OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE **A2 7890 - 2**

Advanced Subsidiary GCE **AS 3890 - 2**

Mark Schemes for the Units

January 2008

3890-2/7890-2/MS/R/08J

Oxford Cambridge and RSA Examinations

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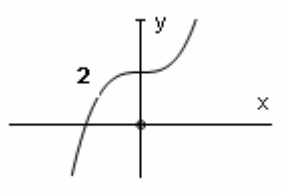
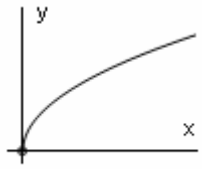
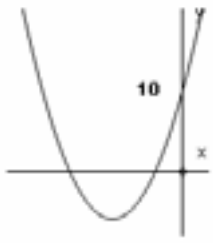
4721 Core Mathematics 1

1	$\frac{4(3+\sqrt{7})}{(3-\sqrt{7})(3+\sqrt{7})}$ $= \frac{12+4\sqrt{7}}{9-7}$ $= 6 + 2\sqrt{7}$	M1 B1 A1 3 **3**	Multiply top and bottom by conjugate 9 ± 7 soi in denominator $6 + 2\sqrt{7}$
2(i)	$x^2 + y^2 = 49$	B1 1	$x^2 + y^2 = 49$
(ii)	$x^2 + y^2 - 6x - 10y - 30 = 0$ $(x-3)^2 - 9 + (y-5)^2 - 25 - 30 = 0$ $(x-3)^2 + (y-5)^2 = 64$ $r^2 = 64$ $r = 8$	M1 A1 2 **3**	3^2 5^2 30 with consistent signs soi 8 cao
3	$a(x+3)^2 + c = 3x^2 + bx + 10$ $3(x^2 + 6x + 9) + c = 3x^2 + bx + 10$ $3x^2 + 18x + 27 + c = 3x^2 + bx + 10$ $c = -17$	B1 B1 M1 A1 4 **4**	$a = 3$ soi $b = 18$ soi $c = 10 - 9a$ or $c = 10 - \frac{b^2}{12}$ $c = -17$
4(i)	p = -1	B1 1	p = -1
(ii)	$\sqrt{25k^2} = 15$ $25k^2 = 225$ $k^2 = 9$ $k = \pm 3$	M1 A1 A1 3	Attempt to square 15 or attempt to square root $25k^2$ $k = 3$ k = -3
(iii)	$\sqrt[3]{t} = 2$ $t = 8$	M1 A1 2 **6**	$\frac{1}{t^{\frac{1}{3}}} = \frac{1}{2}$ or $t^{\frac{1}{3}} = 2$ soi $t = 8$

5(i)		B1 B1 2	+ve cubic +ve or -ve cubic with point of inflection at (0, 2) and no max/min points
(ii)		B1 B1 2	curve with correct curvature in +ve quadrant only completely correct curve
(iii)	Stretch scale factor 1.5 parallel to y-axis	B1 B1 B1 3 7	stretch factor 1.5 parallel to y-axis or in y-direction
6(i)	EITHER $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-8 \pm \sqrt{64 - 40}}{2}$ $x = \frac{-8 \pm \sqrt{24}}{2}$ $x = \frac{-8 \pm 2\sqrt{6}}{2}$ $x = -4 \pm \sqrt{6}$ OR $(x+4)^2 - 16 + 10 = 0$ $(x+4)^2 = 6$ $x + 4 = \pm\sqrt{6}$ M1 A1 $x = \pm\sqrt{6} - 4$ A1	M1 A1 A1 3	Correct method to solve quadratic $x = \frac{-8 \pm \sqrt{24}}{2}$ $x = -4 \pm \sqrt{6}$
(ii)		B1 B1 B1 3	+ve parabola parabola cutting y-axis at (0, 10) where (0, 10) is not min/max point parabola with 2 negative roots
(iii)	$x \le -\sqrt{6} - 4, x \ge \sqrt{6} - 4$	M1 A1 ft 2 8	$x \le$ lower root $x \ge$ higher root (allow < , >) Fully correct answer, ft from roots found in (i)

7(i)	Gradient = $-\frac{1}{2}$	B1 1	$-\frac{1}{2}$
(ii)	$y-5=-\frac{1}{2}(x-6)$	M1	Equation of straight line through (6, 5) with any non-zero numerical gradient
		B1 ft	Uses gradient found in (i) in their equation of line
	$2y-10=-x+6$ $x+2y-16=0$	A1 3	Correct answer in correct form (integer coefficients)
(iii)	EITHER $\frac{4-x}{2}=x^2+x+1$	*M1	Substitute to find an equation in x (or y)
	$4-x=2x^2+2x+2$ $2x^2+3x-2=0$ $(2x-1)(x+2)=0$	DM1	Correct method to solve quadratic
	$x=\frac{1}{2}, x=-2$	A1	$x=\frac{1}{2}, x=-2$
	$y=\frac{7}{4}, y=3$	A1 4	$y=\frac{7}{4}, y=3$
			SR one correct (x,y) pair **www** **B1**
	OR $y=(4-2y)^2+(4-2y)+1$ * M $y=16-16y+4y^2+4-2y+1$ $0=21-19y+4y^2$ $0=(4y-7)(y-3)$ DM1 $y=\frac{7}{4}, y=3$ A1 $x=\frac{1}{2}, x=-2$ A1		
		8	

8(i)	$\frac{dy}{dx} = 3x^2 + 2x - 1$ At stationary points, $3x^2+2x-1=0$ $(3x-1)(x+1)=0$ $x = \frac{1}{3},\ x = -1$ $y = \frac{76}{27},\ y = 4$	*M1 A1 M1 DM1 A1 A1 6	Attempt to differentiate (at least one correct term) 3 correct terms Use of $\frac{dy}{dx} = 0$ Correct method to solve 3 term quadratic $x = \frac{1}{3},\ x = -1$ $y = \frac{76}{27},\ 4$ **SR** one correct (x,y) pair **www** **B1**
(ii)	$\frac{d^2y}{dx^2} = 6x + 2$ $x = \frac{1}{3},\ \frac{d^2y}{dx^2} > 0$ $x = -1,\ \frac{d^2y}{dx^2} < 0$	M1 A1 A1 3	Looks at sign of $\frac{d^2y}{dx^2}$ for at least one of their x-values or other correct method $x = \frac{1}{3}$, minimum point CWO $x = -1$, maximum point CWO
(iii)	$-1 < x < \frac{1}{3}$	M1 A1 2	Any inequality (or inequalities) involving both their x values from part (i) Correct inequality (allow < or ≤)
		11	

9(i)	Gradient of AB = $\frac{-2-1}{-5-3}$ $= \frac{3}{8}$	B1	$\frac{3}{8}$ oe
	$y-1=\frac{3}{8}(x-3)$ $8y-8=3x-9$ $3x-8y-1=0$	M1 A1 3	Equation of line through either A or B, any non-zero numerical gradient Correct equation in correct form
(ii)	$\left(\frac{-5+3}{2}, \frac{-2+1}{2}\right)$ $=(-1, -\frac{1}{2})$	M1 A1 2	Uses $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ $(-1, -\frac{1}{2})$
(iii)	$AC=\sqrt{(-5+3)^2+(-2-4)^2}$ $=\sqrt{2^2+6^2}$ $=\sqrt{40}$ $=2\sqrt{10}$	M1 A1 A1 3	Uses $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\sqrt{40}$ Correctly simplified surd
(iv)	Gradient of AC $=\frac{-2-4}{-5+3} = 3$ Gradient of BC $=\frac{4-1}{-3-3} = -\frac{1}{2}$ $3\times-\frac{1}{2}\neq-1$ so lines are not perpendicular	B1 B1 M1 A1 4 **12**	3 oe $-\frac{1}{2}$ oe Attempts to check $m_1 \times m_2$ Correct conclusion **www**

10(i)	$24x^2 - 3x^{-4}$	B1 B1 B1	$24x^2$ kx^{-4} $-3x^{-4}$
	$48x + 12x^{-5}$	M1 A1 5	Attempt to differentiate their (i) Fully correct
(ii)	$8x^3 + \frac{1}{x^3} = -9$ $8x^6 + 1 = -9x^3$ $8x^6 + 9x^3 + 1 = 0$	*M1	Use a substitution to obtain a 3-term quadratic
	Let $y = x^3$ $8y^2 + 9y + 1 = 0$ $(8y+1)(y+1) = 0$ $y = -\frac{1}{8}, y = -1$	DM1 A1 M1	Correct method to solve quadratic $-\frac{1}{8}$, -1 Attempt to cube root at least one of their y-values
	$x = -\frac{1}{2}, x = -1$	A1 5	$-\frac{1}{2}$, -1
			SR one correct x value **www** **B1**
			SR for trial and improvement: x = -1 B1 $x = -\frac{1}{2}$ B2
		10	Justification that there are no further solutions B2

4722 Core Mathematics 2

		Mark	Total	
1	area of sector = ½ x 11^2 x 0.7	M1		Attempt sector area using (½) $r^2\theta$
	= 42.35	A1		Obtain 42.35, or unsimplified equiv, soi
	area of triangle = ½ x 11^2 x sin0.7 = 38.98	M1		Attempt triangle area using ½*ab*sin*C* or equiv, and subtract from attempt at sector
	hence area of segment = 42.35 – 38.98			
	= 3.37	A1	**4**	Obtain 3.37, or better
			4	
2	area $\approx \frac{1}{2}\times 2\times\left\{2+2\left(\sqrt{12}+\sqrt{28}\right)+\sqrt{52}\right\}$	M1		Attempt *y*-values at *x* = 1, 3, 5, 7 only
		M1		Correct trapezium rule, any *h*, for their *y* values to find area between *x* = 1 and *x* = 7
		M1		Correct *h* (soi) for their *y* values
	≈ 26.7	A1	**4**	Obtain 26.7 or better (correct working only)
			4	
3	**(i)** $\log_a 6$	B1	**1**	State $\log_a 6$ cwo
	(ii) $2\log_{10} x-3\log_{10} y=\log_{10} x^2-\log_{10} y^3$	M1*		Use *b* log *a* = log a^b at least once
	$=\log_{10}\frac{x^2}{y^3}$	M1dep*		Use log *a* - log *b* = log $^a/_b$
		A1	**3**	Obtain $\log_{10}\frac{x^2}{y^3}$ cwo
			4	
4	**(i)** $\frac{BD}{\sin 62}=\frac{16}{\sin 50}$	M1		Attempt to use correct sine rule in Δ*BCD*, or equiv.
	BD = 18.4 cm	A1	**2**	Obtain 18.4 cm
	(ii) $18.4^2 = 10^2 + 20^2 - 2$ x 10 x 20 x cos θ	M1		Attempt to use correct cosine rule in Δ*ABD*
	cos θ = 0.3998	M1		Attempt to rearrange equation to find cos *BAD* (from $a^2=b^2+c^2\pm(2)bc\cos A$)
	θ = 66.4^0	A1	**3**	Obtain 66.4^0
			5	
5	$\int 12x^{\frac{1}{2}}\mathrm{d}x = 8x^{\frac{3}{2}}$	M1		Attempt to integrate
		A1√		Obtain correct, unsimplified, integral following their f(*x*)
		A1		Obtain $8x^{\frac{3}{2}}$, with or without + *c*
	$y = 8x^{\frac{3}{2}} + c \Rightarrow 50 = 8\times 4^{\frac{3}{2}} + c$	M1		Use (4, 50) to find *c*
	$\Rightarrow c = -14$	A1√		Obtain *c* = -14, following $kx^{\frac{3}{2}}$ only
	Hence $y = 8x^{\frac{3}{2}} - 14$	A1	**6**	State $y = 8x^{\frac{3}{2}} - 14$ aef, as long as single power of *x*
			6	

			Mark	Total	
6	**(i)**	$u_1 = 7$	B1		Correct u_1
		$u_2 = 9$, $u_3 = 11$	B1	**2**	Correct u_2 and u_3
	(ii)	Arithmetic Progression	B1	**1**	Any mention of arithmetic
	(iii)	½ N (14 + (N – 1) x 2) = 2200	B1		Correct interpretation of sigma notation
			M1		Attempt sum of AP, and equate to 2200
		$N^2 + 6N – 2200 = 0$	A1		Correct (unsimplified) equation
		$(N – 44)(N + 50) = 0$	M1		Attempt to solve 3 term quadratic in N
		hence $N = 44$	A1	**5**	Obtain $N = 44$ only ($N = 44$ www is full marks)
				8	
7	**(i)**	Some of the area is below the x-axis	B1	**1**	Refer to area / curve below x-axis or 'negative area'…
	(ii)		M1		Attempt integration with any one term correct
			A1		Obtain $^1/_3x^3 – {}^3/_2x^2$
		$\left[\frac{1}{3}x^3 - \frac{3}{2}x^2\right]_0^3 = \left(9 - \frac{27}{2}\right) - (0 - 0)$	M1		Use limits 3 (and 0) – correct order / subtraction
		$= -4\frac{1}{2}$	A1		Obtain (-)4½
		$\left[\frac{1}{3}x^3 - \frac{3}{2}x^2\right]_3^5 = \left(\frac{125}{3} - \frac{75}{2}\right) - \left(9 - \frac{27}{2}\right)$	M1		Use limits 5 and 3 – correct order / subtraction
		$= 8\frac{2}{3}$	A1		Obtain $8^2/_3$ (allow 8.7 or better)
		Hence total area is $13^1/_6$	A1	**7**	Obtain total area as $13^1/_6$, or exact equiv
					SR: if no longer $\int f(x)dx$, then B1 for using [0, 3] and [3, 5]
				8	
8	**(i)**	$u_4 = 10 \times 0.8^3$	M1		Attempt u_4 using ar^{n-1}
		$= 5.12$	A1	**2**	Obtain 5.12 aef
	(ii)	$S_{20} = \frac{10\left(1 - 0.8^{20}\right)}{1 - 0.8}$	M1		Attempt use of correct sum formula for a GP
		$= 49.4$	A1	**2**	Obtain 49.4
	(iii)	$\frac{10}{1 - 0.8} - \frac{10(1 - 0.8^N)}{(1 - 0.8)} < 0.01$	M1		Attempt S_∞ using $\frac{a}{1 - r}$
			A1		Obtain $S_\infty = 50$, or unsimplified equiv
		$50 – 50(1 – 0.8^N) < 0.01$	M1		Link $S_\infty – S_N$ to 0.01 and attempt to rearrange
		$0.8^N < 0.0002$ **A.G.**	A1		Show given inequality convincingly
		$\log 0.8^N < \log 0.0002$	M1		Introduce logarithms on both sides
		$N \log 0.8 < \log 0.0002$	M1		Use $\log a^b = b \log a$, and attempt to find N
		$N > 38.169$, hence $N = 39$	A1	**7**	Obtain $N = 39$ only
				11	

			Mark	Total	
9	**(i)**	$(90^o, 2), (-90^o, -2)$	B1		State at least 2 correct values
			B1	**2**	State all 4 correct values (radians is B1 B0)
	(ii)	**(a)** $180 - \alpha$	B1	**1**	State $180 - \alpha$
		(b) $-\alpha$ or $\alpha - 180$	B1	**1**	State $-\alpha$ or $\alpha - 180$ (radians or unsimplified is B1B0)
	(iii)	$2\sin x = 2 - 3\cos^2 x$			
		$2\sin x = 2 - 3(1 - \sin^2 x)$	M1		Attempt use of $\cos^2 x = 1 - \sin^2 x$
		$3\sin^2 x - 2\sin x - 1 = 0$	A1		Obtain $3\sin^2 x - 2\sin x - 1 = 0$ aef with no brackets
		$(3\sin x + 1)(\sin x - 1) = 0$	M1		Attempt to solve 3 term quadratic in $\sin x$
		$\sin x = -{}^1/_3$, $\sin x = 1$	A1		Obtain $x = -19.5^o$
		$x = -19.5^o, -161^o, 90^o$	A1√		Obtain second correct answer in range, following their x
			A1	**6**	Obtain 90^o (radians or extra answers is max 5 out of 6)
					SR: answer only (and no extras) is B1 B1√ B1
				10	
10	**(i)**	$(2x + 5)^4 = (2x)^4 + 4(2x)^3 5 + 6(2x)^2 5^2 + 4(2x)5^3 + 5^4$	M1*		Attempt expansion involving powers of $2x$ and 5 (at least 4 terms)
		$= 16x^4 + 160x^3 + 600x^2 + 1000x + 625$	M1*		Attempt coefficients of 1, 4, 6, 4, 1
			A1dep*		Obtain two correct terms
			A1	**4**	Obtain a fully correct expansion
	(ii)	$(2x + 5)^4 - (2x - 5)^4 = 320x^3 + 2000x$	M1		Identify relevant terms (and no others) by sign change oe
			A1	**2**	Obtain $320x^3 + 2000x$ cwo
	(iii)	$9^4 - (-1)^4 = 6560$ and $7360 - 800 = 6560$ **A.G.**	B1		Confirm root, at any point
		$320x^3 - 1680x + 800 = 0$	M1		Attempt complete division by $(x - 2)$ or equiv
		$4x^3 - 21x + 10 = 0$	A1√		Obtain quotient of $ax^2 + 2ax + k$, where a is their coeff of x^3
		$(x - 2)(4x^2 + 8x - 5) = 0$	A1		Obtain $(4x^2 + 8x - 5)$ (or multiple thereof)
		$(x - 2)(2x - 1)(2x + 5) = 0$	M1		Attempt to solve quadratic
		Hence $x = ½$, $x = -2½$	A1	**6**	Obtain $x = ½$, $x = -2½$
					SR: answer only is B1 B1
				12	

