Others stated that the total length of the lines above the regression line is equal to the total of those below. A few showed a serious misapprehension by stating that the dotted lines show how 'accurate' the line is.

(iii) This part was well answered on the whole. A few candidates assumed that $S_{xx} = \Sigma x^2$ etc. Those who tried to use $\Sigma(x - \bar{x})^2$ almost always misinterpreted this to mean $(\Sigma x - \bar{x})^2$.

(iv) Most candidates stated that there was good correlation, either basing their statement on the value of *r* or on the diagram. Few explained how the diagram showed good correlation (i.e. because the points are relatively close to being in a straight line). Some thought that the points in the diagram were actually not close to being in a straight line.

4) This question was answered well by many candidates, although some treated it as a question on the binomial distribution. Others miscalculated 1 – 0.3 as 0.6 or as $\frac{2}{3}$.

(i) Answers such as 0.34 and 0.33 × 0.7 were not uncommon.

(ii) Common incorrect answers were 1 – 0.74 and 0.73. Some candidates found 1 – their answer to part (i). Those who used the 'long' method often included an extra term, either 0.7 − 1 × 0.3 or 0.74 × 0.3.

(iii) Common incorrect answers were 0.75, 1 – 0.76 (both gaining partial credit), 1 – 0.75 × 0.3 and 1 – 0.74. Again, those who used the 'long' method often included an extra term, either 0.7 − 1 × 0.3 or 0.75 × 0.3.

5) The comments in the General Comments section, above, should be noted.

(i) Most candidates failed to appreciate that the class width is 10. Some used 9.5 or 9.9. A few multiplied class width by frequency or divided class width by frequency.

(ii) Most candidates used the midpoints for the *x*-coordinates, or used the right hand end without adding 0.5. A few just gave *x*-coordinates. A few others gave frequency rather than cumulative frequency.

(iii) Most candidates saw the point here, although there were a few irrelevant answers such as 'There are gaps between the classes', 'There are only four groups of data' and 'The class widths are different'.

6) (i) Many candidates seemed to be unfamiliar with the formula for standard deviation. Those who attempted to use the most convenient version ($\frac{\sum x^2}{n} - \bar{x}$) sometimes interpreted this as $\frac{770^2 - 70^2}{11}$. Those who used the less convenient version ($\frac{\sum(x-\bar{x})^2}{n}$) often made arithmetical errors or interpreted it as $\frac{(770-70)^2}{11}$. A few candidates just wrote $\sqrt{28.1961}$, derived from the given answer of 5.31.

(ii) This question was designed so that the median and quartiles are obvious by common sense even without using any formulae. Despite this many candidates insisted on using incorrect formulae such as lower quartile = $\frac{11}{4}$th value = 65.75 and median = $\frac{11}{2}$th value (which many found to be 66.5, for some reason).

(iii) Candidates struggled to express their understanding in this part. Frequently they stated that the spread could be less, but did not explain how this could happen without reducing the interquartile range. Typical incorrect answers were 'Lower scores on average, but the difference between them stayed the same. This would cause the standard deviation to drop but the interquartile range to stay the same.' and 'Scores are more grouped around the middle, but extremes are the same so the interquartile range is unchanged.'

(iv) This part was better answered, although some candidates showed some confusion, giving answers such as 'A lower standard deviation shows that she is consistently getting worse scores.'

(v) This part was well answered on the whole, although a few candidates did believe that a lower standard deviation means a lower overall standard.

7) (i) Common incorrect answers were ${}^8P_3$, 8! and 3! × 5!

(ii) Some candidates did not understand how to handle the fact that the letter *P* was included and therefore only two letters have to be chosen from seven. Some omitted to divide by 56. Others found $\frac{7}{56}$. A common error was $\frac{1}{8}+\frac{1}{7}+\frac{1}{6}$.

(iii) Some incorrect attempts were $1 \div \frac{8!}{3!}$, $1 \div {}^8C_3$, ${}^5P_1 \times \frac{1}{{}^8P_3}$, $\frac{{}^8P_3}{8!}$ and $\frac{3}{8}\times\frac{2}{7}\times\frac{1}{6}$.

8) Responses to this question varied a great deal. Some candidates who scored poorly on most of the paper scored well on this question. However, some able candidates drew a correct tree diagram but gained no more marks.

(i)(a) The probability tree was usually drawn correctly. Some candidates added unnecessary branches, but gave correct probabilities on them (e.g. $\frac{18}{18}$ and $\frac{0}{18}$). A few candidates treated the situation as one 'with replacement'. Others omitted labels.

(b) Some common incorrect attempts were $\frac{1}{20}+\frac{1}{19}+\frac{1}{18}$, $\frac{1}{20}+\frac{19}{20}\times\frac{1}{19}+\frac{18}{19}\times\frac{1}{18}$ and $\frac{1}{20}+\frac{19}{20}\times\frac{18}{19}\times\frac{17}{18}$.

(ii)(a) This part was found to be difficult. There were attempts to use the probability tree, such as $\frac{19}{20}\times\frac{18}{19}\times\frac{17}{18}$ and $\frac{19}{20}\times\frac{18}{19}\times\frac{1}{18}$, but many candidates treated this as if it were an example of either a binomial or a geometric distribution. Common attempts were ${}^{20}C_3\left(\frac{1}{20}\right)^3\left(\frac{19}{20}\right)^{17}$ and $\frac{1}{20}\times\left(\frac{19}{20}\right)^2$. Other candidates tried to find a simple link with the probability found in part (i)(b), making attempts such as (i)(b) × $(1 - \text{(i)(b)})^2$ and $(\text{(i)(b)})^3$.

