

ADVANCED GCE

MATHEMATICS (MEI)

Further Methods for Advanced Mathematics (FP2)

4756

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

None

Friday 5 June 2009
Afternoon

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions in Section A and **one** question from Section B.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (54 marks)

Answer all the questions

- 1 (a) (i) Use the Maclaurin series for $\ln(1+x)$ and $\ln(1-x)$ to obtain the first three non-zero terms in the Maclaurin series for $\ln\left(\frac{1+x}{1-x}\right)$. State the range of validity of this series. [4]

- (ii) Find the value of x for which $\frac{1+x}{1-x} = 3$. Hence find an approximation to $\ln 3$, giving your answer to three decimal places. [4]

- (b) A curve has polar equation $r = \frac{a}{1 + \sin \theta}$ for $0 \leq \theta \leq \pi$, where a is a positive constant. The points on the curve have cartesian coordinates x and y .

- (i) By plotting suitable points, or otherwise, sketch the curve. [3]

- (ii) Show that, for this curve, $r + y = a$ and hence find the cartesian equation of the curve. [5]

- 2 (i) Obtain the characteristic equation for the matrix \mathbf{M} where

$$\mathbf{M} = \begin{pmatrix} 3 & 1 & -2 \\ 0 & -1 & 0 \\ 2 & 0 & 1 \end{pmatrix}.$$

Hence or otherwise obtain the value of $\det(\mathbf{M})$. [3]

- (ii) Show that -1 is an eigenvalue of \mathbf{M} , and show that the other two eigenvalues are not real.

Find an eigenvector corresponding to the eigenvalue -1 .

Hence or otherwise write down the solution to the following system of equations. [9]

$$\begin{aligned} 3x + y - 2z &= -0.1 \\ -y &= 0.6 \\ 2x + z &= 0.1 \end{aligned}$$

- (iii) State the Cayley-Hamilton theorem and use it to show that

$$\mathbf{M}^3 = 3\mathbf{M}^2 - 3\mathbf{M} - 7\mathbf{I}.$$

Obtain an expression for \mathbf{M}^{-1} in terms of \mathbf{M}^2 , \mathbf{M} and \mathbf{I} . [4]

- (iv) Find the numerical values of the elements of \mathbf{M}^{-1} , showing your working. [3]

- 3 (a) (i) Sketch the graph of $y = \arcsin x$ for $-1 \leq x \leq 1$. [1]

Find $\frac{dy}{dx}$, justifying the sign of your answer by reference to your sketch. [4]

- (ii) Find the exact value of the integral $\int_0^1 \frac{1}{\sqrt{2-x^2}} dx$. [3]

- (b) The infinite series C and S are defined as follows.

$$C = \cos \theta + \frac{1}{3} \cos 3\theta + \frac{1}{9} \cos 5\theta + \dots$$

$$S = \sin \theta + \frac{1}{3} \sin 3\theta + \frac{1}{9} \sin 5\theta + \dots$$

By considering $C + jS$, show that

$$C = \frac{3 \cos \theta}{5 - 3 \cos 2\theta},$$

and find a similar expression for S . [11]

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

- 4 (i) Prove, from definitions involving exponentials, that

$$\cosh 2u = 2 \cosh^2 u - 1. \quad [3]$$

- (ii) Prove that $\operatorname{arsinh} y = \ln(y + \sqrt{y^2 + 1})$. [4]

- (iii) Use the substitution $x = 2 \sinh u$ to show that

$$\int \sqrt{x^2 + 4} dx = 2 \operatorname{arsinh} \frac{1}{2}x + \frac{1}{2}x\sqrt{x^2 + 4} + c,$$

where c is an arbitrary constant. [6]

- (iv) By first expressing $t^2 + 2t + 5$ in completed square form, show that

$$\int_{-1}^1 \sqrt{t^2 + 2t + 5} dt = 2(\ln(1 + \sqrt{2}) + \sqrt{2}). \quad [5]$$

[Question 5 is printed overleaf.]

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

- 5 Fig. 5 shows a circle with centre $C(a, 0)$ and radius a . B is the point $(0, 1)$. The line BC intersects the circle at P and Q ; P is above the x -axis and Q is below.

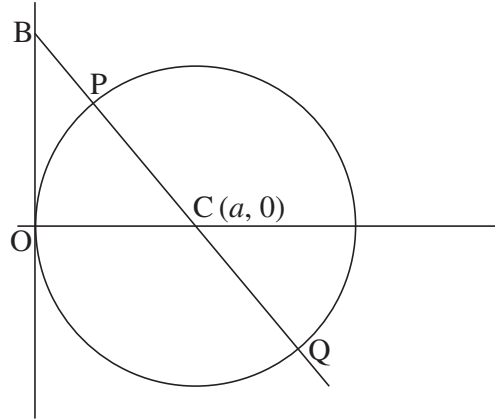


Fig. 5

- (i) Show that, in the case $a = 1$, P has coordinates $\left(1 - \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$. Write down the coordinates of Q . [3]

- (ii) Show that, for all positive values of a , the coordinates of P are

$$x = a \left(1 - \frac{a}{\sqrt{a^2 + 1}}\right), \quad y = \frac{a}{\sqrt{a^2 + 1}}. \quad (*)$$

Write down the coordinates of Q in a similar form. [4]

Now let the variable point P be defined by the parametric equations $(*)$ for all values of the parameter a , positive, zero and negative. Let Q be defined for all a by your answer in part (ii).

- (iii) Using your calculator, sketch the locus of P as a varies. State what happens to P as $a \rightarrow \infty$ and as $a \rightarrow -\infty$.

Show algebraically that this locus has an asymptote at $y = -1$.

On the same axes, sketch, as a dotted line, the locus of Q as a varies. [8]

(The single curve made up of these two loci and including the point B is called a *right strophoid*.)

- (iv) State, with a reason, the size of the angle POQ in Fig. 5. What does this indicate about the angle at which a right strophoid crosses itself? [3]

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations, is given to all schools that receive assessment material and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1PB.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.