4723 Core Mathematics 3

1 (i) Show correct process for composition of functions — **M1** numerical or algebraic; the right way round

Obtain (–3 and hence) –23 — **A1 2**

(ii) Either: State or imply $x^3 + 4 = 12$ — **B1**

Attempt solution of equation involving x^3 — **M1** as far as $x = \ldots$

Obtain 2 — **A1 3** and no other value

Or: Attempt expression for f^{-1} — **M1** involving x or y; involving cube root

Obtain $\sqrt[3]{x-4}$ or $\sqrt[3]{y-4}$ — **A1**

Obtain 2 — **A1 (3)** and no other value

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2 (i) Obtain correct first iterate 2.864 — **B1** or greater accuracy 2.864327…; condone 2 dp here and in working

Carry out correct iteration process — **M1** to find at least 3 iterates in all

Obtain 2.877 — **A1 3** after at least 4 steps; answer required to exactly 3 dp

[3 → 2.864327 → 2.878042 → 2.876661 → 2.876800]

(ii) State or imply $x = \sqrt[3]{31 - \frac{5}{2}x}$ — **B1**

Attempt rearrangement of equation in x — **M1** involving cubing and grouping non-zero terms on LHS

Obtain equation $2x^3 + 5x - 62 = 0$ — **A1 3** or equiv with integers

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3 (a) State correct equation involving $\cos\frac{1}{2}\alpha$ — **B1** such as $\cos\frac{1}{2}\alpha = \frac{1}{4}$ or $\frac{1}{\cos\frac{1}{2}\alpha} = 4$ or …

Attempt to find value of α — **M1** using correct order for the steps

Obtain 151 — **A1 3** or greater accuracy; and no other values between 0 and 180

(b) State or imply $\cot\beta = \frac{1}{\tan\beta}$ — **B1**

Rearrange to the form $\tan\beta = k$ — **M1** or equiv involving $\sin\beta$ only or $\cos\beta$ only; allow missing ±

Obtain 69.3 — **A1**

Obtain 111 — **A1 4** or greater accuracy; and no others between 0 and 180

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4 (i) Obtain derivative of form $kh^5(h^6+16)^n$ — **M1** any constant k; any $n < \frac{1}{2}$; allow if – 4 term retained

Obtain correct $3h^5(h^6+16)^{-\frac{1}{2}}$ — **A1** or (unsimplified) equiv; no –4 now

Substitute to obtain 10.7 — **A1 3** or greater accuracy or exact equiv

(ii) Attempt multn or divn using 8 and answer from (i) **M1**

Attempt 8 divided by answer from (i) — **M1**

Obtain 0.75 — **A1√ 3** or greater accuracy; allow 0.75 ± 0.01; following their answer from (i)

5	**(a)**	Obtain integral of form $k(3x+7)^{10}$	**M1**		any constant k
		Obtain (unsimplified) $\frac{1}{10}\times\frac{1}{3}(3x+7)^{10}$	**A1**		or equiv
		Obtain (simplified) $\frac{1}{30}(3x+7)^{10}+c$	**A1**	**3**	
	(b)	State $\int \pi(\frac{1}{2\sqrt{x}})^2\,\mathrm{d}x$	**B1**		or equiv involving x; condone no dx
		Integrate to obtain $k\ln x$	**M1**		any constant k involving π or not; or equiv such as $k\ln 4x$ or $k\ln 2x$
		Obtain $\frac{1}{4}\pi\ln x$ or $\frac{1}{4}\ln x$ or $\frac{1}{4}\pi\ln 4x$ or $\frac{1}{4}\ln 4x$	**A1**		
		Show use of the log a – log b property	**M1**		not dependent on earlier marks
		Obtain $\frac{1}{4}\pi\ln 2$	**A1**	**5**	or similarly simplified equiv

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6	**(i)**	Either: Refer to translation and reflection	**B1**		in either order; allow clear equivs
		State translation by 1 in negative x-direction	**B1**		or equiv but now using correct terminology
		State reflection in x-axis	**B1**	**3**	using correct terminology
		Or: Refer to translation and reflection	**B1**		in either order; allow clear equivs
		State reflection in y-axis	**B1**		
		State translation by 1 in positive x-direction	**B1**	**(3)**	with order reflection then translation clearly intended
	(ii)	Show sketch with attempt at reflection of 'negative' part in x-axis	**M1**		and curve for $0<x<1$ unchanged
		Show (more or less) correct sketch	**A1**	**2**	with correct curvature
	(iii)	Attempt correct process for finding at least one value	**M1**		as far as x = …; accept decimal equivs (degrees or radians) or expressions involving $\sin(\frac{1}{3}\pi)$
		Obtain $1-\frac{1}{2}\sqrt{3}$	**A1**		or exact equiv
		Obtain $1+\frac{1}{2}\sqrt{3}$	**A1**	**3**	or exact equiv; give **A1A0** if extra incorrect solution(s) provided

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7	**(i)**	Attempt use of product rule for $x\mathrm{e}^{2x}$	**M1**		obtaining … + …
		Obtain $\mathrm{e}^{2x}+2x\mathrm{e}^{2x}$	**A1**		or equiv; maybe within QR attempt
		Attempt use of quotient rule	**M1**		with or without product rule
		Obtain unsimplified $\dfrac{(x+k)(\mathrm{e}^{2x}+2x\mathrm{e}^{2x})-x\mathrm{e}^{2x}}{(x+k)^2}$	**A1**		
		Obtain $\dfrac{\mathrm{e}^{2x}(2x^2+2kx+k)}{(x+k)^2}$	**A1**	**5**	**AG**; necessary detail required
	(ii)	Attempt use of discriminant	**M1**		or equiv
		Obtain $4k^2-8k=0$ or equiv and hence $k=2$	**A1**		
		Attempt solution of $2x^2+2kx+k=0$	**M1**		using their numerical value of k or solving in terms of k using correct formula
		Obtain $x=-1$	**A1**		
		Obtain $-\mathrm{e}^{-2}$	**A1**	**5**	or exact equiv

8 (i)	State or imply $h = 1$	**B1**		
	Attempt calculation involving attempts at y values	**M1**		addition with each of coefficients 1, 2, 4 occurring at least once; involving at least 5 y values
	Obtain $a(1 + 4\times2 + 2\times4 + 4\times8 + 2\times16 + 4\times32 + 64)$	**A1**		any constant a
	Obtain 91	**A1**	**4**	
(ii)	State $e^{x\ln 2}$ or $k = \ln 2$	**B1**		allow decimal equiv such as $e^{0.69x}$
	Integrate e^{kx} to obtain $\frac{1}{k}e^{kx}$	**M1**		any constant k or in terms of general k
	Obtain $\frac{1}{\ln 2}(e^{6\ln 2} - e^{0})$	**A1**		or exact equiv
	Simplify to obtain $\frac{63}{\ln 2}$	**A1**	**4**	allow if simplification in part (iii)
(iii)	Equate answers to (i) and (ii)	**M1**		provided ln 2 involved other than in power of e
	Obtain $\frac{63}{91}$ and hence $\frac{9}{13}$	**A1**	**2**	**AG**; necessary correct detail required

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9 (i)	State at least one of $\cos\theta\cos 60 - \sin\theta\sin 60$ and $\cos\theta\cos 30 - \sin\theta\sin 30$	**B1**		
	Attempt complete multiplication of identities of form $\pm\cos\cos \pm \sin\sin$	**M1**		with values $\frac{1}{2}\sqrt{3}$, $\frac{1}{2}$ involved
	Use $\cos^2\theta + \sin^2\theta = 1$ and $2\sin\theta\cos\theta = \sin 2\theta$	**M1**		
	Obtain $\sqrt{3} - 2\sin 2\theta$	**A1**	**4**	**AG**; necessary detail required
(ii)	Attempt use of 22.5 in right-hand side	**M1**		
	Obtain $\sqrt{3} - \sqrt{2}$	**A1**	**2**	or exact equiv
(iii)	Obtain 10.7	**B1**		or greater accuracy; allow ± 0.1
	Attempt correct process to find two angles	**M1**		from values of 2θ between 0 and 180
	Obtain 79.3	**A1**	**3**	or greater accuracy and no others between 0 and 90; allow ± 0.1
(iv)	Indicate or imply that critical values of $\sin 2\theta$ are -1 and 1	**M1**		
	Obtain both of $k > \sqrt{3} + 2$, $k < \sqrt{3} - 2$	**A1**		condoning decimal equivs, $\leq \geq$ signs
	Obtain complete correct solution	**A1**	**3**	now with exact values and unambiguously stated

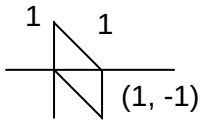
4724 Core Mathematics 4

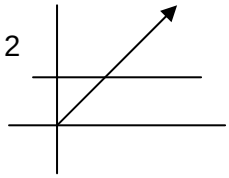
1	Method for finding magnitude of any vector	M1		Expect $\sqrt{14}$ and $\sqrt{6}$
	Method for finding scalar prod of any 2 vectors	M1		Expect $1.2+(-2).1+3.1=3$
	Using $\cos\theta = \dfrac{\mathbf{i}-2\mathbf{j}+3\mathbf{k}\,.\,2\mathbf{i}+\mathbf{j}+\mathbf{k}}{\lvert\mathbf{i}-2\mathbf{j}+3\mathbf{k}\rvert\lvert2\mathbf{i}+\mathbf{j}+\mathbf{k}\rvert}$	M1		Correct vectors only. Expect $\cos\theta = \dfrac{3}{\sqrt{14}\sqrt{6}}$
	70.9 (70.89, 70.893) WWW; 1.24 (1.237)	A1	**4**	Condone answer to nearest degree (71)
2	**(i)** Correct format $\dfrac{A}{x+1}+\dfrac{B}{x+2}$	M1		stated or implied by answer
	$-\dfrac{1}{x+1}$ or $A=-1$	A1		
	$+\dfrac{2}{x+2}$ or $B=2$	A1	**3**	
	(ii) $\int \dfrac{1}{x+1}\,dx = \ln(x+1)$ or $\ln\lvert x+1\rvert$			
	or $\int \dfrac{1}{x+2}\,dx = \ln(x+2)$ or $\ln\lvert x+2\rvert$	B1		
	$A\ln\lvert x+1\rvert + B\ln\lvert x+2\rvert + c$ ISW	√A1	**2**	Expect $-\ln\lvert x+1\rvert + 2\ln\lvert x+2\rvert + c$
3	Method 1 (Long division)			
	Clear correct division method at beginning	M1		x^2 in quot, mult back & attempt subtraction [At subtraction stage, cf $(x^4)=0$]
	Correct method up to & including x term in quot	M1		[At subtraction stage, cf $(x^3)=0$]
	Method 2 (Identity)			
	Writing $(x^2+2x-1)(x^2+bx+2)+cx+7$	M1		Probably equated to $x^4-2x^3-7x^2+7x+a$
	Attempt to compare cfs of x^3 or x^2 or x or const	M1		
	Then:			
	$b=-4$	A1		
	$c=-1$	A1		
	$a=5$	A1	**5**	
4	$\dfrac{d}{dx}(x^2y) = x^2\dfrac{dy}{dx}+2xy$	B1		s.o.i.;
	$\dfrac{d}{dx}(y^3) = 3y^2\dfrac{dy}{dx}$	B1		
	Substitute $(x,y)=(1,1)$ and solve for $\dfrac{dy}{dx}$	M1		or v.v. Solve now or at normal stage. [This
		M1		dep on either/both B1 earned]
	$\dfrac{dy}{dx} = -\dfrac{11}{7}$ WWW	A1		Implied if grad normal = $\dfrac{7}{11}$
	Gradient normal = $-\dfrac{1}{\frac{dy}{dx}}$	M1		Numerical or general, awarded at any stage
	$7x-11y+4=0$ AEF	A1	**6**	No fractions in final answer.

5	**(i)** Use $3\mathbf{i}-4\mathbf{j}+2\mathbf{k}$ and $2\mathbf{i}-\mathbf{j}-5\mathbf{k}$ only	M1		
	Use correct method for scalar prod of <u>any</u> 2 vectors	M1		(indep) May be as part of $\cos\theta = \frac{a.b}{\lvert a\rVert b\rvert}$
	Obtain $6+4-10$, state = 0 & deduce perp **AG**	A1	**3**	
	(ii) Produce 3 equations in s and t	*M1		of the type $5+3s=2+2t$, $-2-4s=-2-t$ and $-2+2s=7-5t$
	Solve 2 of the equations for s and t	dep*M1		*<u>Or</u> Eliminate s (or t) from 2 pairs dep*M1*
	Obtain $(s,t)=\left(\frac{3}{5},\frac{12}{5}\right)$ or $\left(\frac{9}{22},\frac{18}{11}\right)$ or $\left(\frac{3}{19},\frac{33}{19}\right)$	A1		*(5t=12,11t=18,19t=33) <u>or</u> (5s=3,22s= 9,19s=3) A1,A1*
	Substitute their values in 3rd equation	dep*M1		*State/show inconsistency <u>& state non-parallel</u>*
	State/show inconsistency <u>& state non-parallel</u> ∴ skew	A1	**5**	*∴ skew WWW* A1
6	**(i)** $1-4ax+\ldots$	B1		
	$\frac{-4.-5}{1.2}(ax)^2$ or $\frac{-4.-5}{1.2}a^2x^2$ or $\frac{-4.-5}{1.2}ax^2$	M1		Do not accept $\begin{pmatrix}-4\\2\end{pmatrix}$ unless 10 also appears
	$\ldots+10a^2x^2$	A1	**3**	
	(ii) f.t. (their cf x) + b(their const cf) = 1	√B1		Expect $b-4a=1$
	f.t. (their cf x^2) + b(their cf x) $=-2$	√B1		Expect $10a^2-4ab=-2$
	Attempt to eliminate 'b' and produce equation in 'a'	M1		Or eliminate 'a' and produce equation in 'b'
	Produce $6a^2+4a=2$ AEF	A1		Or $6b^2+4b=42$ AEF
	$a=\frac{1}{3}$ and $b=\frac{7}{3}$ only	A1	**5**	Made clear to be only (final) answer
7	**(i)** Perform an operation to produce an equation connecting A and B (or possibly in A or in B)	M1		Probably substituting value of θ, or comparing coefficients of sin x, and/or cos x
	$A=2$	A1		
	$B=-2$	A1	**3**	WW scores 3
	(ii) Write $4\sin\theta$ as $A(\sin\theta+\cos\theta)+B(\cos\theta-\sin\theta)$ and re-write integrand as $A+\frac{B(\cos\theta-\sin\theta)}{\sin\theta+\cos\theta}$	M1		*A* and *B* need not be numerical – but, if they are, they should be the values found in (i).
	$\int A\,\mathrm{d}\theta = A\theta$	√B1		general or numerical
	$\int \frac{B(\cos\theta-\sin\theta)}{\sin\theta+\cos\theta}\mathrm{d}\theta = B\ln(\sin\theta+\cos\theta)$	√A2		general or numerical
	Produce $\frac{1}{4}A\pi+B\ln\sqrt{2}$ f.t. with their A,B	√A1	**5**	Expect $\frac{1}{2}\pi-\ln 2$ (Numerical answer only)
8	**(i)** $\frac{\mathrm{d}x}{\mathrm{d}t}$ or $-kx^{\frac{1}{2}}$ or $kx^{\frac{1}{2}}$ seen	M1		*k* non-numerical; i.e. 1 side correct
	$\frac{\mathrm{d}x}{\mathrm{d}t}=-kx^{\frac{1}{2}}$ or $\frac{\mathrm{d}x}{\mathrm{d}t}=kx^{\frac{1}{2}}$	A1	**2**	i.e. both sides correct
	(ii) Separate variables or invert, + attempt to integrate *	M1		Based <u>only</u> on above eqns or $\frac{\mathrm{d}x}{\mathrm{d}t}=x^{\frac{1}{2}}$, $-x^{\frac{1}{2}}$
	Correct result for their equation after integration	A1		Other than omission of 'c'
	Subst $(t,x)=(0,2)$ into eqn containing k &/or c	dep*M1		or substitute (5,1)
	Subst $(t,x)=(5,1)$ into eqn containing k <u>&</u> c	dep*M1		or substitute (0,2)
	Subst $x=0.5$ into eqn with their k & c subst	dep*M1		
	$t=8.5\,(8.5355339)$	A1	**6**	[1 d.p. requested in question]

