

ADVANCED GCE
MATHEMATICS
Further Pure Mathematics 3

4727

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:

None

Wednesday 20 May 2009
Afternoon

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Find the cube roots of $\frac{1}{2}\sqrt{3} + \frac{1}{2}i$, giving your answers in the form $\cos \theta + i \sin \theta$, where $0 \leq \theta < 2\pi$. [4]

2 It is given that the set of complex numbers of the form $re^{i\theta}$ for $-\pi < \theta \leq \pi$ and $r > 0$, under multiplication, forms a group.

(i) Write down the inverse of $5e^{\frac{1}{3}\pi i}$. [1]

(ii) Prove the closure property for the group. [2]

(iii) Z denotes the element $e^{i\gamma}$, where $\frac{1}{2}\pi < \gamma < \pi$. Express Z^2 in the form $e^{i\theta}$, where $-\pi < \theta < 0$. [2]

3 A line l has equation $\frac{x-6}{-4} = \frac{y+7}{8} = \frac{z+10}{7}$ and a plane p has equation $3x - 4y - 2z = 8$.

(i) Find the point of intersection of l and p . [3]

(ii) Find the equation of the plane which contains l and is perpendicular to p , giving your answer in the form $ax + by + cz = d$. [5]

4 The differential equation

$$\frac{dy}{dx} + \frac{1}{1-x^2}y = (1-x)^{\frac{1}{2}}, \quad \text{where } |x| < 1,$$

can be solved by the integrating factor method.

(i) Use an appropriate result given in the List of Formulae (MF1) to show that the integrating factor can be written as $\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$. [2]

(ii) Hence find the solution of the differential equation for which $y = 2$ when $x = 0$, giving your answer in the form $y = f(x)$. [6]

5 The variables x and y satisfy the differential equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = e^{3x}.$$

(i) Find the complementary function. [3]

(ii) Explain briefly why there is no particular integral of either of the forms $y = ke^{3x}$ or $y = kxe^{3x}$. [1]

(iii) Given that there is a particular integral of the form $y = kx^2e^{3x}$, find the value of k . [5]

6 The plane Π_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -5 \\ -2 \end{pmatrix}$.

(i) Express the equation of Π_1 in the form $\mathbf{r} \cdot \mathbf{n} = p$. [4]

The plane Π_2 has equation $\mathbf{r} \cdot \begin{pmatrix} 7 \\ 17 \\ -3 \end{pmatrix} = 21$.

(ii) Find an equation of the line of intersection of Π_1 and Π_2 , giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. [5]

7 (i) Use de Moivre's theorem to prove that

$$\tan 3\theta \equiv \frac{\tan \theta(3 - \tan^2 \theta)}{1 - 3 \tan^2 \theta}. \quad [4]$$

(ii) (a) By putting $\theta = \frac{1}{12}\pi$ in the identity in part (i), show that $\tan \frac{1}{12}\pi$ is a solution of the equation

$$t^3 - 3t^2 - 3t + 1 = 0. \quad [1]$$

(b) Hence show that $\tan \frac{1}{12}\pi = 2 - \sqrt{3}$. [4]

(iii) Use the substitution $t = \tan \theta$ to show that

$$\int_0^{2-\sqrt{3}} \frac{t(3-t^2)}{(1-3t^2)(1+t^2)} dt = a \ln b,$$

where a and b are positive constants to be determined. [5]

8 A multiplicative group Q of order 8 has elements $\{e, p, p^2, p^3, a, ap, ap^2, ap^3\}$, where e is the identity. The elements have the properties $p^4 = e$ and $a^2 = p^2 = (ap)^2$.

(i) Prove that $a = pap$ and that $p = apa$. [2]

(ii) Find the order of each of the elements p^2, a, ap, ap^2 . [5]

(iii) Prove that $\{e, a, p^2, ap^2\}$ is a subgroup of Q . [4]

(iv) Determine whether Q is a commutative group. [4]

**Copyright Information**

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations, is given to all schools that receive assessment material and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity. For queries or further information please contact the Copyright Team, First Floor, 9 Hills Road, Cambridge CB2 1PB.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.