LEVEL 3 CERTIFICATE

MATHEMATICS FOR ENGINEERING

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Electric Vehicles

Introduction

The sales of electric vehicles in the United Kingdom are currently small; however, their popularity is slowly increasing due to heightened awareness of climate change and recent government incentives. Most car manufacturers are now producing a small range of all-electric or dual-fuel vehicles and it is likely that these will be a common sight on our roads in the future. Contrary to popular belief, electric-powered vehicles can reach speeds and have acceleration performances that match cars powered by conventional internal combustion engines.

Batteries

Many electric cars are powered by a bank of ten or twelve 12 volt, lead-acid batteries connected in series, providing a total of 120 or 144 volts. The electrical power of the battery bank is regulated by a controller. The regulated power is then applied to a single electric motor which drives the car wheels via a gearbox and driveshaft as shown in Fig. 1. A bank of fully-charged batteries has a capacity specified in A h (ampere-hours); this is typically between 100 A h and 250 A h. A 120 volt battery bank with a 100 A h rating will theoretically provide 12 kW h. If the average power needed to propel a vehicle on a journey is known, then a rough calculation can determine the time before the batteries would have to be recharged. The capacity of the battery bank is not its only important property; a battery bank that could deliver 100 amperes for 1 hour, for example, will not necessarily be able to deliver 200 amperes for 30 minutes. In addition to this consideration, power losses in mechanical components further reduce the effective power available to propel the vehicle.

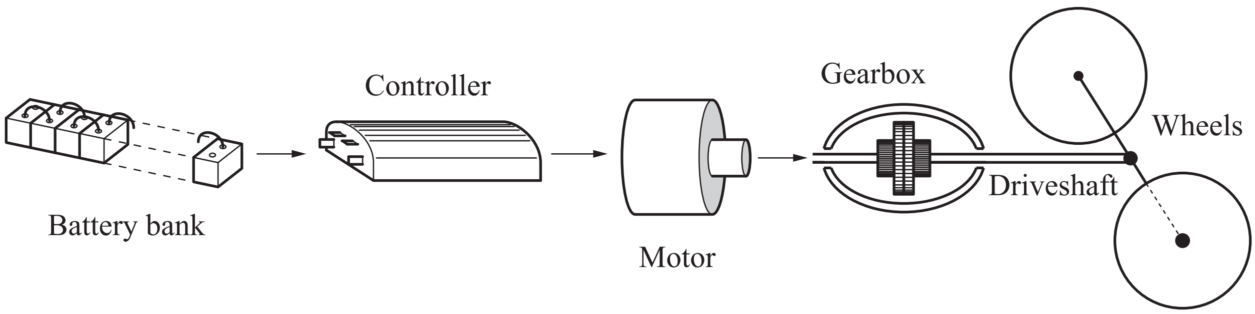

Fig. 1

Force required to propel a vehicle

Fig. 2 shows a vehicle moving in a straight line up a uniform slope. The forces acting on the vehicle parallel to the slope are also shown.

$v\,\mathrm{m\,s^{-1}}$ is the speed of the vehicle up the slope.
D N is the driving force.
F N is the total force acting on the vehicle down the slope.
θ is the angle of the slope with the horizontal.

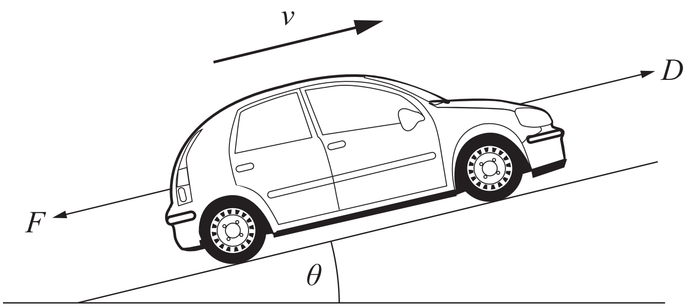

Fig. 2

The force, F N, is given by

$$F = F_d + F_r + F_i$$

where
F_d N is the aerodynamic drag,
F_r N is the rolling resistance,
F_i N is the component down the slope of the weight of the vehicle.