9	**(i)** Use $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ or $\frac{\frac{dy}{dp}}{\frac{dx}{dp}}$	M1		Or conv to cartes form & att to find $\frac{dy}{dx}$ at P
	$= \frac{2t}{3t^2}$ or $\frac{2p}{3p^2}$	A1		
	Find eqn tgt thro (p^3, p^2) or (t^3, t^2) ,their gradient	M1		Using $y - y_1 = m(x - x_1)$ or $y = mx + c$
	$3py - 2x = p^3$ **AG**	A1	**4**	Do not accept t here
	(ii) Substitute $(-10,7)$ into given equation *	M1		to produce a cubic equation in p
	Satis attempt to find at least 1 root/factor dep*	M1		Inspection/factor theorem/rem theorem/t&i
	Any one root	A1		-1 or -4 or 5
	All 3 roots	A1		-1,-4 and 5
	$(-1,1)$, $(-64,16)$ and $(125,25)$	A1	**5**	All 3 sets; no f.t.
10	**(i)** $(1-x^2)^{\frac{3}{2}} \rightarrow \cos^3\theta$	B1		May be implied by $\int \sec^2\theta \, d\theta$
	$dx \rightarrow \cos\theta \, d\theta$	B1		
	$\frac{1}{(1-x^2)^{\frac{3}{2}}} dx \rightarrow \sec^2\theta (d\theta)$ or $\frac{1}{\cos^2\theta}(d\theta)$	B1		
	$\int \sec^2\theta (d\theta) = \tan\theta$	B1		
	Attempt change of limits (expect 0 & $\frac{1}{6}\pi / 30$)	M1		Use with $f(\theta)$; or re-subst & use 0 & $\frac{1}{2}$
	$\frac{1}{\sqrt{3}}$ AEF	A1	**6**	Obtained with no mention of 30 anywhere
	(ii) Use parts with $u = \ln x$, $\frac{dv}{dx} = \frac{1}{x^2}$	*M1		obtaining a result $f(x) + / - \int g(x)(dx)$
	$-\frac{1}{x}\ln x + \int \frac{1}{x^2}(dx)$ AEF	A1		Correct first stage result
	$-\frac{1}{x}\ln x - \frac{1}{x}$	A1		Correct overall result
	Limits used correctly	dep*M1		
	$\frac{2}{3} - \frac{1}{3}\ln 3$	A1	**5**	
	<u>If substitution attempted in part (ii)</u>			
	$\ln x = t$	B1		
	Reduces to $\int t\, e^{-t} \, dt$	B1		
	Parts with $u = t, dv = e^{-t}$	M1		
	$-te^{-t} - e^{-t}$	A1		
	$\frac{2}{3} - \frac{1}{3}\ln 3$	A1		

4725 Further Pure Mathematics 1

1	(i)  (ii) $\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$	M1 A1 B1 B1	 2 2 **4**	For 2 other correct vertices seen, correct direction of shear seen For completely correct diagram, must include scales Each column correct
2	$\frac{a}{6}n(n+1)(2n+1)+bn$ $a = 6$ $b = -3$	M1 A1 M1 A1 A1	 5 **5**	Consider sum as two separate parts Correct answer a.e.f. Compare co-efficients Obtain correct answers
3	(i) $7u^3 + 24u^2 - 3u + 2 = 0$ (ii) *EITHER* correct value is $-\frac{3}{7}$ *OR* correct value is $-\frac{3}{7}$	M1 A1 M1 A1ft M1 A1	 2 2 **4**	Use given substitution Obtain correct equation a.e.f. Required expression related to new cubic Their c / their a Use $\frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}$ or equivalent Obtain correct answer
4	(i) $z^* = 3 + 4i$ $21 + 12i$ (ii) $3 - 5i$ $-16 - 30i$ (iii) $\frac{9}{25} + \frac{12}{25}i$	B1 B1 B1 B1ft B1ft M1 A1 A1	 2 3 3 **8**	Conjugate seen or implied Obtain correct answer Correct $z - i$ or expansion of $(z - I)^2$ seen Real part correct Imaginary part correct Multiply by conjugate Numerator correct Denominator correct
5	(i) $\begin{pmatrix} -13 \\ 1 \\ -10 \end{pmatrix}$ (ii) $\begin{pmatrix} 8 & 16 & -4 \\ 0 & 0 & 0 \\ 6 & 12 & -3 \end{pmatrix}$ (iii) (8)	B1 B1 M1 A1A1A1 M1 A1	 2 4 2 **8**	4**B** seen or implied or 2 elements correct Obtain correct answer Obtain a 3 x 3 matrix Each row (or column) correct Obtain a single value Obtain correct answer, must have matrix

6	(i)  (ii) $2\sqrt{3} + 2\,\mathrm{i}$	B1 B1 B1 B1 B1 B1 M1 A1	 5 3 **8**	Horizontal straight line in 2 quadrants Through (0, 2) Straight line Through *O* with positive slope In 1st quadrant only State or obtain algebraically that $y = 2$ Use suitable trigonometry Obtain correct answer a.e.f. decimals OK must be a complex number
7	(i) $a = -6$ (ii) $\mathbf{A}^{-1} = \frac{1}{a+6}\begin{pmatrix} 1 & -3 \\ 2 & a \end{pmatrix}$ $x = \frac{4}{a+6}, y = \frac{2-a}{a+6}$	M1 A1 B1 B1ft M1 A1ft A1ft	 2 5 **7**	Use det **A** = 0 Obtain correct answer Both diagonals correct Divide by det **A** Premultiply column by $\mathbf{A}^{-1}$, no other method Obtain correct answers from their $\mathbf{A}^{-1}$
8	(i) $u_2 = 4,\ u_3 = 9,\ u_4 = 16$ (ii) $u_n = n^2$ (iii)	M1 A1 B1 B1 M1 A1 A1	 2 1 4 **7**	Obtain next terms All terms correct Sensible conjecture made State that conjecture is true for $n = 1$ or 2 Find u_{n+1} in terms of n Obtain $(n + 1)^2$ Statement of Induction conclusion
9	(i) $\alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$ (ii) *Either* $\alpha + \beta = 5, \alpha\beta = 7$ $\alpha^3 + \beta^3 = 20$ $x^2 - 20x + 343 = 0$ *Or* $u^{\frac{2}{3}} - 5u^{\frac{1}{3}} + 7 = 0$ $u^3 - 20u + 343 = 0$	M1 A1 B1 B1 M1 A1 M1 A1ft M1 A1 M2 A2	 2 6 **8**	Correct binomial expansion seen Obtain given answer with no errors seen State or use correct values Find numeric value for $\alpha^3 + \beta^3$ Obtain correct answer Use new sum and product correctly in quadratic expression Obtain correct equation Substitute $x = u^{\frac{1}{3}}$ Obtain correct answer Complete method for removing fractional powers Obtain correct answer

10	(i)	M1 A1	 2	Attempt to combine 3 fractions Obtain given answer correctly
	(ii)	M1 A1 M1 A1 M1 A1	 6	Express at least first 3 terms using (i) All terms correct Express at least last 2 terms using (i) All terms correct in terms of n Show that correct terms cancel Obtain unsimplified correct answer
	(iii) $\frac{5}{2}$	B1ft	1	Obtain correct answer from their (ii)
	(iv) $\frac{2}{N+1}+\frac{1}{N+2}=\frac{7}{10}$	B1ft		Their (iii) – their (ii)
	$7N^2-9N-36=0$	M1		Attempt to clear fractions & solve equation, Obtain correct simplified equation
	$N=3$	A1 A1	 4 **13**	Obtain only the correct answer

4726 Further Pure Mathematics 2

1	**(i)**	Get $f'(x) = \pm \sin x/(1+\cos x)$	M1	Reasonable attempt at chain at any stage
		Get $f''(x)$ using quotient/product rule	M1	Reasonable attempt at quotient/product
		Get $f(0) = \ln 2$, $f'(0) = 0$, $f''(0) = -½$	B1	Any one correct from correct working
			A1	All three correct from correct working
	(ii)	Attempt to use Maclaurin correctly	M1	Using their values in $af(0)+bf'(0)x+cf''(0)x^2$; may be implied
		Get $\ln 2 - ¼x^2$	A1√	From their values; must be quadratic
2	**(i)**	Clearly verify in $y = \cos^{-1}x$	B1	i.e. $x=½\sqrt{3}$, $y=\cos^{-1}(½\sqrt{3})= {}^1/_6\pi$, or similar
		Clearly verify in $y = ½\sin^{-1}x$	B1	Or solve $\cos y = \sin 2y$
			SR	Allow one B1 if not sufficiently clear detail
	(ii)	Write down at least one correct diff'al	M1	Or reasonable attempt to derive; allow ±
		Get gradient of –2	A1	cao
		Get gradient of 1	A1	cao
3	**(i)**	Get y- values of 3 and $\sqrt{28}$	B1	
		Show/explain areas of two rectangles equal y- value x 1, and relate to A	B1	Diagram may be used
	(ii)	Show $A>0.2(\sqrt{(1+2^3)} + \sqrt{(1+2.2^3)} + \ldots$ ..$\sqrt{(1+2.83)}$)	M1	Clear areas attempted below curve (5 values)
		$= 3.87(28)$	A1	To min. of 3 s.f.
		Show $A<0.2(\sqrt{(1+2.2^3)} + \sqrt{(1+2.4^3)} + \ldots$ $\ldots+ \sqrt{(1+3^3)})$	M1	Clear areas attempted above curve (5 values)
		$= 4.33(11) < 4.34$	A1	To min. of 3 s.f.
4	**(i)**	Correct formula with correct r	M1	May be implied
		Expand r^2 as $A + B\sec\theta + C\sec^2\theta$	M1	Allow B = 0
		Get $C\tan\theta$	B1	
		Use correct limits in their answer	M1	Must be 3 terms
		Limits to ${}^1/_{12}\pi + 2\ln(\sqrt{3}) + {}^{2\sqrt{3}}/_3$	A1	AEEF; simplified
	(ii)	Use $x=r\cos\theta$ and $r^2 = x^2+ y^2$	B1	Or derive polar form from given equation
		Eliminate r and θ	M1	Use their definitions
		Get $(x – 2)\sqrt{(x^2 + y^2)} = x$	A1	A.G.

5 **(i)** Attempt use of product rule M1
Clearly get $x = 1$ A1 Allow substitution of $x=1$

(ii) Explain use of tangent for next approx. B1 Not use of G.C. to show divergence
Tangents at successive approx. give $x>1$ B1 Relate to crossing x-axis; allow diagram

(iii) Attempt correct use of N-R with their derivative M1
Get $x_2 = -1$ A1√
Get –0.6839, -0.5775, (-0.5672…) A1 To 3 d.p. minimum
Continue until correct to 3 d.p. M1 May be implied
Get –0.567 A1 cao

6 **(i)** Attempt division/equate coeff. M1 To lead to some $ax+b$ (allow $b=0$ here)
Get $a = 2$, $b = -9$ A1
Derive/quote $x = 1$ B1 Must be equations

(ii) Write as quadratic in x M1 $(2x^2-x(11+y)+(y-6)=0)$
Use $b^2 \geq 4ac$ (for real x) M1 Allow <, >
Get $y^2 +14y +169 \geq 0$ A1
Attempt to justify positive/negative M1 Complete the square/sketch
Get $(y+7)^2 +120 \geq 0$ – true for all y A1

SC Attempt diff; quot./prod. rule M1
Attempt to solve $dy/dx = 0$ M1
Show $2x^2 – 4x + 17 = 0$ has no real roots e.g. $b^2 – 4ac < 0$ A1
Attempt to use no t.p. M1
Justify all y e.g. consider asymptotes and approaches A1

7 **(i)** Get $x(1+x^2)^{-n} - \int x.(-n(1+x^2)^{-n-1}.2x)\,dx$ M1 Reasonable attempt at parts
Accurate use of parts A1
Clearly get A.G. B1 Include use of limits seen

(ii) Express x^2 as $(1+x^2) – 1$
Get $\frac{x^2}{(1+x^2)^{n+1}} = \frac{1}{(1+x^2)^n} - \frac{1}{(1+x^2)^{n+1}}$ B1 Justified
Show $I_n = 2^{-n} +2n(I_n – I_{n+1})$ M1 Clear attempt to use their first line above
Tidy to A.G. A1

(iii) See $2I_2 = 2^{-1} + I_1$ B1
Work out $I_1 = \frac{1}{4}\pi$ M1 Quote/derive $\tan^{-1}x$
Get $I_2 = \frac{1}{4} + \frac{1}{8}\pi$ A1

8 **(i)** Use correct exponential for sinh x — B1
Attempt to expand cube of this — M1 — Must be 4 terms
Correct cubic — A1
Clearly replace in terms of sinh — B1 — (Allow RHS→ LHS or RHS = LHS separately)

(ii) Replace and factorise — M1 — Or state sinh $x \neq 0$
Attempt to solve for $\sinh^2 x$ — M1 — $(= \frac{1}{4}(k-3))$ or for k and use $\sinh^2 x > 0$
Get $k>3$ — A1 — Not ≥

(iii) Get $x = \sinh^{-1} c$ — M1 — $(c = \pm\frac{1}{2})$; allow sinh $x = c$
Replace in ln equivalent — A1√ — As $\ln(\frac{1}{2}+\sqrt{5}/4)$; their x
Repeat for negative root — A1√ — May be given as neg. of first answer (no need for $x=0$ implied)
SR — Use of exponential definitions
Express as cubic in $e^{2x} = u$ M1
Factorise to $(u-1)(u^2-3u+1)=0$ A1
Solve for $x = 0$, $\frac{1}{2}\ln(\frac{3}{2} \pm \sqrt{5}/2)$ A1

9 **(i)** Get sinh $y \, dy/dx = 1$ — M1 — Or equivalent; allow ±
Allow use of ln equivalent with Chain Rule
Replace sinh $y = \sqrt{(\cosh^2 y - 1)}$ — A1
Justify positive grad. to A.G. — B1 — e.g. sketch

(ii) Get $k \cosh^{-1} 2x$ — M1 — No need for c
Get $k=\frac{1}{2}$ — A1

(iii) Sub. $x = k \cosh u$ — M1
Replace all x to $\int k_1 \sinh^2 u \, du$ — A1
Replace as $\int k_2(\cosh 2u - 1) \, du$ — M1 — Or exponential equivalent
Integrate correctly — A1√ — No need for c
Attempt to replace u with x equivalent — M1 — In their answer
Tidy to reasonable form — A1 — cao $(\frac{1}{2}x\sqrt{(4x^2 - 1)} - \frac{1}{4}\cosh^{-1} 2x \; (+c))$

4727 Further Pure Mathematics 3

1 (a) (i) e.g. $ap \neq pa \Rightarrow$ not commutative	B1 **1**	For correct reason and conclusion
(ii) 3	B1 **1**	For correct number
(iii) e, a, b	B1 **1**	For correct elements
(b) c^3 has order 2	B1	For correct order
c^4 has order 3	B1	For correct order
c^5 has order 6	B1 **3**	For correct order
	6	
2 $m^2 - 8m + 16 = 0$	M1	For stating and attempting to solve auxiliary eqn
$\Rightarrow m = 4$	A1	For correct solution
$\Rightarrow$ CF $(y =) (A + Bx)\mathrm{e}^{4x}$	A1√	For CF of correct form. f.t. from m
For PI try $y = px + q$	M1	For using linear expression for PI
$\Rightarrow -8p + 16(px + q) = 4x$		
$\Rightarrow p = \frac{1}{4} \quad q = \frac{1}{8}$	A1 A1	For correct coefficients
$\Rightarrow$ GS $y = (A + Bx)\mathrm{e}^{4x} + \frac{1}{4}x + \frac{1}{8}$	B1√ **7**	For GS = CF + PI. Requires $\boxed{y =}$. f.t. from CF and PI with 2 arbitrary constants in CF and none in PI
	7	
3 (i) line segment OA	B1	For stating line through O *OR* A
	B1 **2**	For correct description **AEF**
(ii) $(\mathbf{r} - \mathbf{a}) \times (\mathbf{r} - \mathbf{b}) = \overrightarrow{AP} \times \overrightarrow{BP}$	B1	For identifying $\mathbf{r} - \mathbf{a}$ with $\overrightarrow{AP}$ and $\mathbf{r} - \mathbf{b}$ with $\overrightarrow{BP}$ Allow direction errors
$= \lvert AP \rvert \lvert BP \rvert \sin\pi \,.\, \hat{\mathbf{n}} = \mathbf{0}$	B1 **2**	For using × of 2 parallel vectors = **0** *OR* $\sin\pi = 0$ *or* $\sin 0 = 0$ in an appropriate vector expression
(iii) line through O	B1	For stating line
	B1	For stating through O
parallel to AB	B1 **3**	For stating correct direction
		SR For $\overrightarrow{AB}$ or $\overrightarrow{BA}$ allow B1 B0 B1
	7	
4 $(C + \mathrm{i}S =) \int_0^{\frac{1}{2}\pi} \mathrm{e}^{2x}(\cos 3x + \mathrm{i}\sin 3x)(\mathrm{d}x)$		
$\cos 3x + \mathrm{i}\sin 3x = \mathrm{e}^{3\mathrm{i}x}$	B1	For using de Moivre, seen or implied
$\int_0^{\frac{1}{2}\pi} \mathrm{e}^{(2+3\mathrm{i})x}(\mathrm{d}x) = \frac{1}{2+3\mathrm{i}}\left[\mathrm{e}^{(2+3\mathrm{i})x}\right]_0^{\frac{1}{2}\pi}$	M1*	For writing as a single integral in exp form
	A1	For correct integration (ignore limits)
$= \frac{2-3\mathrm{i}}{4+9}\left(\mathrm{e}^{(2+3\mathrm{i})\frac{1}{2}\pi} - \mathrm{e}^{0}\right) = \frac{2-3\mathrm{i}}{13}\left(-\mathrm{i}\,\mathrm{e}^{\pi} - 1\right)$	A1	For substituting limits correctly (unsimplified) (may be earned at any stage)
	M1 (dep*)	For multiplying by complex conjugate of 2+3i
$= \left\{\frac{1}{13}\left(-2 - 3\mathrm{e}^{\pi} + \mathrm{i}(3 - 2\mathrm{e}^{\pi}\right)\right\}$	M1 (dep*)	For equating real and/or imaginary parts
$C = -\frac{1}{13}\left(2 + 3\mathrm{e}^{\pi}\right)$	A1	For correct expression **AG**
$S = \frac{1}{13}\left(3 - 2\mathrm{e}^{\pi}\right)$	A1	For correct expression
	8	

