OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE **4727/01**

MATHEMATICS

Further Pure Mathematics 3

FRIDAY 6 JUNE 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

[Turn over

1 **(a)** A cyclic multiplicative group G has order 12. The identity element of G is e and another element is r, with order 12.

(i) Write down, in terms of e and r, the elements of the subgroup of G which is of order 4. [2]

(ii) Explain briefly why there is no proper subgroup of G in which two of the elements are e and r. [1]

(b) A group H has order mnp, where m, n and p are prime. State the possible orders of proper subgroups of H. [2]

2 Find the acute angle between the line with equation $\mathbf{r} = 2\mathbf{i} + 3\mathbf{k} + t(\mathbf{i} + 4\mathbf{j} - \mathbf{k})$ and the plane with equation $\mathbf{r} = 2\mathbf{i} + 3\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$. [7]

3 **(i)** Use the substitution $z = x + y$ to show that the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x + y + 3}{x + y - 1} \qquad \text{(A)}$$

may be written in the form $\dfrac{\mathrm{d}z}{\mathrm{d}x} = \dfrac{2(z+1)}{z-1}$. [3]

(ii) Hence find the general solution of the differential equation (A). [4]

4 **(i)** By expressing $\cos\theta$ in terms of $\mathrm{e}^{\mathrm{i}\theta}$ and $\mathrm{e}^{-\mathrm{i}\theta}$, show that

$$\cos^5\theta \equiv \tfrac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos\theta). \qquad [5]$$

(ii) Hence solve the equation $\cos 5\theta + 5\cos 3\theta + 9\cos\theta = 0$ for $0 \leqslant \theta \leqslant \pi$. [4]

5 Two lines have equations

$$\frac{x-k}{2} = \frac{y+1}{-5} = \frac{z-1}{-3} \qquad \text{and} \qquad \frac{x-k}{1} = \frac{y+4}{-4} = \frac{z}{-2},$$

where k is a constant.

(i) Show that, for all values of k, the lines intersect, and find their point of intersection in terms of k. [6]

(ii) For the case $k = 1$, find the equation of the plane in which the lines lie, giving your answer in the form $ax + by + cz = d$. [4]

6 The operation $\circ$ on real numbers is defined by $a \circ b = a|b|$.

(i) Show that $\circ$ is not commutative. [2]

(ii) Prove that $\circ$ is associative. [4]

(iii) Determine whether the set of real numbers, under the operation $\circ$, forms a group. [4]

7 The roots of the equation $z^3 - 1 = 0$ are denoted by 1, ω and ω^2.

(i) Sketch an Argand diagram to show these roots. [1]

(ii) Show that $1 + \omega + \omega^2 = 0$. [2]

(iii) Hence evaluate

(a) $(2 + \omega)(2 + \omega^2)$, [2]

(b) $\dfrac{1}{2 + \omega} + \dfrac{1}{2 + \omega^2}$. [2]

(iv) Hence find a cubic equation, with integer coefficients, which has roots 2, $\dfrac{1}{2 + \omega}$ and $\dfrac{1}{2 + \omega^2}$. [4]

8 **(i)** Find the complementary function of the differential equation

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + y = \operatorname{cosec} x.$$ [2]

(ii) It is given that $y = p(\ln \sin x) \sin x + qx \cos x$, where p and q are constants, is a particular integral of this differential equation.

(a) Show that $p - 2(p + q) \sin^2 x \equiv 1$. [6]

(b) Deduce the values of p and q. [2]

(iii) Write down the general solution of the differential equation. State the set of values of x, in the interval $0 \leqslant x \leqslant 2\pi$, for which the solution is valid, justifying your answer. [3]