The aerodynamic drag, F_d, is given by

$$F_d = 0.5\rho C_d A v^2$$

where
$\rho\,\mathrm{kg\,m^{-3}}$ is the density of air,
C_d is the drag coefficient,
$A\,\mathrm{m^2}$ is the area of the front of the vehicle,
$v\,\mathrm{m\,s^{-1}}$ is the velocity of the vehicle.

The rolling resistance, F_r, is given by

$$F_r = MgC_r\cos\theta$$

where
M kg is the total mass of the vehicle,
$g\,\mathrm{m\,s^{-2}}$ is the acceleration due to gravity,
C_r is the rolling resistance coefficient,
θ is the angle of the slope with the horizontal.

The component of the weight of the vehicle down the slope, F_i, is given by

$$F_i = Mg\sin\theta$$

where
M, g and θ are as defined above.

The motion of a vehicle can be modelled by a differential equation established using Newton's second law of motion and the formulae given on the previous page. Solving this equation allows the motion of the vehicle to be analysed.

When the vehicle is travelling with a driving force of magnitude equal to that of the force opposing motion, the speed of the vehicle will be constant. Any increase in driving force subsequently applied will cause the vehicle to accelerate.

Because the vehicle is propelled through its wheels, it is often useful to know the torque required to produce the driving force D N at the wheels. This torque, τ_w N m, is given by

$$\tau_w = Dr$$

where

r m is the total radius of the wheel, including the tyre.

Electric propulsion motors for vehicles

Electric vehicles commonly employ DC motors. The characteristics of such motors are very different from internal combustion engines in that they provide maximum torque at rotational speeds close to zero. It is this feature that allows them to accelerate a vehicle from rest very rapidly.

Within certain operational limits, the general behaviour of such motors can be approximated by the relationship

$$\omega_m \tau_m = \eta V i$$

where

ω_m rad s^{-1} is the rotational speed of the motor,
τ_m N m is the torque of the motor,
V volts is the applied voltage,
i amperes is the current drawn by the motor,
η is an electrical-to-mechanical power efficiency coefficient.

In electric cars the controller varies the voltage thereby ensuring that the motor is not driven beyond its operational limits.

The motor is connected to the wheels via a gearbox, each gear having a specified gear ratio, k:1; this is typically between 14:1 for a low gear and 3:1 for a high gear. Ignoring any efficiency losses in the gearbox and driveshaft, the torque required at the vehicle's wheel, τ_w, and the torque produced by the motor, τ_m, have the following relationship.

$$\tau_w = k\tau_m$$

By analysing the forces required to propel the vehicle, the torque produced by the motor, and the power rating and capacity of the battery bank, it is possible to estimate vehicle speeds, accelerations and typical journey ranges before the battery bank needs recharging.

Example

Consider a 1600 kg vehicle travelling at a constant speed of 25 m s^{-1} in a straight line on level ground.

The area of the front of the vehicle is 2.5 m^2.
The drag coefficient is 0.4.
The rolling resistance coefficient is 0.01.
The density of air is 1.2 kg m^{-3}.

The driving force required is approximately 530 N. For a wheel radius, r, of 0.34 m, the torque required at the wheels is approximately 180 N m. For a gear ratio of 5:1, the torque required at the motor is approximately $\frac{180}{5} = 36$ N m.

The speed of the vehicle, the radius of the wheel and the gear ratio determine how fast the electric motor will rotate. With a wheel radius of 0.34 m and a vehicle speed of 25 m s^{-1}, the wheel will rotate at about 700 rev min^{-1}. For a gear ratio of 5:1, the speed of the motor will be about 3500 rev min^{-1}. Using the relationship

$$\omega_m \tau_m = \eta V i$$

with $\eta = 0.85$, the current required with a constant voltage of 120 volts is about 130 amperes.

A typical battery bank discharged by constant current of this magnitude should last approximately an hour.

The power produced by the motor is ηVi. If there are no other efficiency losses, then

$$Dv = \eta Vi.$$

In this example, Dv =13.26 kW and provides the same value of about 130 amperes, as calculated above.

Future developments

The vehicles described in this document relate to those with conventional chassis designs, powered by lead-acid batteries and DC motors. Developments in all these areas continue; lighter vehicles with lithium polymer batteries and a combination of DC and AC motors will provide higher performance characteristics in the future. These factors, coupled with others such as the common availability of public battery charging points, could mean that by the end of the decade the majority of cars on UK roads could be either dual-fuel or all-electric.

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