5 (i) IF $e^{\int \frac{1}{x}dx} = e^{\ln x} = x$ *OR* $x\frac{dy}{dx} + y = x\sin 2x$	M1	For correct process for finding integrating factor *OR* for multiplying equation through by x
$\Rightarrow \frac{d}{dx}(xy) = x\sin 2x$	A1	For writing DE in this form (may be implied)
$\Rightarrow xy = \int x\sin 2x(dx)$	M1	For integration by parts the correct way round
$xy = -\frac{1}{2}x\cos 2x + \frac{1}{2}\int \cos 2x(dx)$	A1	For 1st term correct
$xy = -\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x\ (+c)$	M1	For their 1st term and attempt at integration of $\frac{\cos}{\sin} kx$
$\Rightarrow y = -\frac{1}{2}\cos 2x + \frac{1}{4x}\sin 2x + \frac{c}{x}$	A1 **6**	For correct expression for y
(ii) $\left(\frac{1}{4}\pi, \frac{2}{\pi}\right) \Rightarrow \frac{2}{\pi} = \frac{1}{\pi} + \frac{4c}{\pi} \Rightarrow c = \frac{1}{4}$	M1	For substituting $\left(\frac{1}{4}\pi, \frac{2}{\pi}\right)$ in solution
$\Rightarrow y = -\frac{1}{2}\cos 2x + \frac{1}{4x}\sin 2x + \frac{1}{4x}$	A1 **2**	For correct solution. Requires $\boxed{y =}$.
(iii) $(y \approx) -\frac{1}{2}\cos 2x$	B1√ **1**	For correct function **AEF** f.t. from **(ii)**
	9	
6 (i)		***Either coordinates or vectors may be used*** Methods 1 and 2 may be combined, for a maximum of 5 marks
METHOD 1		
State $B = (-1, -7, 2) + t(1, 2, -2)$	M1	For using vector normal to plane
On plane $\Rightarrow (-1+t) + 2(-7+2t) - 2(2-2t) = -1$	M1	For substituting parametric form into plane
	M1	For solving a linear equation in t
$\Rightarrow t = 2 \Rightarrow B = (1, -3, -2)$	A1	For correct coordinates
$AB = \sqrt{2^2 + 4^2 + 4^2}$ *OR* $2\sqrt{1^2 + 2^2 + 2^2} = 6$	A1 **5**	For correct length of AB
METHOD 2		
$AB = \left\lvert \frac{-1-14-4+1}{\sqrt{1^2 + 2^2 + 2^2}} \right\rvert = 6$	M1	For using a correct distance formula
	A1	For correct length of AB
OR $AB = \mathbf{AC} \,.\, \mathbf{AB} = \frac{[6, 7, 1] \,.\, [1, 2, -2]}{\sqrt{1^2 + 2^2 + 2^2}} = 6$		
$B = (-1, -7, 2) \pm 6\frac{(1, 2, -2)}{\sqrt{1^2 + 2^2 + 2^2}}$	M1	For using $B = A$ + length of AB × unit normal
$B = (-1, -7, 2) \pm (2, 4, -4)$	B1	For checking whether + or – is needed (substitute into plane equation)
$B = (1, -3, -2)$	A1	For correct coordinates (allow even if B0)
(ii) Find vector product of any two of $\pm[6, 7, 1]$, $\pm[6, -3, 0]$, $\pm(0, 10, 1)$	M1	For finding vector product of two relevant vectors
Obtain $k[1, 2, -20]$	A1	For correct vector **n**
$\theta = \cos^{-1} \frac{\lvert[1, 2, -2] \,.\, [1, 2, -20]\rvert}{\sqrt{1^2 + 2^2 + 2^2}\sqrt{1^2 + 2^2 + 20^2}}$	M1*	For using scalar product of two normal vectors
	M1 (dep*)	For stating both moduli in denominator
$\theta = \cos^{-1} \frac{45}{\sqrt{9}\sqrt{405}} = 41.8°\ (41.810...°, 0.72972...)$	A1√	For correct scalar product. f.t. from **n**
	A1 **6**	For correct angle
	11	

7 (i) (a) $\sin\frac{6}{8}\pi = \frac{1}{\sqrt{2}}$, $\sin\frac{2}{8}\pi = \frac{1}{\sqrt{2}}$	B1 **1**	For verifying $\theta = \frac{1}{8}\pi$
(b) 	M1	For sketching $y = \sin 6\theta$ and $y = \sin 2\theta$ for $0 \,,, \theta \,,, \frac{1}{2}\pi$ *OR* any other correct method for solving $\sin 6\theta = \sin 2\theta$ for $\theta \neq k\frac{\pi}{2}$ *OR* appropriate use of symmetry *OR* attempt to verify a reasonable guess for θ
$\theta = \frac{3}{8}\pi$	A1 **2**	For correct θ
(ii) $\text{Im}\,(c+\text{i}s)^6 = 6c^5s - 20c^3s^3 + 6cs^5$	M1 A1	For expanding $(c+\text{i}s)^6$; at least 3 terms and 3 binomial coefficients needed For 3 correct terms
$\sin 6\theta = \sin\theta\left(6c^5 - 20c^3(1-c^2) + 6c(1-c^2)^2\right)$	M1	For using $s^2 = 1-c^2$
$\sin 6\theta = \sin\theta\left(32c^5 - 32c^3 + 6c\right)$	A1	For any correct intermediate stage
$\sin 6\theta = 2\sin\theta\cos\theta\left(16c^4 - 16c^2 + 3\right)$	A1	For obtaining this expression correctly
$\sin 6\theta = \sin 2\theta\left(16\cos^4\theta - 16\cos^2\theta + 3\right)$	**5**	**AG**
(iii) $16c^4 - 16c^2 + 3 = 1$	M1	For stating this equation **AEF**
$\Rightarrow c^2 = \frac{2 \pm \sqrt{2}}{4}$	A1	For obtaining both values of c^2
$-$ sign requires larger $\theta = \frac{3}{8}\pi$	A1 **3**	For stating and justifying $\theta = \frac{3}{8}\pi$ Calculator OK if figures seen
	11	

8 (i) Group A: $e = 6$
Group B: $e = 1$
Group C: $e = 2^0$ *OR* 1
Group D: $e = 1$

B1
B1
2

For any two correct identities
For two other correct identities
AEF for D, but not "$m = n$"

(ii) *EITHER* *OR*

A	2	4	6	8
2	4	8	2	6
4	8	6	4	2
6	2	4	6	8
8	6	2	8	4

orders of elements
1, 2, 4, 4
OR cyclic group

B	1	5	7	11
1	1	5	7	11
5	5	1	11	7
7	7	11	1	5
11	11	7	5	1

orders of elements
1, 2, 2, 2
OR non-cyclic group
OR Klein group

C	2^0	2^1	2^2	2^3
2^0	2^0	2^1	2^2	2^3
2^1	2^1	2^2	2^3	2^0
2^2	2^2	2^3	2^0	2^1
2^3	2^3	2^0	2^1	2^2

orders of elements
1, 2, 4, 4
OR cyclic group

For showing group table
OR sufficient details of orders of elements
OR stating cyclic / non-cyclic / Klein group
(as appropriate)

B1* for one of groups A, B, C
B1* for another of groups A, B, C

$A \not\cong B$
B1 (dep*) For stating non-isomorphic

$B \not\cong C$
B1 (dep*) For stating non-isomorphic

$A \cong C$
B1 (dep*) For stating isomorphic

with sufficient detail relating to the first 2 marks

5

(iii) $\dfrac{1+2m}{1+2n} \times \dfrac{1+2p}{1+2q} = \dfrac{1+2m+2p+4mp}{1+2n+2q+4nq}$

$= \dfrac{1+2(m+p+2mp)}{1+2(n+q+2nq)} \equiv \dfrac{1+2r}{1+2s}$

M1* For considering product of 2 distinct elements of this form
M1 (dep*) For multiplying out
A1 For simplifying to form shown
A1 **4** For identifying as correct form, so closed

SR $\dfrac{\text{odd}}{\text{odd}} \times \dfrac{\text{odd}}{\text{odd}} = \dfrac{\text{odd}}{\text{odd}}$ earns full credit
SR If clearly attempting to prove commutativity, allow at most M1

(iv) Closure not satisfied
Identity and inverse not satisfied

B1 For stating closure
B1 **2** For stating identity and inverse
SR If associativity is stated as not satisfied, then award at most B1 B0 *OR* B0 B1

13

4728 Mechanics 1

1	70 x 9.8 or 70g 70 x 0.3 686 + 21 707 N	B1 B1 M1 A1 [4]	=686 =21 + cvs [70(9.8+0.3) gets B1B1M1]

2	+/-(40 x 4 - 60 x 3) +/-([40 + 60] v +/-(40 x 4 - 60 x 3) = +/-([40 + 60] v Speed = 0.2 ms^{-1} Same as heavier or opposite lighter/"she"	B1 B1 M1 A1 B1 [5]	Difference of terms, accept with g Sum of terms, accept with g. Accept inclusion of g in equation. Not if g used. ***SR*** 40x4-60x3=[40 + 60] v; v=0.2, as heavier, award 5 marks "Left" requires diagram for B1 If same direction before collision award B0B1M1A0B0

3i	 $\sqrt{(12^2 + 15^2)}$ 19.2 N tanθ =12/15, tanθ =15/12, sinθ =12/19.2,cosθ =15/19.2 Bearing = 038.7°	M1 A1 A1 M1 A1 A1 [6]	Applies Pythagoras, requires +. trig and R included between X and Y Accept cv 19.2 Accept 039 or 39 or art 39 from below (not given if X and Y transposed)
3ii	*E* = 19.2 Bearing = 180 + 38.7 = 219°	B1ft B1ft [2]	ft cv 19.2 180+cv 38.7(-360) or correct answer

4i	v = dx/dt $v = 4t^3 - 8 \times 2t$ $v(2) = 4x2^3 - 8x2x2$ = 0 AG $x(2) = 2^4 - 8 \times 2^2 + 16 = 0$ AG	M1 A1 M1 A1 B1 [5]	Uses differentiation, may be seen in (ii) Accept with +c Substitutes 2 in cv v, explicit A0 if +c Substitutes 2 in displacement, explicit
4ii	a = dv/dt $a = 12t^2 - 16$ $a(2) = 12 \times 2^2 - 16 = 32\ ms^{-2}$	M1 A1 A1 [3]	Uses differentiation of v formula Accept with +c A0 with +c

5ia	250a = -150 $a = -0.6\ ms^{-2}$ AG	M1 A1 [2]	Values used in N2L for trailer F=+/-150 Or -ve convincingly argued
5ib	 900 x -0.6 = D -600 or (900+250)x-0.6 = D -600 -150 D = 60 N	M1 A1 A1 [3]	Applies N2L to car or car/trailer with correct number of forces (including T if T=0 used later)
5ic	$15^2 = 18^2 + 2x(-0.6)s$ s = 82.5 m	M1 A1 [2]	Uses $v^2 = u^2 + 2(+/-0.6)s$ with 15, 18 Positive, allow from $18^2 = 15^2 + 2x0.6s$
5iia	 (900+250)a = 980 - 600 -150	M1 A1	Applies N2L to car+trailer with F(driving) F(resisting), F(wt cmpt-allow without g), or each part, as above and T. 900a = 980 - 600 +/- 900x9.8sin3 - T
5iib	+ /-(900+250)x9.8sin3 $a = 0.713\ ms^{-2}$	A1 A1 [4]	250a = T - 150 +/- 250x9.8sin3 Allow (art) 0.71 from correct work
	  250 x 0.713 = T - 150 + 250x9.8sin3 T = 200 N	M1 A1 A1 [3]	N2L for trailer, cv a, with correct number of forces of correct type. Or for car 900x0.713 = -T-600 + 900x9.8sin3 + 980 Anything rounding to 200 (3sf)

6i	$4.9 = \mu \times 14.7$	M1	Uses $F = \mu R$
	$\mu = 1/3$ AG	A1	Allow 0.333 or 0.3 recurring
		[2]	
6iia		M1	3 force vertical equation
	$R + 4.9\sin30 = 14.7$	A1	
	$R = 12.25$ N	A1	Accept 12.2 or 12.3
	$F = 12.25 \times 1/3$	M1	Uses $F = \mu R$ with new R {may be seen in
	$F = 4.08(333..)$ N [or 49/12 N]	A1	{part b
		[5]	
6iib	$m = 14.7/9.8 = 1.5$kg	B1	
		M1	N2L horizontally with 2 relevant forces, including 4.9sin/cos30
	$4.9\cos30 - 4.08(333..) = 1.5a$	A1	Allow cv(F) ***SR*** Award A1 if m=14.7 used
	$a = 0.107$ ms^{-2}	A2	***SR*** A1 for 0.11, 0.109
6iii		[5]	or art 0.011 from m = 14.7
	$\mu R = (14.7 - 4.9\cos30)/3$	B1	3.49, accept 3.5
	Horizontal component of force = $4.9\sin30$	B1	2.45, accept 2.4 or 2.5
	Horizontal component of force < ③R	M1	Comparing two values
	Friction = 2.45 N	A1	Not 2.4 or 2.5; Explicit (M1 essential)
		[4]	

7i	$s = 0.5 \times 1.4 \times 0.8^2$	M1	Uses $s = 0.5x1.4t^2$
	$s = 0.448$ m	A1	Not 0.45
	$v = 1.4 \times 0.8$	M1	Uses $v = 1.4t$
	$v = 1.12$ ms^{-1}	A1	
		[4]	
7ii	$0^2 = 1.12^2 - 2 \times 9.8s$	M1	Uses $0^2 = u^2 - 2gs$ or $u^2 = 2gs$
	$s = 0.064$ m	A1	Allow verification
	$0 = 1.12 - 9.8t$ ($t = 0.114$s)	M1	or $0.064 = 1.12t - 4.9t^2$
	$t = (0.114 + 0.8) = 0.914$s	A1	Allow 0.91 {or $0 = 1.12t - 4.9t^2$ and halve t
		[4]	
7iii	Scalene triangle, base on t axis	B1	NB Award A1 for 0.91 on t axis if total
	right edge steeper and terminates on axis, or crosses	B1	time not given in (ii)
	axis at t = 0.91	[2]	
7iv		M1	Uses N2L for A or B with attempt at 2 forces
		A1	Either
	$1.4xA = 9.8xA - 5.88$ or $1.4xB = 5.88 - 9.8xB$	A1	
	$A = 0.7$	A1	Not 0.53
7va	$B = 0.525$	[4]	
		M1	Uses tension and 0.5g without particle weights
	$T = 0.5 \times 9.8 + 2 \times 5.88$		
7vb	$T = 16.66$ N	A1	Allow 16.7
		[2]	
	$T = 4.9$ N	B1	
		[1]	

4729 Mechanics 2

1 (i)	12 x cos55° 6.88 m s^{-1}	M1 A1 **2**	
(ii)	12 x cos55° x 0.65 (±) 4.47 m s^{-1} ✓	M1 A1 **2**	✓ 0.65 x their (i) **4**