(b) Many candidates attempted Σxp but with incorrect probabilities, frequently not having a total of 1, for example $1\times\frac{1}{20}+2\times\frac{1}{20}+3\times\frac{1}{20}$. Other candidates attempted to use a formula for the mean of a binomial or geometric distribution, for example $1 \div \frac{1}{20}$, $1 \div \frac{3}{20}$ and $3 \times \frac{1}{20}$.

9) (i) Although this question is fairly standard, most candidates seemed not to have met one like it before. A few used a correct method but stopped at $n = 23.4$. On the whole, however, attempts showed little understanding. Some candidates produced solutions such as $0.95 \div 0.12 = 8.996 \Rightarrow n = 9$. and $0.12 \times 8 = 0.96$, $0.12 \times 7 = 0.84 \Rightarrow$ smallest $n = 8$.

Even amongst those who understood a little more, the phrase 'at least one' was not correctly interpreted by most candidates. Many attempted to use $1 - P(X = 1)$. Others just used $P(X = 1)$. Examples of such attempts were $n \times 0.12 \times 0.88^{n-1} < 0.05$ and $0.12 \times 0.88^n > 0.95$.

When such attempts were followed by trial and improvement, they sometimes produced nonsense such as:
$0.12 \times 0.88^n = 0.95$, T & I: $0.12 \times 0.88^{-16.2} = 0.9518...$ $\Rightarrow$ n = -16.2.

However, there were some candidates who started with a correct first statement and then used either logarithms or trial and improvement successfully.

(ii) Many candidates did not know how to start this question. Some just found P(3 successes in 7 trials). Others recognised the need to start with 2 successes in 6 trials, but did not recognise this as a binomial distribution and therefore made errors such as omitting the combination. These obtained $0.12^2 \times 0.88^4 \times 0.12$, gaining 3 marks out of 5.

A few able candidates gave $\frac{{}^6C_2}{{}^7C_3}$, which is a good attempt, but sadly makes the assumption that there are three successes in the first seven trials.

4733 Probability & Statistics 2

General Comments

Candidates could *do* this paper more easily than some of its predecessors, but they couldn't get much more of it *right*. As usual, centres that have taken note of previous reports were clearly distinguishable from those that have not; it is no doubt preaching to the converted to say that big improvements can be made by noting the requirements of examiners.

It is worth reminding all centres that answers to questions on modelling assumptions should *never* be couched in general terms about 'events' but always interpreted in the context given by the question. Candidates should attempt to state exactly what things are independent (and they are not probabilities). Nor should such questions be answered in terms of parameter values (for example, 'the sample is large'); these are the conditions for one distribution to be approximated by another, not for the original distribution to apply in the first place.

At several centres a large proportion of candidates lost marks through showing insufficient detail of their probability calculations using the normal distribution. Candidates with calculators that give normal probabilities directly are strongly advised to show the standardisation line, for instance (in question 2) $\frac{20-25}{\sqrt{20}}$. Although this is wrong (a continuity correction is needed), this would gain a method mark, whereas a candidate who wrote nothing more than 'N(20, 20), P(≥ 25) = 0.1319', as was seen on several scripts, would not score that mark.

Most Centres seem aware now that conclusions to hypothesis tests that are expressed in too assertive a manner, for example 'the proportion claimed by the company *is* not too high', will lose a mark. '*There is insufficient evidence that* the proportion claimed by the company is too high' is much better.

As usual, questions on hypothesis testing were among the least well done. The concepts involved seem widely misunderstood.

There seemed a large number of significantly under-prepared candidates, but few seemed to run out of time.

Comments on Individual Questions

1) A routine start in principle, but the particular probabilities given were not that easy to handle correctly. A large number of candidates equated $(105 - \mu)/\sigma$ to 0.7 or $(110 - \mu)/\sigma$ to 0.5, instead of -0.7 and -0.5, respectively. It is surprising that a significant number of candidates either use the raw probabilities in their equations, or use the tables forwards instead of backwards, finding, for instance, $\Phi(0.2420)$; this type of question could hardly be more predictable.

2) This was largely well done. Most knew that the correct approximation was N(20, 20), and there was a pleasing proportion of correct continuity corrections. The justification for using the normal approximation was that μ is 'large'. Candidates who specify 'μ must be bigger than [a number]' must use the condition given in the specification, namely 15, and not 20 or some other number.

3) This question on hypothesis tests using a binomial distribution was perhaps better done than in recent years, no doubt because it involved the easier left-hand tail. Nevertheless as usual a large number attempted to use an invalid normal approximation, or to find $P(R = 4)$ or $P(R < 4)$ instead of $P(R \leq 4)$. These candidates scored few marks.
The scenario ('Is a claim of at least 60% justified?') encouraged incorrect statements of the alternative hypothesis, which should be '$H_1 : p < 0.6$', and not '$p > 0.6$'.

4) (i) Many could produce two sensible comments here, though some gave only one. As one candidate put it, 'it is necessary to get opinions not just from the high street but from the low and the middle streets too'. The answer 'this method is slow' did not gain credit; it is about the quickest and simplest there could be.

(ii) As has been stated in previous reports, this specification tests only one method of sampling, namely simple random, using random numbers. Candidates should specify that a list of residents (or houses) should be obtained, that they should be numbered *sequentially* (in particular, not randomly), and then random numbers be used to select. Hence neither just 'number all residents' nor 'select numbers randomly' scored a mark. Other methods of sampling, including systematic, score fewer marks. Sending out questionnaires and hoping that they will be returned is a seriously biased method.

