

**Mathematics**

Advanced GCE

Unit **4734**: Probability and Statistics 3

**Mark Scheme for January 2011**

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Any enquiries about publications should be addressed to:

OCR Publications  
PO Box 5050  
Annesley  
NOTTINGHAM  
NG15 0DL

Telephone: 0870 770 6622  
Facsimile: 01223 552610  
E-mail: [publications@ocr.org.uk](mailto:publications@ocr.org.uk)

1(i)	Est $\mu$ = sample mean = 5.25	B1 1	
(ii)	Use (i) $\pm zSD$ SD = $0.19/\sqrt{5}$ $z = 1.96$ $5.083 < \mu < 5.417$	M1 B1 B1 A1 4 [5]	With $\sqrt{5}$ seen  Rounding to 5.08, 5.42
2	Use $G - M \sim N(-6.23, \sigma^2)$ $\sigma^2 = 6.87^2 + 10.25^2$ $z = (16.23)/\sigma$ $= 1.315$ Probability = 0.0942 or 0.0943	M1 A1 M1 A1 A1 [5]	<b>Or <math>G-M-10 \sim N(-16.23, \sigma^2)</math></b>  Accept 0.094
3(i)	$\int_0^2 ae^{-t} dt + \int_2^\infty ae^{-\frac{1}{2}t} dt = 1$ [ $ae^{-t}$ ] + [ $-2ae^{-\frac{1}{2}t}$ ] $\Rightarrow a = \frac{1}{4}e$ AG	M1  A1 A1 3	Properly obtained
(ii)	$\int_{q_3}^\infty \frac{1}{4}e^{-\frac{1}{2}t} dt = \frac{1}{4}$ [ $-\frac{1}{2}e^{-\frac{1}{2}t}$ ] $-\frac{1}{2}e^{-\frac{1}{2}q_3} + 1 = -\ln 2$ $\Rightarrow q_3 = 2(\ln 2 + 1)$ or 3.39	M1 B1 M1 A1 4 [7]	OR $\int_0^2 \frac{1}{4} dt + \int_2^q \frac{1}{4} e^{-t/2} dt = \frac{3}{4}$ AEF For taking logs (not $\ln(-)$ ) AEF
4	$\hat{p}_2 = 106/143, \hat{p}_1 = 61/107$ $= 0.7413 \quad = 0.5701$ Pooled est $p = 167/250$ Variance est = $(\frac{167}{250})(\frac{83}{250})(\frac{143^{-1}+107^{-1}}{250})$ Test statistic $z = (0.7413 - 0.5701)/SD$ $= 2.84(35)$ Smallest significance level = 0.23% SR: No pe, B1B0B0M1A1(2.84)M1A1 Max 5/7	B1  B1 B1 M1 A1 M1 A1√ [7]	For both  Only if used  ART 0.22 or 0.23 Accept 0.0023 $\sqrt{z}$ M1A0 if 0.25%
5(i)	$s^2 = 0.2 \times 0.8/90$ $p_s \pm zs$ $z = 1.645$ $0.1306 < p_y < 0.2693$	B1 M1 B1 A1 4	OR /89  Art (0.131, 0.269)
(ii)	$0.7306 < p_p < 0.8694$	B1ft 1	ft (i) Art (0.731, <b>0.869</b> )
(iii)	If a large number of such intervals were calculated from independent samples, approximately 90% of all such intervals would contain $p$	B2 2	Or: Probability that such an interval contains $p$ is 0.9 B1 for right idea
(iv)	(0.131, 0.269) encloses 0.25 so Mendel's theory is supported	M1 A1 √ 2 [9]	Or equivalent Ft CI(i)