2	F = 0.2 mg cos30° 0.2mgcos30° x d mg x d x sin30° d=½x25/(0.2x9.8cos30°+9.8xsin30°) 1.89 m	M1 A1 B1 B1 M1 A1 **6**	= = (1.6974m) (49√3/50m) a=0.2gcos30°+gsin30° a= (±) 6.60 $0 = 5^2 - 2x6.60d$ **6**

3	direction of R perp. to wall R at 70° to rod 0.8 x 25cos60° = 1.6 x R sin70° 0.8 x 25 cos60° 1.6 x R sin70° R = 6.65 N	B1 B1 M1 A1 A1 A1 **6**	 10° to horiz. moments about A **6**

4 (i)	45 000/v = kv k = 50	M1 A1 **2**	 **AG**
(ii)	45 000/20 – 50x20 = 1200a a = 1.04 m s^{-2}	M1 A1 A1 **3**	
(iii)	P/15 = 50x15 + 1200x9.8sin10° 41 900 W	M1 A1 A1 **3**	 **8**

5 (i)	2mu – 3kmu = –mu + kmv v = ….. v = 3u(1 – k)/k (0 <) k < 1	M1 M1 A1 A1 **4**	 attempting to make v the subject 3u/k – 3u not ≤ 1
(ii)	I = mu – – 2mu 3mu	M1 A1 **2**	or km(3u/k - 3u + 3u) + only
(iii)	v = ± 3u e = (u/2 + 3u)/4u e = 7/8 or 0.875	B1 M1 A1 **3**	 **9**

6 (i)(a)	T cos 45° = 2.94 T = 4.16 N	M1 A1 **2**	Resolving vertically **AG**
(b)	Tcos45° + T = 0.3x1.96ω^2 (res. horiz.) ω = 3.47 rad s^{-1}	M1 A1 A1 **3**	calculates v = 6.81 (Max 2/3)
(ii)(a)	Tcos30° + Tcos60° = 2.94 T = 2.15 N	M1 A1 A1 **3**	Resolving vertically
(b)	Tcos30° + Tcos60° =0.3v^2/1.5 (res. horiz.) v = 3.83 m s^{-1}	M1 A1 A1 **3**	calculates ω = 2.56 (Max 2/3) **11**

7 (i)	0 = $(175\sin\theta)^2$ – 2x9.8x650 θ = 40.2°	M1 A1 A1 **3**	
(ii)	Attempt at t_1 , t_2 , t_{top} or t_{total} 5.61, 23.65, 14.63, 29.26 $t_2 - t_1$ or $2(t_{top} - t_1)$ or $t_{total} - 2t_1$ time difference = 18.0	M1 A1 M1 A1 A1 **5**	650 = 175sin55°.t - 4.9t^2 etc
(iii)	v_h = 175cos55° (100.4) v_v = 175sin55° – 9.8 x 5.61 speed = $\sqrt{(88.4^2 + 100.4^2)}$ 134 m s^{-1}	B1 M1 M1 A1 **4**	or KE ½mv^2 (B1) PE mx9.8x650 v = $\sqrt{(175^2 - 2x9.8x650)}$ **12**

8 (i)	(2x4xsinΠ/2)/3xΠ/2 1.70	M1 A1 **2**	or 4r/3Π **AG**
(ii)(a)	$\bar{x}$ xd(8x20–Πx4^2/2)=10x8x20d– 12xΠx4^2/2xd 10x8x20(d) (1600) (8x20–Πx4^2/2) (d) (134.9) (12xΠx4^2/2) (d) (301.6) $\bar{x}$ = 9.63 cm	M1 A1 A1 A1 A1 **5**	or 134.9$\bar{x}$ = 64x4+38.9x12+32x18 (1298.8) 64x4 38.9x12 32x18 **AG**
(ii)(b)	$\bar{y}$ xd(8x20–Πx4^2/2)=4x8x20d– 1.7xΠx4^2/2xd 4x8x20 (d) 1.7d x Π x 4^2 /2 (13.6Π) $\bar{y}$ = 4.43 cm	M1 A1 A1M1 A1 **4**	or 64x4=42.7+38.9$\bar{y}$ $\bar{y}$ = 5.49 135$\bar{y}$ =32x4+38.9x5.49+64x4
(iii)	20cos10° x T 15cos10° x 9.63 15sin10° x 4.43 20cos10°.T=15cos10°x9.63– 15sin10°x4.43 (needs 3 parts) T = 6.64 N	B1 B1 B1 M1 A1 **5**	= or 10.6 (A to com) 34.7° ∠ comAH =15x10.6xcos34.7° **16**

4730 Mechanics 3

1	(i) $[0.5(v_x - 5) = -3.5, 0.5(v_y - 0) = 2.4]$	M1		For using I = m(v – u) in x or y direction
	Component of velocity in x-direction is $-2ms^{-1}$	A1		
	Component of velocity in y-direction is $4.8ms^{-1}$	A1		
	Speed is $5.2ms^{-1}$	A1	4	AG
SR For candidates who obtain the speed without finding the required components of velocity (max 2/4)				
	Components of momentum after impact are -1 and 2.4 Ns	B1		
	Hence magnitude of momentum is 2.6 Ns and required speed is $2.6/0.5 = 5.2ms^{-1}$	B1		
	(ii)	M1		For using $I_y = m(0 - v_y)$ or I_y = -y-component of 1st impulse
	Component is –2.4Ns	A1	2	

2	(i)	M1		For 2 term equation, each term representing a relevant moment
	$50x1\sin\beta = 75x2\cos\beta$	A1		
	$\tan\beta = 3$	A1	3	AG
	(ii) Horizontal force is 75N	B1		
	Vertical force is 50N	B1	2	
	(iii)	M1		For taking moments about A for the whole or for AB only
	For not more than one error in $Wx1\sin\alpha + 50(2\sin\alpha + 1\sin\beta) = 75(2\cos\alpha + 2\cos\beta)$ or $Wx1\sin\alpha + 50x2\sin\alpha = 75x2\cos\alpha$	A1		Where $\tan\alpha = 0.75$
	$0.6W + 107.4... = 167.4\ldots$ or $0.6W + 60 = 120$	A1		
	$W = 100$	A1	4	

3	(i)	M1		For using the principle of conservation of momentum in the **i** direction
	$6x4 - 3x8 = 6a + 3b$ $(0 = 2a + b)$	A1		
		M1		For using NEL
	$(4 + 8)e = b - a$ $(12e = b - a)$	A1		
	Component is $4e\ ms^{-1}$ to the left	A1	5	'to the left' may be implied by a = -4e and arrow in diagram
	(ii) $b = 8e\ ms^{-1}$	B1ft		ft b = -2a or b = a + 12e
		M1		For using '**j** component of A's velocity remains unchanged'
	$(8e)^2 = (4e)^2 + v^2$	A1ft		ft $b^2 = a^2 + v^2$
	$v = 4$	A1	4	

4	(i) $[mg - 0.49mv = ma]$	M1		For using Newton's second law
	$mv\frac{dv}{dx} = mg - 0.49mv$	A1		
	$\left[\frac{v(dv/dx)}{g - 0.49v} = 1\right]$	M1		For relevant manipulation
	$\left[\frac{v}{9.8 - 0.49v} \equiv \frac{-1}{0.49}\left(\frac{(9.8 - 0.49v) - 9.8}{9.8 - 0.49v}\right)\right]$	M1		For synthetic division of v by g - 0.49v, or equivalent
	$\left(\frac{20}{20 - v} - 1\right)\frac{dv}{dx} = 0.49$	A1	5	AG
	(ii)	M1		For separating the variables and integrating
	$\int \frac{20}{20 - v}dv = -20\ln(20 - v)$	B1		
	$-20\ln(20 - v) - v = 0.49x$ (+C)	A1ft		
	$[-20\ln 20 = C]$	M1		For using v = 0 when x = 0
	$x = 40.8(\ln 20 - \ln(20 - v)) - 2.04v$	A1	5	Accept any correct form

5	(i)	M1		For using Newton's second law with a = 0
	$mgsin30^o = 0.75mgx/1.2$	A1		
	Extension is 0.8m	A1	3	AG
	(ii) PE loss = $mg(1.2 + 0.8)\sin 30^o$ (mg)	B1		
	EE gain = $0.75mg(0.8)^2/(2x1.2)$ (0.2mg)	B1		
	[½ $mv^2 = mg - 0.2mg$]	M1		For an equation with terms representing PE, KE and EE in linear combination
	Maximum speed is $3.96ms^{-1}$	A1	4	
	(iii) PE loss = $mg(1.2 + x)sin30^o$ or $mgdsin30^o$	B1ft		ft with x or d – 1.2 replacing 0.8 in (ii)
	EE gain = $0.75mgx^2/(2x1.2)$ or $0.75mg(d - 1.2)^2/(2x1.2)$	B1ft		ft with x or d – 1.2 replacing 0.8 in (ii)
	[$x^2 - 1.6x - 1.92 = 0$, $d^2 - 4d + 1.44 = 0$]	M1		For using PE loss = EE gain to obtain a 3 term quadratic in x or d
	Displacement is 3.6m	A1	4	
Alternative for parts (ii) and (iii) for candidates who use Newton's second law and a = v dv/dx: In the following x, y and z represent displacement from equil. pos^n, extension, and distance OP respectively.				
	[$mv\,dv/dx = mgsin30^o - 0.75mg(0.8 + x)/1.2$, $mv\,dv/dy = mgsin30^o - 0.75mgy/1.2$, $mv\,dv/dz = mgsin30^o - 0.75mg(z - 1.2)/1.2$]	M1		For using N2 with a = v dv/dx
	$v^2/2 = -5gx^2/16 + C$ or $v^2/2 = gy/2 - 5gy^2/16 + C$ or $v^2/2 = 5gz/4 - 5gz^2/16 + C$	A1		
	[$C = 0.6g + 5g(-0.8)^2/16$ or $C = 0.6g$ or $C = 0.6g - 5g(1.2/4) + 5g(1.2)^2/16$	M1		For using $v^2(-0.8)$ or $v^2(0)$ or $v^2(1.2)$ = $2(g\sin30^o)1.2$ as appropriate
	$v^2 = (-5x^2/8 + 1.6)g$ or $v^2 = (y - 5y^2/8 + 1.2)g$ or $v^2 = (5z/2 - 5z^2/8 - 0.9)g$	A1		
	(ii) [$v_{max}^2 = 1.6g$ or $0.8g - 0.4g + 1.2g$ or $5g - 2.5g - 0.9g$]	M1		For using $v_{max}^2 = v^2(0)$ or $v^2(0.8)$ or $v^2(2)$ as appropriate
	Maximum speed is $3.96ms^{-1}$	A1		
	(iii) [$5x^2 - 12.8 = 0 \rightarrow x = 1.6$, $5y^2 - 8y - 9.6 = 0 \rightarrow y = 2.4$, $5z^2 - 20z + 7.2 = 0 \rightarrow z = 3.6$]	M1		For solving v = 0
	Displacement is 3.6m	A1	8	
Alternative for parts (ii) and (iii) for candidates who use Newton's second law and SHM analysis.				
	[$m\ddot{x} = mgsin30^o - 0.75mg(0.8 + x)/1.2 \rightarrow \ddot{x} = -\omega^2 x$; $v^2 = \omega^2(a^2 - x^2)$]	M1		For using N2 with $v^2 = \omega^2(a^2 - x^2)$
	$v^2 = 5g(a^2 - x^2)/8$	A1		
		M1		For using $v^2(-0.8) = 2(gsin30^o)1.2$
	$v^2 = 5g(2.56 - x^2)/8$	A1		
	(ii) [$v_{max}^2 = 5g \times 2.56 \div 8$]	M1		For using $v_{max}^2 = v^2(0)$
	Maximum speed is $3.96ms^{-1}$	A1		
	(iii) [$2.56 - x^2 = 0 \rightarrow x = 1.6$]	M1		For solving v = 0
	Displacement is 3.6m	A1		

6	(i) [½m7^2 = ½mv^2 + 2mg]	M1		For using the principle of conservation of energy
	Speed is 3.13ms^{-1}	A1		
	[T = mv^2/r]	M1		For using Newton's second law horizontally and a = v^2/r
	Tension is 1.96N	A1ft	4	
	(ii) [T – mgcosθ = mv^2/r]	M1		For using Newton's second law radially
		M1		For using T = 0 (may be implied)
	v^2 = -2gcosθ	A1		
		M1		For using the principle of conservation of energy
	½m7^2 = ½mv^2 +mg(2 – 2cosθ)	A1		
	[-2gcosθ = 49 –4g + 4gcosθ]	M1		For eliminating v^2
	6gcosθ = -9.8	A1		May be implied by answer
	θ = 99.6	A1	8	
Alternative for candidates who eliminate v^2 before using T = 0.				
	(ii) [T – mgcosθ = mv^2/r]	M1		For using Newton's second law radially
		M1		For using the principle of conservation of energy
	½m7^2 = ½mv^2 +mg(2 – 2cosθ)	A1		
	[T - mgcosθ = m(49 –4g + 4gcosθ)2]	M1		For eliminating v^2
		M1		For using T = 0 (may be implied)
	-2g cosθ = 49 –4g + 4gcosθ	A1ft		ft error in energy equation
	6gcosθ = -9.8	A1		May be implied by answer
	θ = 99.6	A1	8	

7	(i) T = 4mg(4 + x – 3.2)/3.2	B1		
	[ma = mg – 4mg(0.8 + x)/3.2]	M1		For using Newton's second law
	4$\ddot{x}$ = -49x	A1	3	AG
	(ii) Amplitude is 0.8m	B1		(from 4 + A = 4.8)
	Period is $2\pi/\omega$ s where ω^2 = 49/4	B1		
		M1		String is instantaneously slack when shortest (4 - A = 3.2 = L). Thus required interval length = period.
	Slack at intervals of 1.8s	A1	4	AG
	(iii) [ma = -mgsinθ]	M1		For using Newton's second law tangentially
	mL$\ddot{\theta}$ = -mgsinθ	A1		
	For using sin$\theta \approx \theta$ for small angles and obtaining $\ddot{\theta} \approx$ –(g/L)θ	A1	3	AG
	(iv) [θ = 0.08cos(3.5x0.25)] (= 0.05127..)	M1		For using = $_o$cosωt where ω^2=12.25 (may be implied by $\dot{\vartheta}$ = -ω $_o$sinωt)
	[$\dot{\theta}$ = -3.5(0.08)sin(3.5x0.25), $\dot{\theta}^2$ = 12.25(0.08^2 – 0.05127..^{2})]	M1		For differentiating = $_o$cosωt and using $\dot{\vartheta}$ or for using $\dot{\theta}^2 = \omega^2(\theta_o{}^2 - \theta^2)$ where ω^2=12.25
	$\dot{\theta} = \mp 0.215$	A1		May be implied by final answer
	[v = 0.215x9.8/12.25]	M1		For using v = L $\dot{\vartheta}$ and L =g/ω^2
	Speed is 0.172 ms^{-1}	A1	5	

4732 Probability & Statistics 1

Note: "(3 sfs)" means "answer which rounds to ... to 3 sfs". If correct ans seen to ≥ 3sfs, ISW for later rounding
Penalise over-rounding only once in paper.