(iii) The specification requires knowledge of the benefits of randomness in choosing a sample. These include the two conditions that validate the use of a binomial distribution: each member of the population is equally likely to be chosen, and they are chosen independently of one another.

5 (i) As usual questions on the modelling assumptions for a Poisson distribution were poorly answered. Candidates can regurgitate learnt phrases but many seem not to understand them enough to be able to apply them to a particular context. It is yet again emphasised that uncontextualised answers, such as '*They* must be independent', do not score full marks, and the word 'events' should be avoided altogether. Candidates must explain what 'they' or 'events' are. Some wrote that 'the probability that a brick is found must be constant', but this is wrong, confused with the conditions for a binomial distribution. Some said 'the number of bricks in any given area must be constant', which is also wrong; the numbers are bound to vary randomly, and what is necessary is that the *average* number per unit area remains the same. 'Singly' can be considered a relevant condition here, but is simply part of the more general condition 'bricks must occur independently of one another'. As usual, candidates would do well to avoid using the word 'random' or the 'singly' condition in this type of question.

(ii) There were plenty of good answers here, and also many errors.
$P(8 \leq X \leq 14) = P(X \leq 14) - P(X \leq 8)$ was a common mistake. Some failed to convert the mean to 12.

(iii) Although this question has been asked, in various forms, several times before, many candidates used the inaccurate method of finding a value of λ from tables, instead of using logarithms. Most, however, were able to divide their value of λ by 3. Some found the value corresponding to 0.04 instead of 0.4. Oddly, some very weak candidates answered this question correctly, suggesting that their grasp of pure mathematics was better than their statistics.

6 (i) Weaker candidates made a variety of errors with these basic calculations – multiplying the sample mean by 100/99, or trying to use $\sigma^2 = \Sigma x^2/n$, or putting 99 in the wrong place. Candidates are advised *not* to use a single formula for the unbiased variance estimate, nor to use the formula on page 8 of MF1, but to calculate the sample variance (as in unit S1) and multiply by $n/(n-1)$.

(ii) Many failed to divide the variance by 9 (some dividing by 100 instead), and many could not decide correctly which tail of the distribution was needed.

(iii) As usual the Central Limit Theorem remains widely misunderstood. Also many failed to understand the difference between whether it *needs* to be used, and whether it *can* be used. The correct answer is, 'It is not needed as the distribution of *R* is already known to be normal.' It could not be used in any case as *n* is only 9, but that is not the point. Some thought that it could be used as *n* was 100, which is wrong on two counts.

7 (i) As usual, weaker candidates found the question on continuous random variables brought them quite a lot of marks. The first part is entirely routine, which nevertheless did not prevent weaker candidates from making mistakes such as not multiplying f(x) by x, or x^2, before integrating.

(ii) As this has a given answer, candidates had to take care to demonstrate it fully. Those who said that the value of the integral was 0.074… had not shown that it equalled exactly 2/27; decimal approximations should not be used here.

(iii) This was found quite difficult, although the idea, using a probability obtained from one distribution as the value of *p* in another distribution, is a common exam question. Many confused the scenarios in parts (iii) and (iv).

(iv) Conversely, many here thought that the distribution was N(8, 50/729) instead of N(1.5, 0.45/108). The figures should be derived from part (i) and not part (iii). Oddly, some who could not do part (iii) at all got part (iv) right, and vice versa; these candidates seemed to find the mental gear-changes beyond them.

8 (i) This hypothesis test on a normal distribution was generally done quite well, although a distressing number continue to use the sample mean, 76.4, in place of the assumed population mean, 78. Thus $\overline{X} \sim$ N(76.4, 68.9/120) is completely wrong. Likewise the attempt to find a critical value as if it were a confidence interval ($76.4 + 2.576\sigma/\sqrt{n}$, instead of the correct $78.0 - 2.576\sigma/\sqrt{n}$) also scores few marks.

(ii) The last part was quite hard. It was surprising that many who had correctly used the 2-tailed *z*-value 2.576 in part (i) here used the 1-tailed value of 2.326. Many tried to bring 76.4 into the calculation, perhaps attempting to answer it as if it were 'last year's question' on Type II errors. The final rider was often omitted altogether and rarely correct; the answer is only an estimation because it is based on an estimated variance (but not on an estimated mean).

4734 Probability & Statistics 3

General Comments

Candidates found this paper relatively difficult and a number failed to finish. The calculations in question 5 were relatively intricate, though standard enough. Several of the marks proved remarkably difficult to obtain, for instance in questions 4 (ii), 5 (i), 6 (i) and 7 (iii). The overall impression, nevertheless, was that the number of candidates who really knew what they were doing was fairly small, and there were many who were not ready for the examination.

As with S2, many centres have taken note of the requirement that conclusions to hypothesis tests should be given not only in context but in a way that acknowledges the uncertainty of the conclusion. Thus '*There is* a difference in the standard of difficulty' loses a mark, compared with 'There is *significant evidence of* a difference in the standard of difficulty'.

Many candidates write in such poor English that their answers to verbal questions are all but incomprehensible, and indeed many clearly do not really understand what they are being asked. (Several answered question 3 (ii) by explaining what a confidence interval means, which is no doubt what they were expecting to answer.) One has sympathy with the many for whom English is obviously not their first language, and the greatest efforts are made to give credit for proper understanding even when poorly expressed, but without comprehension and explanation, Statistics becomes a meaningless exercise.

Comments on Individual Questions

1) (i) For many a straightforward start, though there were some errors handling fractions.