<p>6(i)</p>	<p><math>G(y) = P(Y \leq y)</math>  <math>= P(X \geq 1/y)</math>  <math>= 1 - F(1/y)</math>  <math>= (2y - 1)/(y+1)</math>                  For <math>\frac{1}{2} \leq 1/y \leq 2 \Rightarrow \frac{1}{2} \leq y \leq 2</math>                  X and Y have identical distributions</p> <p>SR: CDF not used.                  y decreases with x                  Use <math>g(y) = f(x(y) dx/dy)</math>  <math>f(x) = 3/(x+1)^2</math>  <math> dx/dy  = 1/y^2</math>  <math>g(y) = [3/(y^{-1}+1)^2][1/y^2] = 3/(y+1)^2</math>; for <math>\frac{1}{2} \leq y \leq 2</math>                  So X and Y have identical distributions</p> <hr/> <p>(ii)</p> <p><math>f(x) = F'(x) = 3/(x+1)^2, \frac{1}{2} \leq x \leq 2</math>  <math>E(X+1) = \int_{\frac{1}{2}}^2 \frac{3}{x+1} dx</math>  <math>= 3 \ln 2</math> (2.08)</p> <p><math>E(1/X) = E(X)</math>  <math>= 3 \ln 2 - 1</math> (1.08)</p>	<p>M1                  A1                  M1                  A1                  B1                  B1 <b>6</b></p> <p>M1                  M1A1                  B1                  M1A1B1                  B1 <b>8</b></p> <hr/> <p>M1A1                  M1                  A1                  M1                  A1 <b>6</b>  <b>[12]</b></p>	<p>Seen</p> <hr/> <p>Must have range of x                  AEF Not if awarded in (i)</p>
<p>7(i)</p>	<p>In a 2x2 contingency table</p> <hr/> <p>(ii)</p> <p><math>H_0</math>: Vaccine type and outcome are independent  <math>H_1</math>: They are not independent                  E-values: 10.81 12.19                  318.19 358.81  <math>\chi^2 = 7.69^2(10.81^{-1} + 12.19^{-1} + 318.19^{-1} + 358.81^{-1})</math>  <math>= 10.67</math>                  CV = 6.635  <b>10.67 &gt; CV</b>                  Reject <math>H_0</math>, there is sufficient evidence at the 1% significance level that the outcome of the test depends on the vaccine used</p> <p>The results is significant at a level less than <math>\frac{1}{2}</math> %, so the evidence is very strong</p>	<p>B1 <b>1</b></p> <hr/> <p>B1M*dep</p> <p>M1                  A1                  M1                  M1                  A1                  B1                  M1</p> <p>A1√                  dep*M</p> <p>A1√ <b>10</b>  <b>[11]</b></p>	<p>Or equivalent Accept df=1</p> <hr/> <p>Accept omission of <math>H_1</math></p> <p>1 correct E value                  Accept 1 dp                  1 correct <math>\chi^2</math> value ft E values                  Using Yates' correctly                  Accept 10.7</p> <p>√ 10.67</p> <p>Sensible comment.√ 10.67</p>

8(i)	When independent samples are drawn from populations having a common variance	B1 1	For common variance
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(ii)	<p><b>(a)</b> Lung capacities should have normal distributions with a common variance</p> <p><b>(b)</b> <math>H_0: \mu_1 = \mu_2, H_1: \mu_1 &gt; \mu_2</math>  <math>s_1^2 = \frac{1}{19}(90.43 - 42.4^2 / 20)</math>  <math>s_2^2 = \frac{1}{21}(82.93 - 42.5^2 / 22)</math>  <math>\bar{x}_1 = 2.12 \quad \bar{x}_2 = 1.93(2)</math>  <math>PEV, s^2 = (19s_1^2 + 21s_2^2) / (20 + 22 - 2)</math>  <math>= 0.03424(3)</math></p> <p>Test statistic = <math>\frac{2.12 - 1.932}{\sqrt{s^2(20^{-1} + 22^{-1})}}</math>  <math>= 3.29(15)</math></p> <p>CV = 2.423                  TS &gt; CV</p> <p>There is sufficient evidence at the 1% SL that the mean lung capacity is greater for children whose parents do not smoke than for children whose parents do smoke                  SR1: For 2-tail test Lose 1<sup>st</sup> B1 and last 3. Max 8/11                  SR2: If <math>s^2 = s_1^2/20 + s_2^2/22</math>, B1M1A0A0M1A0A1(3.32)                  B1M1A1 Max 8/11</p> <p><b>(c)</b> <math>t = 2.704</math>  <math>0.1882 \dots \pm t_s(20^{-1} + 22^{-1})^{1/2}</math>                  (0.0336, 0.3423)</p>	<p>B1 1</p> <p>B1 1</p> <p>B1</p> <p>M1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1A1</p> <p>A1√</p> <p>B1</p> <p>M1</p> <p>A1 11</p> <p>B1</p> <p>M1</p> <p>A1 3</p> <p><b>[16]</b></p>	<p>Normal distributions required                  In context here</p> <p>Or equivalent</p> <p>For 1 correct <math>s^2</math></p> <p>For both</p> <p>ft <math>s^2</math></p> <p>Accept answer rounding to 3.3                  If z used the B0M0A0                  Compare with CV</p> <p>Or equivalent, in context</p> <p>Accept 0.19                  (0.033-0.036, 0.342-0.346)</p>

**OCR (Oxford Cambridge and RSA Examinations)**  
**1 Hills Road**  
**Cambridge**  
**CB1 2EU**

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Telephone: 01223 553998

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