1ia	5! or ${}^5P_5$ = 120	M1 A1 2	
b	4! or ${}^4P_4$ seen 4! × 2 48	M1 M1dep A1 3	or 2 × 3! or 2! × 3! or 2! × ${}^3P_3$ 2 × 3! × 4
ii	${}^1/_5C_2$ or ${}^1/_5 \times {}^1/_4 \times 2$ or 0.4×0.25 or ${}^2/_{5P2}$ = ${}^1/_{10}$	M1 A1 2	Allow M1 for ${}^5C_2$ or ${}^1/_5$ x ¼ or ${}^1/_{20}$ or ${}^1/_5 \times {}^1/_5 \times 2$ or ${}^2/_{25}$ oe
Total		**7**	
2i	$({}^4/_5)^3 \times ({}^1/_5)$ oe = ${}^{64}/_{625}$ or 0.102 (3 sfs)	M1 A1 2	Allow M1 for $({}^4/_5)^4$ x $({}^1/_5)$
ii	$({}^4/_5)^4$ alone or $1-({}^1/_5 + {}^4/_5 x\ {}^1/_5 + ({}^4/_5)^2 x\ {}^1/_5 + ({}^4/_5)^3 x\ {}^1/_5)$ = ${}^{256}/_{625}$ or 0.410 (3 sfs)	 M1 A1 2	Allow $({}^4/_5)^3$ or $({}^4/_5)^5$; not 1 - $({}^4/_5)^4$ Allow one term omitted or wrong or "correct" extra Allow 0.41
iii	5	B1 1	
Total		**5**	
3i	$r = \dfrac{212 - \dfrac{24 \times 39}{5}}{\sqrt{(130 - \dfrac{24^2}{5})(361 - \dfrac{39^2}{5})}}$	B2 2	$\frac{24.8}{\sqrt{14.8 \times 56.8}}$ or $\frac{24.8}{\sqrt{840.64}}$ or $\frac{24.8}{3.85 \times 7.54}$ or $\frac{24.8}{29}$ B2 for correct subst in r B1 for correct subst in any S
ii	$R = 0.7$ or (B) Definition of r_s is PMCC for ranks	B1 B1 2	(A) and (B) true: B0B0 dep 1st B1
iii	$r = 0.855$ $r_s = 0.7$	B1 B1 2	or "unchanged": B1B1 Interchanged: B1
Total		**6**	
4i	0.4 x p = 0.12 or ${}^{0.12}/_{0.4}$ or ${}^{12}/_{40}$ oe p = 0.3 oe	M1 A1 2	
ii	0.4 x (1 – their 0.3) oe eg ${}^{40}/_{100} \times {}^{28}/_{40}$ 0.28 or 28% oe	M1 A1ft 2	or 0.4 – 0.12 or 0.28 or 28 seen Not 0.4×0.88 unless ans to (i) is 0.12
Total		**4**	
5ia	Binomial stated or implied 0.9806	B1 B1 2	by use of tables or 0.2^a x 0.8^b, $a+b = 12$
b	0.5583 seen 1 – 0.5583 = 0.442 (3 sfs)	M1 M1 A1 3	add 10 corr terms or 1-(add 3 corr terms): M2 or 1– 0.7946 or 0.205 or 1-0.6774 or 0.323 or 1-0.3907 or 0.609 or add 9 terms or 1-(add 2 or 4 terms): M1
ii	${}^{15}C_4$ x 0.3^4 x 0.7^{11} = 0.219 (3 sfs)	M2 A1 3	${}^{15}C_4$ x 0.3^{11} x 0.7^4 : M1
Total		**8**	

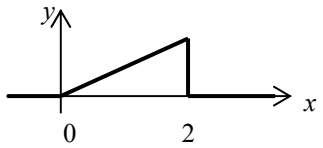
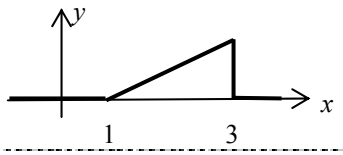
6i	Σyp $= 2.3$ $\Sigma y^2 p$ (= 5.9) $- (\Sigma yp)^2$ $= 0.61$ oe	M1 A1 M1 M1 A1 5	$\geq$ 2 terms added ÷ 3 or ÷ 6 etc: M0 $\geq$ 2 terms added ÷ 3 or ÷ 6 etc: M0 dep +ve result $(-1.3)^2\times0.2+(-0.3)^2\times0.3+0.7^2\times0.5$: M2 one term correct: M1 Use of Z: MR, lose last A1 (2.55, 0.4475)
ii	0.2x0.25 + 0.3x0.1 or 0.05 + 0.03 alone $= 0.08$ oe	M2 A1 3	M1 for one product eg correct×2: M1 or clearly ident (1,2), (2,1): M1
iii	$0.3\times0.1 + 0.3\times0.25 + 0.3\times0.65$ $+ 0.25\times0.2 + 0.25\times0.5$ alone or 0.03 + 0.075 + 0.195 + 0.05 + 0.125 $= 0.475$ or $^{19}/_{40}$ oe	 M2 A1 3	 M1 : any 3, 4 of these prods alone or these 5 prods plus 1 extra or repeat or (ii) + prod or 0.3 + prod or 0.25 + prod or clearly identify (1,2) (3,2) (2,2) (2,1) (2,3) M2 for 0.3 + (0.2 + 0.5) × 0.25 or 0.25 + (0.1 + 0.65) × 0.3 or 0.3 + 0.25 – 0.3 × 0.25 or 1 – (0.2+ 0.5)(0.1+0.65) M1 for (0.2+ 0.5)(0.1+0.65)
Total		**11**	
7ia	Results or matches are indep Prob of winning is constant	B1 B1 2	allow "wins" indep; not "trials" indep not "success"
ib	No of wins (or losses)	B1 1	
ii	$^{21}C_{10} p^{10} q^{11} = {}^{21}C_9 p^9 q^{12}$ $\frac{12}{10} p = q$ or $\frac{12}{10} p(1-p)^{-1} = 1$ or similar $1.2p = 1 - p$ oe eg $p = 0.833(1-p)$ or $352716p = 293930(1-p)$ $p = {}^5/_{11}$ or 0.455 (3 sfs) oe	 M1 M1M1 M1 A1 5	or $(1 - p)$ for q & allow omit bracket or $352716\,p^{10}q^{11} = 293930\,p^9q^{12}$ M1 for $^{12}/_{10}$ or $^6/_5$ or 1.2 or $^5/_6$ or 0.833 M1 for p & q cancelled correctly or equiv equn in p or q (cancelled) nos not nec'y cancelled; not alg denom
Total		**8**	

8i	m = 26.5		B1	
	LQ = 22	or 21.5	or 21.75	
	UQ = 39	40	39.5	M1 M1 for either LQ or UQ
	IQR = 17	18.5	17.75	A1 3 A1 must be consistent LQ, UQ & IQR
ii	Ave or overall or med or "it" similar		B1f	or F med (or ave) higher or F mean less or M & F both have most in 20s
	Male spread greater or M more varied oe		B1f 2	or male range greater or more younger F or more older M
iii	Med less (or not) affected by extreme(s) or Mean (more) affected by extreme(s)		B1 1	oe; not "anomalies" ignore eg "less accurate"
iv				must consistently decode last or first
	Decode last			
	245/49		M1	
	= 5		A1	
	mean = 205		B1f	200 + "5"
	$\sqrt{(9849/49 - (^{245}/_{49})^2)}$		M1	dep √+ve
	= 13.3 (3sfs) or 4√11		A1	
	sd = 13.3 or 4√11		B1f 6	dep M1 or ans 176; award if not +200
	Decode first			
	245 + 200×49 or 10045	B1		
	$^{10045}/_{49}$	M1		allow $^{445}/_{49}$ or 9.08 seen
	= 205	A1		
	$\Sigma x^2 = 9849+400\times10045-49\times40000$ or 2067849	B1		
	$\sqrt{\frac{"\Sigma x^2"}{49} - "\bar{x}^2"}$	M1		dep √+ve
				Σx^2 must be: attempt at Σx^2 >9849 not involve 9849^2 not $(\Sigma x)^2$ eg10045^2, 445^2
				$\bar{x}$ must be decoded attempt, eg 9.08
	= 13.3 or 4√11	A1		
Total			**12**	
9i	Because growth may depend on pH oe or expt is investigating if y depends on x		B1 1	In context. Not x is controlled or indep
ii	S_{xy} = 17082.5 – 66.5 x 1935/8 (= 997.8125) S_{xx} = 558.75 – 66.5^2/8 (= 5.96875)			
	$b = S_{xy}/S_{xx}$		M1	Correct sub into any correct b formula
	= 167 (3 sfs)		A1	
	y – 1935/8 = "167"(x – 66.5/8)		M1	or a =1935/8 – "167" x 66.5/8
	y = -1150 + 167x		A1 4	cao NB 3 sfs
iii	y = -1150 + 167 x 7		M1	ft their eqn for M1 only
	= 19 to 23		A1 2	
iv	No (or little) relationship or correlation		B1 1	or weak or small corr'n. Not "agreement"
va	Reliable as r high oe		B1 1	Allow without "interpolation" oe, but must include r high
b	Unreliable as extrapolation oe		B1 1	or unreliable as gives a neg value
vi	Unreliable (or No) because r near 0 or because little (or no or small) corr'n (or rel'n)		B1 1	or No because Q values vary widely for pH = 8.5
Total			**11**	

Total 72 marks

4733 Probability & Statistics 2

1		$\frac{80-\mu}{\sigma} = \Phi^{-1}(0.95) = 1.645$ $\frac{\mu-50}{\sigma} = \Phi^{-1}(0.75) = 0.674(5)$ Solve simultaneously $\mu = 58.7$, $\sigma = 12.9$	M1 B1 A1 M1 A1 A1	**6**	Standardise once with Φ^{-1}, allow σ^2, cc Both 1.645 (1.64, 1.65) and [0.674, 0.675], ignore signs Both equations correct apart from wrong z, *not* 1–1.645 Solve two standardised equations μ, a.r.t 58.7 σ, a.r.t. 12.9 [*not* σ^2] [σ^2: M1B1A0M1A1A0]
2	(i)	Let R denote the number of choices which are 500 or less. $R \sim B(12, \frac{5}{6})$ $P(R = 12) = (\frac{5}{6})^{12}$ [=0.11216] = **0.112**	M1 M1 A1	**3**	B(12, $\frac{5}{6}$) stated or implied, allow 501/600 etc p^{12} or q^{12} or equivalent Answer, a.r.t. 0.112 [SR: $\frac{500}{600} \times \frac{499}{599} \times \frac{498}{598} \times \ldots$; 0.110: M1A1] [M1 for 0.910 or 0.1321 or vague number of terms]
	(ii)	Method unbiased; unrepresentative by chance	B1 B1	**2**	State that method is unbiased Appropriate comment (e.g. "not unlikely") [SR: partial answer, e.g. not necessarily biased: B1]
3	(i)	$P(\leq 1) = 0.0611$ $P(\geq 9) = 1 - P(\leq 8) = 1 - 0.9597$ $= 0.0403$ $0.0611 + 0.0403$ [= 0.1014] = **10.1%**	B1 M1 A1 M1 A1	**5**	0.0611 seen Find $P(\geq 9)$, allow 8 or 10 [0.0866, 0.0171] 0.0403 correct Add probabilities of tails, *or* 1 tail × 2 Answer [10.1, 10.2]% or probability
	(ii)	$P(2 \leq G \leq 8)$ $= 0.8944 - 0.0266$ [= 0.8678] = **0.868**	M1 M1 A1	**3**	Attempt at $P(2 \leq G \leq 8)$, *not* isw, allow $1 \leq G \leq 9$ etc Po(5.5) tables, P(≤ top end) – P(≤ bottom end) Answer, a.r.t. 0.868, allow %
4	(i)	$\hat{\mu} = \bar{y} = \frac{3296.0}{40} = 82.4$ $\frac{286800.4}{40} - 82.4^2$ [= 380.25] $S^2 \times \frac{40}{39}$; = 390	B1 M1 M1 A1	**4**	Mean 82.4, c.a.o. Use correct formula for biased estimate Multiply by $n/(n-1)$ [SR: all in one, M2 or M0] Variance 390, c.a.o.
	(ii)	$\Phi\left(\frac{60-82.4}{\sqrt{390}}\right) = \Phi(-1.134)$ = 1 – 0.8716 = **0.128**	M1 A1	**2**	Standardise, allow 390, cc or biased estimate, +/–, do not allow $\sqrt{n}$ Answer in range [0.128, 0.129]
	(iii)	No, distribution irrelevant	B1	**1**	"No" stated or implied, any valid comment
5	(i)	$H_0 : \mu = 500$ where μ denotes $H_1 : \mu < 500$ the population mean α: $z = \frac{435-500}{100/\sqrt{4}} = -1.3$ Compare –1.282	B2 M1 A1 B1		Both hypotheses stated correctly [SR: 1 error, B1, but $\bar{x}$ etc: B0] Standardise, use $\sqrt{4}$, can be + $z = -1.3$ (allow –1.29 from cc) *or* $\Phi(z) = 0.0968$ (.0985) Compare z & –1.282 *or* p (< 0.5) & 0.1 or equivalent
		β: $500 - 1.282 \times 100/\sqrt{4}$ = 435.9; compare 435	M1 A1√;B1		$500 - z \times 100/\sqrt{4}$, allow √ errors, any Φ^{-1}, must be – CV correct, √ on their z; 1.282 correct and compare
		Reject H_0 Significant evidence that number of visitors has decreased	M1√ A1√	**7**	Correct deduction, needs $\sqrt{4}$, $\mu = 500$, like-with-like Correct conclusion interpreted in context
	(ii)	CLT doesn't apply as n is small So need to know distribution	M1 B1	**2**	Correct reason ["n is small" is sufficient] Refer to distribution, e.g. "if not normal, can't do it"

6	(i)	(a) $1 - 0.8153$ $= 0.1847$ (b) $0.8153 - 0.6472$ $= \mathbf{0.168}$	M1 A1 **2** M1 A1 **2**		Po(3) tables, "1 –" used, e.g. 0.3528 or 0.0839 Answer 0.1847 or 0.185 Subtract 2 tabular values, or formula [$e^{-3}\ 3^4/4!$] Answer, a.r.t. 0.168
	(ii)	N(150, 150) $1 - \Phi\left(\frac{165.5 - 150}{\sqrt{150}}\right)$ $= 1 - \Phi(1.266) = \mathbf{0.103}$	B1 B1 M1 A1 A1 **5**		Normal, mean 3×50 stated or implied Variance or SD = 3 × 50, or same as μ Standardise 165 with λ, √λ or λ, any or no cc √λ and 165.5 Answer in range [0.102, 0.103]
	(iii)	(a) The sale of one house does not affect the sale of any others (b) The average number of houses sold in a given time interval is constant	B1 B1 **2**		Relevant answer that shows evidence of correct understanding [but *not* just examples] Different reason, in context [Allow "constant rate" or "uniform" but not "number constant", "random", "singly", "events".]
7	(i)	$\int_0^2 kx\,dx = \left[\frac{kx^2}{2}\right]_0^2 = 2k$ $= 1$ so $k = ½$	M1 A1 **2**		Use $\int_0^2 kx\,dx = 1$, or area of triangle Correctly obtain $k = ½$ **AG**
	(ii)		B1 B1 **2**		Straight line, positive gradient, through origin Correct, some evidence of truncation, no need for vertical
	(iii)	$\int_0^2 \frac{1}{2}x^2\,dx = \left[\frac{1}{6}x^3\right]_0^2 = \frac{4}{3}$ $\int_0^2 \frac{1}{2}x^3\,dx = \left[\frac{1}{8}x^4\right]_0^2$ [= 2] $2 - \left(\frac{4}{3}\right)^2 = \frac{2}{9}$	M1 A1 M1 M1 A1 **5**		Use $\int_0^2 kx^2\,dx$; $\frac{4}{3}$ seen or implied Use $\int_0^2 kx^3\,dx$; subtract their mean^2 Answer $\frac{2}{9}$ or a.r.t. 0.222, c.a.o.
	(iv)		M1 A1√ **2**		Translate horizontally, allow stated, or "1, 2" on axis One unit to right, 1 and 3 indicated, nothing wrong seen, no need for vertical or emphasised zero bits [If in doubt as to → or ↓, M0 in this part]
	(v)	$\frac{7}{3}$ $\frac{2}{9}$	B1√ B1√ **2**		Previous mean + 1 Previous variance [If in doubt as to → or ↓, B1B1 in this part]

8	(i)	$H_0: p = 0.65$ OR $p \geq 0.65$ $H_1: p < 0.65$ B(12, 0.65)	B2 M1	Both hypotheses correctly stated, in this form [One error (but not r, x or $\bar{x}$): B1] B(12, 0.65) stated or implied
		α: P(≤ 6) = 0.2127 Compare 0.10	A1 B1	Correct probability from tables, *not* P(= 6) Explicit comparison with 0.10
		β: Critical region ≤ 5; 6 > 5 Probability 0.0846	B1 A1	Critical region ≤5 or ≤6 or {≤4} ∩ {≥11} & compare 6 Correct probability
		Do not reject H_0 Insufficient evidence that proportion of population in favour is not at least 65%	M1√ A1√ **7**	Correct comparison and conclusion, needs correct distribution, correct tail, like-with-like Interpret in context, e.g. "consistent with claim" [SR: N(7.8, 2.73): can get B2M1A0B1M0: 4 ex 7]
	(ii)	Insufficient evidence to reject claim; test and p/q symmetric	B1√ B1 **2**	Same conclusion as for part (i), don't need context Valid relevant reason, e.g. "same as (i)"
	(iii)	$R \sim B(2n, 0.65)$, $P(R \leq n) > 0.15$ B(18, 0.65), $p = 0.1391$ Therefore $n = 9$	M1 A1 A1 A1 **4**	B(2n, 0.65), P($R \leq n$) > 0.15 stated or implied Any probability in list below seen $p = 0.1391$ picked out (i.e., not just in a list of > 2) Final answer $n = 9$ only [SR <n: M1A0, n = 4, 0.1061 A1A0] [SR 2-tail: M1A1A0A1 for 15 or 14] [SR: 9 only, no working: M1A1] [MR B(12, 0.35): M1A0, n = 4, 0.1061 A1A0]

3	0.3529	7	0.1836	12	0.0942
4	0.2936	8	0.1594	13	0.0832
5	0.2485	9	0.1391	14	0.0736
6	0.2127	10	0.1218	15	0.0652