(ii) Some made this unnecessarily hard for themselves by attempting to do $1 - P(X \leq \mu)$. Again, weaker candidates tended to make arithmetic mistakes. A common error was $P(X \geq 2) = 1 - P(X \leq 1)$.

2) (i) Good candidates found this straightforward, and most got as far as Po(1.6). However, some then found P(> 5) instead of P(≥ 5), and some failed to recognise that a binomial distribution was necessary. In the expression $q^{20} + 20pq^{19}$ the factor of 20 was often omitted. It is disappointing to find at this level a number of candidates who think that the probability of no more than 1 infection out of 20 patients is 20 times the probability of an infection in one patient.

(ii) The comments made were generally sensible and relevant. Candidates could focus on either the independence of drugs *A* and *B*, or the conditions needed for the binomial distribution (patients independent, or each equally likely to be infected).

3) (i) Again good candidates found this very straightforward, but many struggled even over so basic a matter as an estimate of the population variance (which is in S2). Many used a *z*-value from the normal distribution (usually 1.96) instead of a *t*-value.

(ii) Most knew that all they had to do was to multiply their previous answers by 2π. A few thought that the answer was unchanged.

4) (i) The hypotheses were well stated. The usual mistake here was not to use a common proportion (of 39/80). Most put $1/n$ factors in the right place.

(ii) Relatively few seemed to realise that this was now a one-tailed test, and even fewer did the right thing, which was simply to find the probability of the tail to the right of their t-value from part (i). Most tried to find a nearby ‘tabular’ value, such as 2.5%.

5) (i) With three marks available, candidates should have expected to give three conditions, and to contextualise them, but very few scored full marks. The conditions were that the hours exercised by men and women should be random variables with independent normal distributions, and with the same variance. Many gave about two of these, often not referring to the context, and added that the variance had to be unknown, which is not a ‘condition’ to use t in the same sense.

(ii) Much the same comments apply here as in question 4. The hypotheses were well stated but many failed to use a pooled variance estimate, or used it wrongly (even though it is in the formula book). In addition, there was a lot of premature rounding, which meant that accurate values of the appropriate t statistic were comparatively rare. Many then used a normal critical value such as 1.96 instead of the correct t_{30} value of 2.042.

(iii) The obvious comment was made by many: the sample would not be representative of the university as a whole. More specifically, the samples would almost certainly not be independent. However, it is incorrect to say that they are not random as, first, one has no guarantee that that is the case and, second, even if it were true, it wouldn’t matter in itself. There were some vaguely entertaining suggestions about the amount of exercise taken by mathematicians.

6) (i) Very few got both marks here. In addition to showing that F(0) = 0 and F(π/2) = 1, it was necessary to assert that F is an increasing function. Some differentiated, perhaps because they could only think in terms of the PDF, but almost nobody remarked that their answer was always positive. Weaker candidates confused F and f throughout.

(ii) By contrast, many found the answer of $\pi/4$ with no difficulty.

(iii) Most candidates were able to get as far as G(y) = F($\sin^{-1} y$), but the notation $\sin^4 (\sin^{-1} y)$ defeated many. Some attempted to use this expression unsimplified; some thought that it was $\sin^3 y$. They could get a method mark for differentiating to find g(y) but inevitably ended up with a completely impossible integral in part (iv). However, most got the correct range for g and stated that it was zero outside that range.

(iv) Only the better candidates got as far as the correct integrand. Some attempted to find $1/[E(Y^3) + 2E(Y^4)]$.

7) (i) Most went through the calculations with admirable care, though it was necessary to ensure that sufficient working was seen (z = 0.674 was virtually essential). Some cut corners and duly lost marks.

(ii) Almost all competent candidates could get $\chi^2 = 10$. However, the vast majority made the wrong decision as to the number of degrees of freedom. The question says 'whether a normal distribution fits the data'. The null hypothesis should therefore refer to 'a normal distribution', and specifically *not* to 'the distribution $N(8.592, 0.7534^2)$'. The mean and variance used for the test are found from the data, rather than imposed from outside, and therefore $\nu = 4 - 3 = 1$. Most assumed that 8.592 and 0.7534 were imposed from outside and took $\nu = 4 - 1 = 3$.

With $\nu = 1$ there is controversy as to whether Yates's correction should be used. The weight of expert opinion is that it should not be, but candidates who did so did not lose a mark on this occasion.

(iii) Most worked out $8.592 \pm 2.576 \times 0.7534/\sqrt{n}$. But what is n? The value 0.7534 is the *sample* standard deviation and therefore n should be 49 rather than 50.

4735 Probability & Statistics 4

General Comments

There were slightly more candidates this year than last. Most of the candidates were well-prepared for the paper. Most candidates used the formula booklet successfully in question 2, but not so successfully elsewhere. The best answered question was question 1, whereas question 7 proved to be the hardest.

Comments on Individual Questions

1) Almost all candidates scored full marks. A few failed to use suffixes and were allowed two marks out of five, for multiplying the moment generating functions (MGF) and identifying that the result was Normal.

2) (i) Most knew the circumstances for the use of non-parametric tests.

(ii) Almost all were successful at using the Wilcoxon rank sum test. There were some errors in the use of the sign test. Some compared with 5%, rather than 2.5%. Some of those who produced the critical region omitted the upper tail. There were some over-assertive conclusions.

(iii) Most identified that the Wilcoxon test is more powerful than the sign test.

3) (i) There were some problems with the limits of integration. The integrations themselves were generally well done.

(ii) Almost all identified that $E(X) = 0$. Most found $Var(X)$ using $M''(0)$ rather than using the binomial expansion, usually successfully.