4734 Probability & Statistics 3

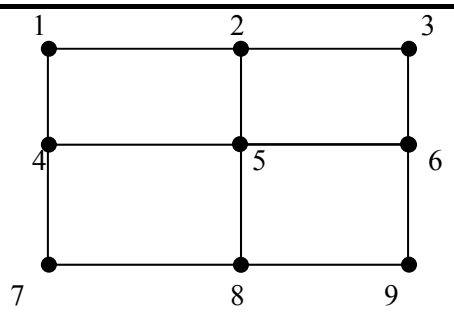
1(i)	$s^2 = 0.00356/80+0.00340/100$ $= 7.85 \times 10^{-5}$	M1 A1 **2**	Sum of variances Or pooled, giving 7.81×10^{-5}
(ii)	---	---	---
	$(1.36-1.24) \pm zs$ z=1.96 (0.103, 0.137)	M1 B1 A1 **3**	Must be s, accept t
	---	---	---
(iii)	Not necessary since sample sizes are large	B1 **1 (6)**	Or equivalent. Nothing wrong
2 (i)	Use $\bar{x} \pm z\dfrac{\sigma}{\sqrt{n}}$	M1	
	$\bar{x} = 337.5 / 20$	**B1** B1	
	z =2.326 (14.9,18.9)	A1 4	3 or 4 SF
	---	---	---
(ii)	$1- 0.98^3$ 0.0588	M1 A1 **2**	Use B(3,0.02) or B(3,0.98) for M.
	---	---	---
(iii)	Unbiased estimate of σ^2 required t – distribution used to obtain CV	B1 B1 2 **(8)**	
3 (i)	H_0: $p_W = p_N$, H_1: $p_W > p_N$	B1	For both hypotheses. Or π.
	Pooled $\hat{p} = \dfrac{71+73}{80+90}$ $\left(=\dfrac{144}{170}\right)$	B1	SR: from $p_1q_1/n_1 + p_2q_2/n_2 = 0.00295$ z = 1.406 B1M1A1M1A1 Max 5/7
	s^2 = (144/170)(26/170)(1/80+1/90)	B1	
	z = (71/80-73/90)/s	M1	
	=1.381	A1	If no explicit comparison and correct conclusion then M1A0.
	1.381 < 1.645 Do not reject H_0, there is insufficient evidence that the proportion of on-time Western trains exceeds the proportion of on-time Northern trains	M1 A1 **7**	Or use P-value or CR In context, not too assertive
(ii)	---	---	---
	s^2= 71×9/80^3+73×17/90^3 = 0.00295	M1 A1 **2 (9)**	AEF Allow one error Accept 0.0029
4(i)	Use $L - S_1 - S_2$	M1	Or equivalent, or implied
	μ = 0.7	B1	
	σ^2 = 0.58^2+ 0.31^2+0.31^2 = 0.5286	M1 A1	May be implied later
	(1-0.7)/σ	M1	Correct numerator
	0.340	A1 **6**	
	---	---	---
(ii)	Use $L - 2S$ with μ=0.7	M*1	M0 if as (i) unless correct
	σ^2 = 0.58^2 + 4$(0.31)^2$	B1	
	- 0.7/ σ	Dep*M1	Accept +
	- 0.824(5)	A1	
	0.2048	A1 **5** **(11)**	0.205 (3SF)

5(i)	Population of differences is normal H_0:$\mu_A = \mu_B$, H_1: $\mu_A < \mu_B$ where μ_A and μ_B denote the population means $\bar{x}_D = 3.222$ $s_D = 5.019$ $t = 3.222/(5.019/3)$ $=1.926$ CV = 1.860 1.926 > 1.860 Reject H_0 , there is evidence that brand *A* takes less time than brand *B* ----------	B1 B1 B1 M1A1 M1 A1 B1 M1 A1 **10** ----------	*Not "independent"* Or $\mu_D = 0$, $\mu_D > 0$ From formula ,or B2 from calculator Accept 1.93. M1A0 if t = - 1.926 ----------
(ii)	One valid reason	B1 **1 (11)**	Data are clearly paired Data not independent
6(i)	37×58/120 17.883.. , 17.88 AG ----------	M1 A1 **2** ----------	Or equivalent ----------
(ii)	H_0: Gender and shade are independent (H_1:--are not independent $3.02^2(14.02^{-1}+14.98^{-1})$ + $6.12^2(17.88^{-1}+19.12^{-1})$ $+3.1^2(26.1^{-1}+27.9^{-1})$ =6.03 EITHER: CV 5.991 6.03 > 5.991, reject H_0 and accept that gender and shade are not independent OR: $P(\chi^2 > 6.03) = 0.049$ < 0.05 , reject H_0 and accept that gender and shade are not independent ----------	B1 M1 A1 A1 B1 M1 A1√ **7** B1 M1 A1√ ----------	 At least two correct All correct Ft X^2 . Can be assertive. Ft X^2 ----------
(iii)	G1 G2 G3 O 29 37 54 E 40 40 40 121/40 + 9/40+196/40 = 8.15 Using df = 2 2.5% tables, 1.7% calculator	 M1 A1 M1 A1 M1 A1 **6 (15)**	 For combining

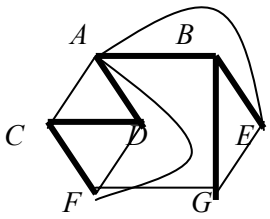
7(i)	$F(t) = \begin{cases} 0 & t \le 0, \\ t^4 & 0 < t \le 1, \\ 1 & \text{otherwise.} \end{cases}$	B1 B1 **2**	For t^4 For rest
	---	---	---
(ii)	$G(h) = P(H \le h)$ $= P(T \ge 1/h^{1/4})$ $= 1 - F((1/h^{1/4})$ $= 1 - 1/h$ $g(h) = G'(h)$ $= 1/h^2$ $h \ge 1$,(0 otherwise)	M1 A1 A1 A1 M1 A1 **B**1 **7**	Accept < With attempt at differentiation Only from G obtained correctly
	---	---	---
(iii)	EITHER: $\int_1^\infty (h^{-2} + 2h^{-3})\mathrm{d}h$ $= \left[-h^{-1} - h^{-2}\right]_1^\infty$ $= 2$ OR: $= 1 + 2\int_1^\infty \frac{1}{h^3}\mathrm{d}h$ $= 1 + 2\left[-\frac{1}{2h^2}\right]_1^\infty$ $= 2$ OR: $E(1+2T^4) = 1 + \int_0^1 8t^7\,\mathrm{d}t$ $= 1 + [t^8]$ $= 2$	M1 B1 A1 M1 B1 A1 M1 B1 A1 **3 (12)**	For integrating $(1+2h^{-1})g(x)$, with limits from (ii) Limits not required Limits not required Limits not required

4736 Decision Mathematics 1

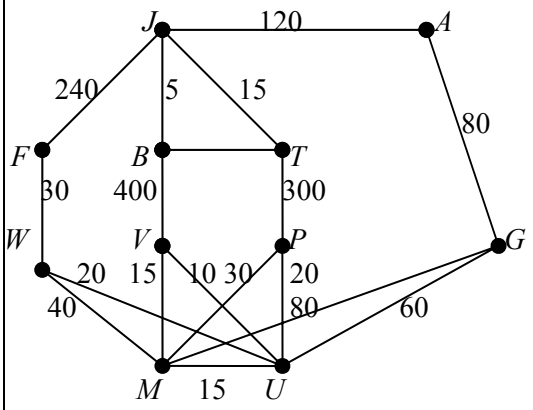
1	**(i)**	5 2 4 3 8 Bin 1: 5 2 3 Bin 2: 4 Bin 3: 8	M1 A1	First bin correct All correct in three bins	[2]
	(ii)	8 5 4 3 2 Bin 1: 8 2 Bin 2: 5 4 Bin 3: 3	M1 A1	First bin correct All correct in three bins	[2]
	(iii)	The heaviest box is originally at the bottom of the stack	B1	Referring to the physical act of sorting the weights into decreasing order	[1]
	(iv)	Bins in any order and boxes in any order Bin 1: 8 or 8 Bin 2: 5 3 5 2 Bin 3: 4 2 4 3	B1	Any valid packing into three bins of capacity 8 kg.	[1]
				Total =	**6**

2	**(i)**		M1 A1	A connected graph with nine vertices labelled 1 to 9 Correct graph	
		4 moves	B1	Stating 4	[3]
	(ii)	Neither It has four odd nodes The nodes 2, 4, 6, 8 each have three arcs joined to them whereas an Eulerian graph has no odd nodes and a semi-Eulerian graph has exactly two odd nodes	M1 A1	'Neither', together with an attempt at a reason A correct reference to the number of odd nodes for this graph. Be careful about whether 'odd' refers to the parity or the value. However, just defining Eulerian and semi-Eulerian, without reference to this graph, is not enough	[2]
				Total =	**5**

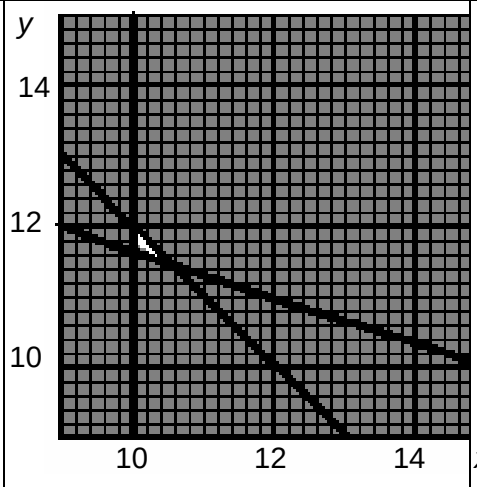
ANSWERED ON INSERT

3	(i)	*AD* = 16 *CD* = 18 *CF* = 21 ~~*AC* = 23~~ ~~*DF* = 34~~ *BE* = 35 *BG* = 46 *AB* = 50 ~~*EG* = 55~~ ~~*FG* = 58~~ ~~*AE* = 80~~ ~~*AF* = 100~~  Total weight = 186	M1 A1 M1 A1 B1	Using Kruskal: Not selecting *AC* and *DF* Selecting correct arcs in list, or implied (16+18+21+35+46+50, in this order with no others, can imply M1, A1) Drawing a spanning tree for these six vertices Correct (minimum) spanning tree drawn 186 (cao)	[5]
	(ii)	Delete *BG* from spanning tree 186 – 46 = 140 Two shortest arcs from *G* are *BG* and *EG* 140 + 46 + 55 = 241 Lower bound = 241	 B1 M1 A1	Correct working for wrong vertex deleted can score B1, M1, A0 Weight of MST on reduced network (ft from part (i) Adding two shortest arcs to MST 241 (cao)	[3]
	(iii)	*A–D–C–F–G–* … or 16+18+21+58+ … *A – D – C – F – G – B – E - A* Upper bound = 274	M1 A1 B1	Using nearest neighbour Correct closed tour listed, not just weights added 274 (cao)	[3]
				Total =	**11**

ANSWERED ON INSERT

4 (i)		B1	Times for flying route, $JA = 120$ $AG = 80$ $GU = 60$ $UM = 15$ $GM = 80$	
		B1	Times for train route correct $JT = 15$ $JB = 5$ $BT = 20$ $TP = 300$ $PU = 20$ $PM = 30$	
		B1	Times for coach route and driving route correct $BV = 400$ $VU = 10$ $VM = 15$ $JF = 240$ $FW = 30$ $WU = 20$ $WM = 40$	**[3]**
	Strictly, these are directed arcs, but they are shown as undirected arcs		Follow through their arc weights if reasonable	
	J 1 | 0 A 4 | 120 (120) F 6 | 240 (240) B 2 | 5 (5) T 3 | 15 (15) V (405) P (315) W 8 | (270) G 5 | 200 (200) M 9 | 275 (280 275) U 7 | 260 (260)	M1	Permanent values correct at A, F, B, T $A = 120$, $F = 240$, $B = 5$, $T = 15$	
		M1 d	Both 280 and 275 seen at M (updating at M)	
		A1 ft	All temporary labels correct (or implied) and no extras	
		B1 ft	All permanent labels correct (or implied) (condone labelling past M)	
		B1 ft	Order of labelling correct (condone labelling past M)	
	Alternatively, if treating as undirected: J, A, F, B and T are unchanged, then			
	Or $V = 8^{th}$ and $W = 9^{th}$ V 9 | 270 (405 270) P (315 280) W 8 | 270 (270) G 5 | 200 (200) M 10 | 275 (280 275) U 7 | 260 (260)		Marked as above	
	Route: J - A - G - U - M	B1	Correct answer only	**[6]**

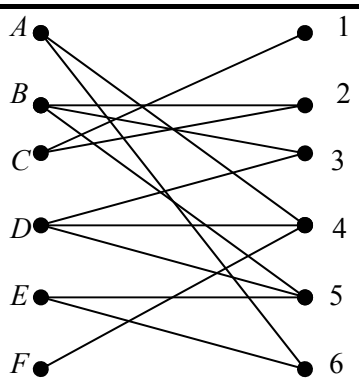
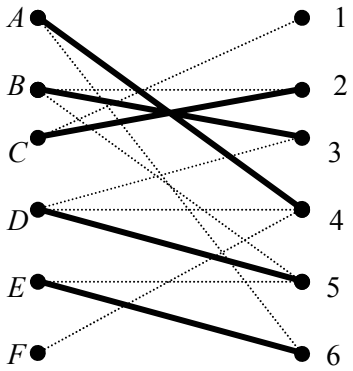
(ii)	The quickest journey time from Jenny's house to the meeting venue	B1	Quickest journey / least travel time or equivalent	**[1]**
(iii)	Does not allow for waiting for connections There may be delays at the airport She may not want to fly because of the 'carbon footprint' She may want to choose the cheapest route rather than the quickest route She may not like flying She may want to see her friend She may want to break the journey overnight	B1 B1	Any reasonable suggestion for why she may not want to use the drive/fly/underground route or why she may want to use a different route Any second reasonable suggestion	**[2]**
				Total = 12

5	**(i)**	x = area of wall to be panelled (m^2) y = area to be painted z = area to be covered with pinboard	B1 B1	Reference to area or m^2 (at least once) Identifying x as panelling, y as paint and z as pinboard, in any way	**[2]**
	(ii)	Cost $\leq$ £150 $\Rightarrow 8x + 4y + 10z \leq 150$ $\Rightarrow 4x + 2y + 5z \leq 75$ (given)	B1 B1	Use of word 'cost' or equivalent $8x + 4y + 10z \leq 150$ seen or explicitly referred to	**[2]**
	(iii)	(Minimise P =) $15x + 30y + 20z$	B1 ft	Any positive multiple of this eg $3x + 6y + 4z$ or $\frac{1}{4}x + \frac{1}{2}y + \frac{1}{3}z$	**[1]**
	(iv)	(Minimise $P = 480 +$) $- 5x + 10y$ Subject to $x + 3y \geq 45$ $x \geq 10$ $y \geq 0$ $x + y \leq 22$	B1 ft B1 B1	Any positive multiple of this, eg $2y–x(+ c)$ - or maximise a negative multiple Any equivalent simplified form $x \geq 10$ may be implied $y \geq 0$ may be implied $x + y \leq 22$, any equivalent simplified form	**[3]**
	(v)		M1 M1 M1 A1	ANSWERED ON GRAPH PAPER $x = 10$ drawn accurately with a sensible scale $x + y = 22$ drawn accurately with a sensible scale Their $x + 3y = 45$ drawn accurately with a sensible scale Shading correct or identification of the feasible region (triangle with (10, $11\frac{2}{3}$), (10, 12) and ($10\frac{1}{2}$, $11\frac{1}{2}$) as vertices)	**[4]**
					Total = 12

6	**(i)**	P | x | y | z | s | t | 1 | -25 | -14 | 32 | 0 | 0 | 0 0 | 6 | -4 | 3 | 1 | 0 | 24 0 | 5 | -3 | 10 | 0 | 1 | 15	B1 B1	Rows and columns may be in any order Objective row with -25, -14, 32 Constraint rows correct (condone omission of P column)	**[2]**
	(ii)	x column has a negative value in objective row Cannot use y column since it has negative entries in all the other rows $24 \div 6 = 4$ $15 \div 5 = 3$ Least non-negative ratio is 3, so pivot on 5	B1 B1 B1	'negative in top row', '-25', or similar 'most negative in top row' $\Rightarrow$ bod B1 Correct reason for not choosing y column Both divisions seen and correct choice made (or both divisions seen and correct choice implied from pivoting)	**[3]**
	(iii)	1 | 0 | -29 | 82 | 0 | 5 | 75 0 | 0 | -0.4 | -9 | 1 | -1.2 | 6 0 | 1 | -0.6 | 2 | 0 | 0.2 | 3 New row 3 = $\frac{1}{5}$ row 3 New row 1 = row 1 + 25×new row 3 oe New row 2 = row 2 - 6×new row 3 oe $x = 3$, $y = 0$, $z = 0$ $P = 75$	M1 A1 B1 B1 B1 B1 ft B1 ft	Follow through their sensible tableau (with two slack variable columns) and pivot Pivot row correct (no numerical errors) Other rows correct (no numerical errors) Calculation for pivot row Calculation for objective row Calculation for other row x, y and z from their tableau P from their tableau, provided $P \geq 0$	**[2]** **[3]** **[2]**
	(iv)	Problem is unbounded No limit to how big y (and hence P) can be Only negative in objective row is y column, but all entries in this column are negative	B1	Any one of these, or equivalent. If described in terms of pivot choices, must be complete and convincing	**[1]**
				Total =	**13**