4) (i) Some candidates made the question more difficult by using the quotient rule and/or producing a complicated expression in t before substituting $t = 1$.

(ii) Most candidates found $Var(Y)$ successfully.

(iii) Many scored the first mark and no more.
However, there were several correct solutions.

5) The first two parts were generally answered well. In the third part the formula booklet was not used well to produce the correct formula. Part (iv) was answered well by only the better candidates.

6) Again, there were some problems with the limits. The integration by parts proved difficult for some. In parts (ii) and (iii), the multiplier $n - 2$ was missed by many. There were many errors in finding the variances; $(1 + 1 + 2 - 1 - 1)Var(X)$ was surprisingly common. Most answered part (iv) correctly.

7 Those who used tree diagrams generally did better than those who did not. Part (i) was answered correctly by many candidates. Very few made progress in (ii). In part (iii) there were some correct answers, usually using the method in the right hand column of the mark scheme. Omission of one or both of the ‘5’s led to partially correct solutions.

4736 Decision Mathematics 1

General Comments

A good proportion of the candidates were able to achieve most of the marks on the paper, although only a few gained full marks.
Most candidates showed a good understanding of the algorithms needed, but often they were not able to apply them with full accuracy. Several candidates struggled to interpret the output from the algorithms.
A few candidates seemed to have run out of time, but the majority of these had wasted time, for example by writing out multiple copies of the simplex tableau.

Some candidates did not number their answers to show which question, or part of a question, they were answering. In some questions candidates had made multiple attempts at one or more parts; in these circumstances it is helpful if they indicate which attempt is their final answer.

Comments on Individual Questions

1) (i) This was a straightforward application of a standard algorithm. Several candidates did not achieve the correct packing because once they had started to use the second folder they forgot to go back and check whether each file would fit in the first folder before checking the second. The effect of this was that the file of size 220 KB was often put into the second folder instead of the first, with the last file sometimes ending up in the first folder and sometimes in the third.

There was no need to illustrate the packing using a diagram, an ordered list showing which files went into which folders was sufficient.

(ii) Many correct answers. A few candidates sorted the list into decreasing order instead of increasing order, although some did then go on to apply first-fit decreasing correctly. Some candidates did not show the sorted list.

(iii) Most candidates were able to calculate that it would take approximately 130 seconds. A few candidates just scaled the time by a factor of 10, and others used a factor of 10^4, instead of 10^2. Some candidates squared the time and then multiplied by the scale factor instead of squaring the scale factor and multiplying by the time. The units were sometimes omitted.

2) (i) Several candidates were able to say that a graph cannot have an odd number of odd vertices, and many of these also went on to explain why. A few candidates claimed that the sum of the vertex orders must be twice the number of vertices (instead of being twice the number of arcs) and some candidates seemed to think that the result depended on there being an even number of vertices.

A general argument was needed rather than an attempt to describe how the arcs joined the vertices together for some specific case. Invariably candidates who tried this approach had assumed that the graph needed to be simple.

There was evidence from some candidates of confusion between the terms ‘vertex’ and ‘arc’.

(ii) (a) Some candidates drew a graph that fitted all the requirements given; rather more drew graphs that had five vertices and were neither simple nor connected but did not have the given vertex orders (usually because of counting a loop from a vertex to itself as 1 arc ending instead of 2); some candidates drew graphs with the required orders but which were either simple or connected or both; a few had loose arcs that appeared to only have one end.

(b) Candidates should have responded that the graph was not semi-Eulerian because it was not connected. Some candidates managed this even when the graph that they had drawn was already connected. Sensible alternative ways of describing connectedness were accepted. There were several interesting, but wrong, suggestions about why the graph was not semi-Eulerian.

(c) Many candidates were able to draw an appropriate graph with the required vertex orders.

(iii) Several candidates did not attempt this part of the question. Of those who did a few seemed to relate it to their own social networks rather than the problem specified.

Some candidates clearly struggled with the idea of colouring the arcs of a complete graph and assumed that if two vertices were connected then the corresponding people had met.

Although many candidates realised that Ann's vertex would have order 5, some talked about a total of 6 arcs and 3 being half of 6. Other candidates started talking about bipartite graphs, or drew the graph $K_{3,3}$ when they should have been working with K_6.

(a) The best answers came from candidates who either listed out all the possibilities (5 red, 0 blue; 4 red, 1 blue; 3 red, 2 blue; 2 red, 3 blue; 1 red, 4 blue; 0 red, 5 blue) or who explained in words that either she had met at least three of the others, in which case at least 3 arcs were blue; or she had met fewer than three, in which case she had not met at least three, and so at least 3 arcs were red.

(b) Some candidates thought that because the arcs AB, AC and AD were blue this meant that there was a blue triangle. Others just claimed that the result was obvious without giving any explanation. A number of candidates assumed that because it was stated that Ann had met Bob, Caz and Del it meant that she had not met Eric and Fran and then gave arguments involving who else had or had not met Eric and Fran.

Several candidates regarded 'has met' as being a transitive operation, so 'if Bob has met Ann and Ann has met Caz then Bob must have met Caz'.

The best answers came from candidates who considered just the arcs connecting A, B, C and D and then considered the case when at least two of Bob, Caz and Del have met one another and the case when none of Bob, Caz and Del have met.

3) (i) As in previous sessions, it seems that some candidates have difficulties with forming the equation of a straight line. A number thought that $y = 1$ or $x + y = 0$ were boundaries of the feasible region. Even those candidates who had the boundary lines correct often had at least one of the inequality signs reversed (or occasionally all three, so that they had described the shaded region rather than the unshaded region).

(ii) Most candidates were able to read the coordinates from the graph, although a few misread (1, 7) as (1, 8) or (1, 1) as (0, 0).

(iii) Most candidates were able to either substitute their x and y values into the objective or use lines of gradient -2/3 to find the optimal point and corresponding maximum value. Some candidates found the optimal point but did not explicitly state the optimal value, or found the optimal value but did not explicitly state the optimal point.

(iv) There were few good answers to this part. Several candidates got as far as substituting $x = 1$, $y = 7$ into $2x + ky$, but often this was then equated to 23 to give $k = 3$. Some candidates achieved $k = 2$ as the critical case, but then gave a strict inequality, $k > 2$, rather than realising that when $k = 2$ the point (1, 7) is still an optimal point.

4) (i) For most candidates this was their most successful question.

Dijkstra's algorithm was usually attempted well, although several candidates made arithmetic slips, omitted some temporary labels or wrote down unnecessary temporary labels. Often the order of assigning permanent labels was switched for two of the nodes.

Most candidates were able to find the shortest path from *A* to *H* and its length.

(ii) Simon needed to solve the route inspection problem. The answer 'Chinese postman' was also accepted. Several candidates thought that Simon needed to solve the travelling salesperson problem, although often then went on to use the route inspection algorithm in part (iii).

(iii) Most candidates who knew that they were meant to be solving a route inspection problem were able to identify the odd nodes correctly and to write down appropriate pairings with their weights. They were then able to sum the weights for the three pairings and then add the smallest total to 67.5 to get the required distance. Several candidates made mistakes with the weight of one or more of the pairings, frequently stating that $AD = 6$ and $EH = 5$ so $AD + EH = 11$, despite having shown that the shortest route from *A* to *D* has weight 5 in part (i).

Some candidates just wrote down a route and tried to add up its length. This did not fully satisfy the requirements of the question, even when they managed to get the distance correct.

A few candidates just picked $DE = 3.5$ and $DH = 4.5$ as being the two shortest routes joining odd vertices.

(iv) Candidates usually found the length of the shortest path, but instead of recording the arcs that needed to be travelled twice they recorded that *DE* had to be repeated. This was an instance where the question had not been read carefully enough.

(v) Several candidates were able to apply the nearest neighbour method correctly as far as vertex *D*, although some slipped up very near the beginning. A few then tried to continue beyond vertex *D* but usually they were able to explain that the method had stalled at *D*, or that it could not continue beyond *D* without starting to repeat vertices that had already been visited.

(vi) Most candidates were able to apply the nearest neighbour algorithm but then omitted to complete the cycle by returning to the start. Some candidates made a shortcut, using *EFG* instead of the direct route *EH*; although this gave a better upper bound they had not shown the application of the nearest neighbour method as the question had asked.

(vii) Several candidates did not show their tree on the diagram in the insert, although they appeared to have carried out Prim's algorithm. Many candidates gave the order in which the arcs were added rather than the nodes as the question had asked. Some candidates then doubled the weight of the minimum spanning tree rather than just adding on the weights of the two shortest arcs from *A* to their tree.

5) (i) Several candidates found all four constraints correctly, although some candidates over-simplified and ended up with decimal coefficients.

(ii) The other constraint on the variables is the trivial or non-negativity constraint that x, y and z are all > 0. Some candidates generated spurious additional constraints or referred to the restriction that x, y and z needed to be integers (or, more correctly, that $100x$, $100y$ and $100z$ needed to be integers).

(iii) Most candidates were able to write down a correct objective function.

(iv) The majority of candidates knew what was required here. Common errors were omitting the row corresponding to the given constraint, misreading z as 2; changing the coefficient of z to 1 in the given constraint and giving positive coefficients for x, y and z in the objective row.

(v) The method was generally known, but arithmetic errors prevented several candidates from achieving full marks. A few candidates misunderstood the instruction 'Use the Simplex algorithm, pivoting first on a value chosen from the x-column and then on a value chosen from the y-column' to mean pivot on the x-column and then go back and start again with a pivot choice from the y-column.

Few candidates interpreted their solution beyond writing down what values they thought the variables now took. A few candidates interpreted the value of the objective as being a profit in £, and some described how many badges (or in one case, badgers) of each type should be made (or sold). Only a few candidates put their values back through the original constraints, before the constraints were simplified, to correctly find how many seconds of printing time, stamping out time, fixing pin time and checking time were not used.

4737 Decision Mathematics 2

General Comments

Several candidates achieved good marks on this paper. The candidates were, in general, well prepared and were able to show what they knew.

Comments on Individual Questions

1) (a)(i) Nearly all the candidates were able to draw the bipartite graph correctly; those who did not had usually omitted one of the arcs into *K*. Some candidates put the people on the left-hand side and the fillings on the right-hand side, but this was condoned. A tiny minority of candidates put the people on a horizontal axis and the fillings on a vertical axis and then joined the corresponding points with lines, this was not accepted.

(ii) The majority of candidates drew a second bipartite graph correctly showing the incomplete matching. Some candidates then drew their alternating path on this graph which sometimes meant that the original solution could not be seen.

(iii) Most candidates were able to write down an alternating path, although some did not give the shortest alternating path. Some candidates just gave label numbering on their diagram, this was condoned. Most candidates wrote their alternating path out as a string. A few candidates gave a list of which arcs had been added in without really saying about the arcs that had been removed. Some candidates just wrote the numbers by the vertices on the graph or showed their alternating path on their graph, such solutions should be followed up with a written statement of the alternating path.