		$F = N \div B$ $G = \text{INT}(F)$ $H = B \times G$ $C = N - H$ $N = G$						For reference only	
7	**(i)**	*F*	*G*	*H*	*C*	*N*	M1	A reasonable attempt at first pass (presented in any form)	
		2.5	2	4	1	2	A1	$F = 2.5$ and $G = 2$	
							A1	$H = 4$ (or double their G value) and $C = 5 -$ their H	
		1	1	2	0	1	A1	F, G, H, C and N correct for second pass (ft their N value)	
		0.5	0	0	1	0	A1	F, G, H, C and N correct for third pass (ft their N value)	[5]
	(ii)	*F*	*G*	*H*	*C*	*N*	M1	A reasonable attempt	
		-2.5	-3	-6	1	-3	M1 d	First pass correct (or implied)	
		-1.5	-2	-4	1	-2			
		-1	-1	-2	0	-1			
		-0.5	-1	-2	1	-1	A1	Reaching two lines with the same value for G	
		-0.5	-1	-2	1	-1			
								If described in words only, then M1 for a correct statement; M1 d for all correct statements (sufficient to guarantee result), and A1 for convincingly correct explanation of how they know these to be true and why the result follows	
		Does not terminate					B1	Saying 'does not stop', or equivalent	[4]
	(iii)	*F*	*G*	*H*	*C*	*N*			
		3.7	3	30	7	3	M1	First pass correct	
		0.3	0	0	3	0	A1	All correct	
		The first value is the units digit of N, the second value is the tens digit, the third value is the hundreds digit, and so on.					M1 A1	Outputs are digits of N In reverse order	[4]
									Total = 13

4737 Decision Mathematics 2

1	**(i)**		M1 A1	Any three stars paired to the correct rooms All correct $A \rightarrow 4, 6$ $D \rightarrow 3, 4, 5$ $B \rightarrow 2, 3, 5$ $E \rightarrow 5, 6$ $C \rightarrow 1, 2$ $F \rightarrow 4$	**[2]**
	(ii)	Faye 	B1 B1	Accept *F* Incomplete matching shown correctly on a second diagram (need not see other arcs) Arc $F \rightarrow 1$ must NOT be shown as part of the matching	**[2]**
	(iii)	F=4 – A=6 – E=5 – D=3 – B=2 – C=1 Arnie = Room 6 Diana = Room 3 Brigitte = Room 2 Edward = Room 5 Charles = Room 1 Faye = Room 4	B1 B1	This path indicated clearly This matching <u>listed</u> in any form (but NOT just shown as a bipartite graph)	**[2]**

(iv)

	1	2	3	4	5	6
A	3	6	4	1	5	2
B	5	3	2	4	1	6
C	2	1	3	4	5	6
D	5	4	1	3	2	6
E	5	6	4	3	2	1
F	5	6	4	1	3	2

For reference only

Reduce rows

M1

2	5	3	0	4	1
4	2	1	3	0	5
1	0	2	3	4	5
4	3	0	2	1	5
4	5	3	2	1	0
4	5	3	0	2	1

Then reduce columns

M1

1	5	3	0	4	1
3	2	1	3	0	5
0	0	2	3	4	5
3	3	0	2	1	5
3	5	3	2	1	0
3	5	3	0	2	1

Or reduce columns

1	4	3	0	4	1
3	2	1	3	0	5
2	0	2	3	4	5
3	3	0	2	1	5
3	5	3	2	1	0
3	5	3	0	2	1

Then reduce rows

1	4	3	0	4	1
3	2	1	3	0	5
2	0	2	3	4	5
3	3	0	2	1	5
3	5	3	2	1	0
3	5	3	0	2	1

A1 cao with rows reduced first

Follow through their reasonable reduced cost matrix if possible **[3]**

Cross out 0's using 5 lines

M1 Any valid choice of lines (max for theirs)

Augment by 1 to get a complete allocation

M1 Augmenting appropriately

A1 Augmentation completely correct (ft) **[3]**

$A = 1 \quad B = 5 \quad C = 2 \quad D = 3 \quad E = 6 \quad F = 4$

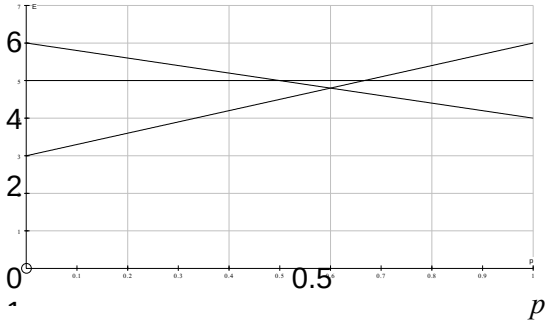
B1 This allocation <u>listed</u> in any form, cao

Arnie

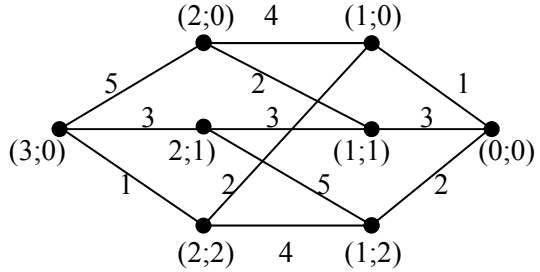
B1 Arnie named (not just *A*), cao **[2]**

Total = 14

2	**(i)**	6	B1	6 **[1]**
	(ii)	The total number of points for each combination is 10, subtracting 5 from each entry gives a total of 0 for each entry.	B1	Total = 10 changes to total = 0 or subtracting 5 gives total = 0 for every cell **[1]**
	(iii)	<table><tr><th></th><th>Liam</th><th>Mike</th><th>Nicola</th><th>row min</th></tr><tr><td>Philip</td><td>-1</td><td>0</td><td>1</td><td>-1</td></tr><tr><td>Sanjiv</td><td>-2</td><td>-3</td><td>-1</td><td>-3</td></tr><tr><td>Tina</td><td>1</td><td>0</td><td>-2</td><td>-2</td></tr><tr><td>col max</td><td>1</td><td>0</td><td>1</td><td></td></tr></table> Play-safe for R is Philip Play-safe for C is Mike Not stable since $-1 \neq 0$ If Team R play safe then Team C should choose Liam	M1 B1 A1 B1 B1	Row for Sanjiv is optional Writing out pay-off matrix for zero-sum game (or explaining that the given matrix will give the same play safes since each entry is a constant 5 more than in the zero-sum game P, cao, row minima need not be seen M, cao, col maxima need not be seen Accept any reasonable identification Any equivalent reasoning Their row maximin ≠ their col minimax 'Liam' or 'L', or follow through their choice of play safe for Team R **[5]**
	(iv)	If the entry for row P column L is increased the col max for Liam is at least as big as at present so column M is still the column minimax and the row min for Philip is at least as big as at present so row P is still the row maximin.	M1 A1	Using either original values or augmented values. A reasonable explanation of either part A correct explanation of both (in play safe row and not in play safe column, without further explanation ⇒ M1, A0) **[2]**
	(v)	Sanjiv's scores are dominated by Philip's. Sanjiv scores fewer hits than Philip <u>for each choice</u> of captains from Team C	B1	Identifying dominance by *P* <u>and</u> explaining it or showing the three comparisons **[1]**
	(vi)	$4p + 6(1-p)$ or $-1p + 1(1-p) + 5$ $= 6-2p$ M: $5p + 5(1-p)$ or $0(p) + 0(1-p) + 5 = 5$ N: $6p + 3(1-p)$ or $1p + -2(1-p) + 5 = 3p+3$	M1 A1 B1	Using original or reduced values correctly Achieving given expression from valid working 5 and $3p+3$, cao **[3]**

	(vii)	E  $3p + 3 = 6 - 2p \Rightarrow p = 0.6$ Expect at least 4.8 hits	 M1 A1 B1 B1	MAY BE ON GRAPH PAPER Appropriate scales and line E = 6-2p drawn correctly (Their) other lines drawn correctly Solving for their p or from graph Their E for chosen value of p or from graph	 **[2]** **[2]**
					Total = 17

ANSWERED ON INSERT

3	**(i)**	 Stage / State / Action / Working / Minimax 0 / 0 / 1 / **1** 1 / 1 / 0 / 3 / **3** 2 / 0 / 2 / **2** 2 / 0 / 0 / (4, **1**)= **4** 1 / (2, **3**)= **3** / **3** 1 / 1 / (3, **3**)= **3** / **3** 2 / (5, **2**)= **5** 2 / 0 / (2, **1**)= **2** / **2** 2 / (4, **2**)= **4** 3 / 0 / 0 / (5,**3**)= **5** 1 / (3,**3**)= **3** 2 / (1,**2**)= **2** / **2**	 B1 M1 M1 A1 M1 A1	 Minimax column for stage 1 shows 1, 3, 2 identified in some way 1, 3, 2 transferred to working column for stage 2 correctly Calculating maximum values in working column for stage 2 Minimax column for stage 2 shows 3, 3, 2 identified in some way (cao) Calculating maximum values in working column for stage 3, correct method Minimax column for stage 3 shows 2 identified in some way (cao)	 **[4]** **[2]**
	(ii)	Minimax value = 2 Minimax route = (3;0) – (2;2) – (1;0) – (0;0) (or in reverse)	B1 M1 A1	2, cao Tracing their route (whatever problem solved) This route from correct working (using network ⇒ M0)	 **[3]**
	(iii)	 	B1 M1 A1	All vertices labelled correctly Arcs correct, need not be directed Condone stage boundaries shown Arc weights correct (be generous in interpretation of which weight is attached to which arc)	 **[3]**
					Total = 12

ANSWERED ON INSERT

4	**(i)**	A single source that joins to S_1 and S_2 Directed arcs with weights of at least 90 and 110, respectively T_1 and T_2 joined to a single sink Directed arcs with weights of at least 100 and 200, respectively	 B1 B1	 Condone no directions shown Condone no directions shown	**[2]**
	(ii)	If AE and BE were both full to capacity there would be 50 gallons per hour flowing into E, but the most that can flow out of E is 40 gallons per hour.	M1 A1	Considering what happens at E (50 into E) At most 40 out	**[2]**
	(iii)	40 + 60 + 60 + 140 = 300 gallons per hour	B1	300	**[1]**
	(iv)	30 + 20 + 30 + 20 + 40 + 40 + 20 + 40 = 240 gallons per hour	M1 A1	Evidence of using correct cut 240	**[2]**
	(v)	A feasible flow through network Flow = 200 gallons per hour Cut through arcs S_1A, S_1B, S_1C, S_2B, S_2C and S_2D or cut $X = \{S_1, S_2\}$, $Y = \{A, B, C, D, E, F, G, T_1, T_2\}$	M1 A1 B1	 Cut indicated in any way (May be on diagram for part (i))	**[3]**
	(vi)	 Flows into C go to C_{IN}, arc of capacity 20 from C_{IN} to C_{OUT}, and flows out of C go from C_{OUT}. Cut $X = \{S_1, S_2, C_{IN}\}$ or $X = \{S_1, S_2, C_{IN}, D\}$ shows max flow = 140 gallons per hour	 B1 B1 B1 B1	May have working or cut shown on diagram Into C ($S_1 = 40$, $S_2 = 40$, $D = 20$) Through C Out of C ($F = 60$, $G = 60$) 140 (cut not necessary)	**[4]**
					Total = 14

ANSWERED ON INSERT

5 (i)

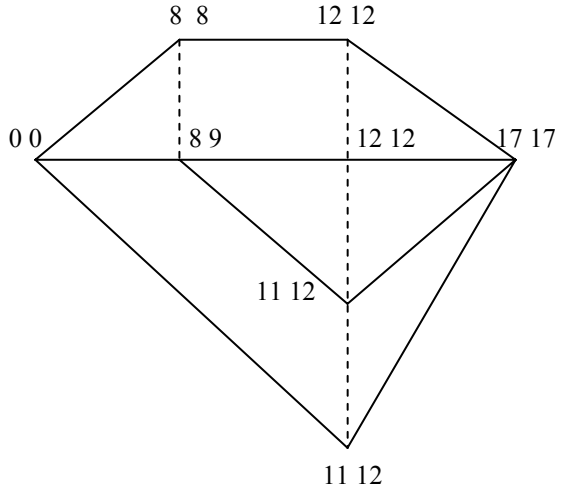
Activity	Duration (days)	Immediate predecessors
A	8	-
B	6	-
C	4	-
D	4	***A***
E	2	***A B***
F	3	***A B***
G	4	***D***
H	5	***D E F***
I	3	***F***
J	5	***C F***

B1 Precedences correct for *A*, *B*, *C*, *D*

B1 Precedences correct for *E*, *F*, *G*

B1 Precedences correct for *H*, *I*, *J* **[3]**

(ii)



Minimum project duration = 17 days

Critical activities = *A D H*

M1 Forward pass, no more than one independent error

A1 Forward pass correct (cao)

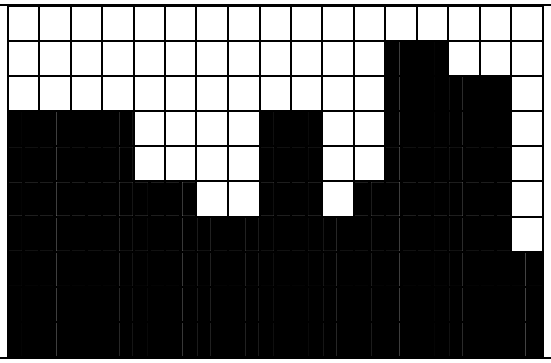
M1 Backward pass, no more than one independent error

A1 Backward pass correct (cao) **[4]**

B1 17, cao

B1 *A D H*, cao **[2]**

(iii)



ANSWERED ON GRAPH PAPER

M1 A plausible histogram, with no holes or overhanging blocks

A1 Correct shape

[2]

(iv)

Example:

Start *A* and *B* as before but delay *C* to day 6

Start *D* and *F* as before but delay *E* to day 11

Then, for example, start *G* on day 12, *H* on day 13, and *I* and *J* on day 16

Precedences not violated, durations correct

B1 Dealing with *A*, *B* and *C*

B1 Dealing with *D*, *E* and *F*

M1 Dealing with *G*, *H I* and *J*

A1 A valid solution using 6 workers for 21 days **[4]**

Total = 15

Grade Thresholds

Advanced GCE Mathematics (3890-2, 7890-2)
January 2008 Examination Series

Unit Threshold Marks

7892		Maximum Mark	A	B	C	D	E	U
4721	Raw	72	58	50	42	35	28	0
	UMS	100	80	70	60	50	40	0
4722	Raw	72	60	52	45	38	31	0
	UMS	100	80	70	60	50	40	0
4723	Raw	72	51	44	37	31	25	0
	UMS	100	80	70	60	50	40	0
4724	Raw	72	57	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4725	Raw	72	56	49	42	36	30	0
	UMS	100	80	70	60	50	40	0
4726	Raw	72	49	43	37	31	25	0
	UMS	100	80	70	60	50	40	0
4727	Raw	72	55	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
4728	Raw	72	59	52	45	38	31	0
	UMS	100	80	70	60	50	40	0
4729	Raw	72	57	49	41	33	25	0
	UMS	100	80	70	60	50	40	0
4730	Raw	72	50	43	36	29	22	0
	UMS	100	80	70	60	50	40	0
4732	Raw	72	55	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
4733	Raw	72	55	48	41	34	28	0
	UMS	100	80	70	60	50	40	0
4734	Raw	72	52	45	38	31	25	0
	UMS	100	80	70	60	50	40	0
4736	Raw	72	57	51	45	40	35	0
	UMS	100	80	70	60	50	40	0
4737	Raw	72	59	52	45	39	33	0
	UMS	100	80	70	60	50	40	0

Specification Aggregation Results

Overall threshold marks in UMS (ie after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
3890	300	240	210	180	150	120	0
3891	300	240	210	180	150	120	0
3892	300	240	210	180	150	120	0
7890	600	480	420	360	300	240	0
7891	600	480	420	360	300	240	0
7892	600	480	420	360	300	240	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
3890	25.5	49.6	70.9	84.3	96.0	100	478
3892	28.6	71.4	100	100	100	100	7
7890	33.0	58.3	79.1	92.2	97.4	100	115
7892	11.1	44.4	100	100	100	100	9

For a description of how UMS marks are calculated see:
http://www.ocr.org.uk/learners/ums_results.html

Statistics are correct at the time of publication.

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