The candidates who had found the alternating path were able to set up the complete matching; a few who had not given the alternating path also gave the matching.

(iv) Most candidates who had given the matching in part (iii) were also able to find the other complete matching.

(b) Most candidates made a reasonable attempt at reducing the rows and columns, a few only reduced the rows and then started augmenting without having reduced the columns.

A minority of candidates decided that they had a maximisation problem and subtracted all the entries in the table from 9 or 10. Some candidates started dropping rows or columns from the matrix, and a few decided to carry on reducing but now ignoring any zero entries.

There were fewer errors in the augmenting than in previous sessions. As in previous series, some candidates carried out numerous augmentations by 1 instead of augmenting by a larger value when this was possible, and some appeared to think that the entries that were crossed out twice should be increased by 1 irrespective of the value being used for the augmentation.

2) (i) There were some good answers to this part, but several candidates were unsure of the headings for the table and a significant number of candidates worked forwards through the network rather than working backwards.

The table should have had a column showing the stage (2, 1, 0); a separate column showing the state (0, 1, 2 alongside stage 2; 0, 1, 2 alongside stage 1; 0 alongside stage 0); the third column is for the action – the state label of the vertex from the previous stage; the final columns should have been for working and identifying the suboptimal maxima (the maximum for each (stage, state) entry).

Some candidates calculated the maximin or minimax instead of the maximum weight route in the table.

Most candidates were able to write down the maximum weight route and give its total weight. A few candidates omitted either the start or the finish vertex and some gave a weight of 7, which was the maximum weight on the route rather than the total weight of the maximum weight route.

(ii) The question told candidates that the same network was being used as in part (i) and had asked them to make a large copy of the network, suitably relabelled. Most candidates gave tiny networks on which it was extremely difficult to read their working values, and several gave networks that were nothing like the network in part (i), usually because large numbers of unnecessary dummy activities were introduced between the stages.

Most candidates achieved a reasonable attempt at both a forward pass and a backward pass. There were several arithmetic errors, and quite a few candidates chose the wrong value when faced with a choice between two or more times. On the forward pass the early event time is the largest value when two or more paths merge (the earliest time at which all the activities leading to this event are complete). On the backward pass the late event time is the smallest of the values available (the latest time to still be able to complete all the activities leading from this event and finish in the minimum project completion time).

Most candidates were able to list the critical activities and minimum project completion time correctly.

(iii) Several candidates realised that the weight of the maximum weight route from part (i) and the minimum project completion time for part (ii) were the same; they often then claimed something to the effect of ‘the minimum and maximum are equal so this is the answer’. What was intended was that candidates would realise that the actions that made up the maximum weight route corresponded to the critical activities. This is because the critical activities will always form a continuous path with no slack, which is therefore the ‘longest’ path.

3) (i) Some candidates gave excellent explanations, such as calling the points for the dog owned by the Rovers x and the corresponding points for the dog owned by the Collies $(10 - x)$, then observing that the scores would be $x - 5$ and $(10 - x) - 5 = 5 - x$, which add together to give 0. Most candidates who attempted this part either gave an incomplete explanation or else just gave a specific example.

(ii) Many candidates confused the scores and the points, often giving the row or column sum as the number of points won in the competition.

For the cell described in the question the table shows that *P* scored 1, so *W* must have scored -1. Then, reversing the effect of the subtraction, *P* must have been given 6 points and *W* must have been given 4 points.

(iii) Most candidates realised that column *W* was dominated by column *Y*, and usually they were also able to show how they knew this. A few candidates gave general statements to support their choice, rather than explicitly showing the three relevant inequalities. A small number of candidates insisted that column *W* dominated column *Y*, having not appreciated that the scores for the dogs owned by the Collies were the negatives of the entries in the table.

(iv) The majority of candidates wrote out the reduced table correctly and found the play-safe strategies. Some candidates did not show their working, that is the row minima and column maxima, or equivalent. A few candidates did not state the play-safe strategies.

(v) Many candidates were able to find the expression $6p - 4$ as the expected score of the Rovers when the Collies choose Xena. Most of these candidates were also able to find the expressions $1 - 2p$ and $3p$ for *Y* and *Z* respectively.

(vi) Few candidates used graph paper but most drew diagrams that were sufficiently well-scaled that they could be used to find the optimal value of p. Some candidates read off the p value from their graph rather than using the equations to calculate the exact value.

(vii) Several candidates did not attempt the last two parts of this question. The expressions given in the question were obtained by first adding 4 throughout the matrix to make all the values non-negative and then using the columns of the augmented matrix to say that if the Collies play *X* then the Rovers expect $6p_1 + 5p_2$, if they play *Y* then $3p_1 + p_2 + 5p_3$ and if they play *Z* then $7p_1 + 3p_2 + 4p_3$.

The linear programming formulation tells you that M is a maximin solution because, for each value of the probability vector, m is the minimum expected value, and M is the maximum value of $m – 4$. Some candidates gave descriptions in terms of finding the highest point under the lower boundary of the graph.

(viii) Most candidates who attempted this part were able to put the given values into the constraints. Some candidates did not then appreciate that the optimal value of p_3 is the limiting value and the corresponding m value (and hence M value) is the smaller of the limiting values.

4) (i) There were fewer errors in calculating the capacity of the given cut than in previous series, but there were still some candidates who either omitted one of the arcs or who did not address the issue of the directions of the flows.

(ii) Most candidates appreciated that the maximum flow into vertex *A* was 6 gallons per litre and that if *AC* and *AD* were both full to capacity there would need to be 7 gallons per litre leaving it. Some candidates then claimed that the flows through vertex *D* prevented *DF* and *EF* from both being full to capacity, instead of considering the flow into and out of vertex *F*.

(iii) A few candidates showed flow through one or both of *A* and *D*. A small number of candidates did not appear to understand what a flow is and just wrote down the capacities of the pipes.

Some candidates tried to show excess capacities and potential backflows instead of the flow that had been asked for. Others showed the flow and then replaced it with the augmented flow after the flow augmenting route had been applied in the next part of the question.

Most candidates realised that 12 gallons per minute was the maximum that could flow through vertex *E*, although some gave the maximum flow for the entire network instead. Even though there was no flow through *A* and *D*, the arcs connecting these vertices should have been shown.

(iv)(a) Several candidates said that there could be no flow augmenting route that passes through vertex *A* but not through vertex *D,* because '*C* is full' (when there could have been further flow through *C*) or because 'arc *CE* is saturated' (which may have been true for their flow pattern but did not prevent flow from being re-routed − for example, using *SBE* rather than *SCE*). The crucial issue was that any flow that passed through *A* and not *D* had to use one of the routes *SACET* or *SACEFT*, but there could be no more flow though *E*.

(b) Candidates had been asked to write down a flow augmenting route. Some candidates instead gave a list of arcs and how much flowed through each, or gave flow augmenting routes starting from a position of zero flow instead of augmenting the flow from part (iii). A few candidates had already given the maximum flow of 13 gallows per minute in part (iii) and so could not now augment their flow.

(v) Few candidates achieved all four marks on this part which asked them to prove that the augmented flow is the maximum flow. The answer required firstly stating the need to find a flow of 13 gallons per minute, then finding a cut of 13 gallons per minute (this was the cut through *ET* and FT) and finally explaining how this proves that 13 gallons per minute is the maximum flow.

Every cut forms a restriction, so having a cut of 13 gallons per minute means that there is no flow that is greater than 13 gallons per minute and hence the maximum flow must be less than or equal to 13 gallons per minute. But we have already found a flow of 13 gallons per minute, so this must be the maximum flow.

(vi) Several of the candidates who attempted this part were able to give well-reasoned arguments to support the required value of 3 gallons per minute.

(vii) There were many accurate and correct diagrams. As above, however, some candidates missed off arcs that had no flow, and some tried to show excess capacities and potential backflows instead of a flow. Quite a number of candidates just gave their flow of 13 gallons per minute again, without regard to the blockage preventing arc *BE* from being used, or went back to having 12 gallons per minute passing through vertex *E* instead if the reduced flow of 9 gallons per minute.

Grade Thresholds

Advanced GCE Mathematics (3890-2, 7890-2)
June 2009 Examination Series

Unit Threshold Marks

Units		Maximum Mark	A	B	C	D	E	U
4721	Raw	72	58	51	44	38	32	0
	UMS	100	80	70	60	50	40	0
4722	Raw	72	56	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4723	Raw	72	53	46	39	33	27	0
	UMS	100	80	70	60	50	40	0
4724	Raw	72	53	46	39	33	27	0
	UMS	100	80	70	60	50	40	0
4725	Raw	72	49	43	37	32	27	0
	UMS	100	80	70	60	50	40	0
4726	Raw	72	53	46	40	34	28	0
	UMS	100	80	70	60	50	40	0
4727	Raw	72	55	49	43	38	33	0
	UMS	100	80	70	60	50	40	0
4728	Raw	72	62	52	42	33	24	0
	UMS	100	80	70	60	50	40	0
4729	Raw	72	57	48	39	31	23	0
	UMS	100	80	70	60	50	40	0
4730	Raw	72	61	51	41	32	23	0
	UMS	100	80	70	60	50	40	0
4731	Raw	72	55	46	38	30	22	0
	UMS	100	80	70	60	50	40	0
4732	Raw	72	54	47	40	33	27	0
	UMS	100	80	70	60	50	40	0
4733	Raw	72	57	49	41	33	26	0
	UMS	100	80	70	60	50	40	0
4734	Raw	72	55	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
4735	Raw	72	52	45	38	32	26	0
	UMS	100	80	70	60	50	40	0
4736	Raw	72	57	50	44	38	32	0
	UMS	100	80	70	60	50	40	0
4737	Raw	72	52	46	40	34	29	0
	UMS	100	80	70	60	50	40	0

Specification Aggregation Results

Overall threshold marks in UMS (i.e. after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
3890	300	240	210	180	150	120	0
3891	300	240	210	180	150	120	0
3892	300	240	210	180	150	120	0
7890	600	480	420	360	300	240	0
7891	600	480	420	360	300	240	0
7892	600	480	420	360	300	240	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
3890	37.64	54.75	68.85	80.19	88.46	100	18954
3892	58.92	74.42	85.06	91.87	96.04	100	2560
7890	47.57	68.42	83.78	93.17	98.15	100	11794
7892	60.58	80.66	90.76	95.89	98.72	100	2006

For a description of how UMS marks are calculated see:
http://www.ocr.org.uk/learners/ums_results.html

Statistics are correct at the time of publication.

List of abbreviations

Below is a list of commonly used mark scheme abbreviations. The list is not exhaustive.

AEF Any equivalent form of answer or result is equally acceptable
AG Answer given (working leading to the result must be valid)
CAO Correct answer only
ISW Ignore subsequent working
MR Misread
SR Special ruling
SC Special case
ART Allow rounding or truncating
CWO Correct working only
SOI Seen or implied
WWW Without wrong working
Ft or √ Follow through (allow the A or B mark for work correctly following on from previous incorrect result.)

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