

Mathematics (MEI)

Advanced GCE **A2 7895-8**

Advanced Subsidiary GCE **AS 3895-8**

Report on the Units

January 2009

3895-8/7895-8/MS/R/09J

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of pupils of all ages and abilities. OCR qualifications include AS/A Levels, GCSEs, OCR Nationals, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new syllabuses to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support which keep pace with the changing needs of today's society.

This report on the Examination provides information on the performance of candidates which it is hoped will be useful to teachers in their preparation of candidates for future examinations. It is intended to be constructive and informative and to promote better understanding of the syllabus content, of the operation of the scheme of assessment and of the application of assessment criteria.

Reports should be read in conjunction with the published question papers and mark schemes for the Examination.

OCR will not enter into any discussion or correspondence in connection with this Report.

© OCR 2009

Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

CONTENTS

- Advanced GCE Further Mathematics (MEI) (7896)**
- Advanced GCE Further Mathematics (Additional) (MEI) (7897)**
- Advanced GCE Mathematics (MEI) (7895)**
- Advanced GCE Pure Mathematics (MEI) (7898)**

- Advanced Subsidiary GCE Further Mathematics (MEI) (3896)**
- Advanced Subsidiary GCE Further Mathematics (Additional) (MEI) (3897)**
- Advanced Subsidiary GCE Mathematics (MEI) (3895)**
- Advanced Subsidiary GCE Pure Mathematics (MEI) (3898)**

REPORT ON THE UNITS

Unit/Content	Page
GCE Mathematics and Further Mathematics Certification	1
4751 Introduction to Advanced Mathematics (C1)	3
4752 Concepts for Advanced Mathematics (C2)	7
4753 Concepts in Pure Mathematics (C3) (Written Examination)	10
4754 Applications of Advanced Mathematics (C4)	13
4755 Further Concepts for Advanced Mathematics (FP1)	16
4756 Further Methods for Advanced Mathematics (FP2)	18
4758 Differential Equations (Written Examination)	22
4761 Mechanics 1	24
4762 Mechanics 2	29
4763 Mechanics 3	32
4766 Statistics 1 (G241 Z1)	34
4767 Statistics 2	37
4768 Statistics 3	40
4771 Decision Mathematics 1	43
4776 Numerical Methods (Written Examination)	45
Coursework	47
Grade Thresholds	51

GCE Mathematics and Further Mathematics Certification

Optimising Grades for GCE Mathematics Qualifications

Centres are reminded that when candidates certificate for a GCE qualification in Mathematics they are strongly advised to recertify for any GCE Mathematics qualification for which they have previously certificated.

For example

- a candidate certifying for A level Mathematics is advised to recertify for AS Mathematics if this has been certificated in a previous session.
- a candidate certifying for A level Further Mathematics is advised to recertify (or certificate) for AS Mathematics, A level Mathematics and AS Further Mathematics.

The reason for this is to ensure that all units are made available to optimise the grade for each qualification.

Certification entries are free of charge.

Manual Certification for Further Mathematics

It is permissible for candidates to enter for GCE Further Mathematics with the OCR (MEI) specification if they have previously entered (or are simultaneously entering) for GCE Mathematics with another specification or Awarding Body. In this case OCR has to check that there is no overlap between the content of the units being used for the GCE Mathematics qualification and the GCE Further Mathematics qualification.

A Manual Certification Form must be completed for each candidate. A copy of the form is available on the GCE Mathematics pages on the OCR website. If you wish to have an electronic copy of the form email your request to fmathsmancert@ocr.org.uk

The table below summarises this.

Qualification	
7895	Candidates are strongly advised to apply for recertification for 3895 in the same series as certificating for 7895.
3896	Candidates are strongly advised to apply for recertification (or certification) for 3895 (and 7895 if enough units have been sat) in the same series as certificating for 3896. If a candidate has certificated or is certificating for AS Mathematics or A-level Mathematics with a different specification or Awarding Body then a Manual Certification form* must be completed and returned to OCR.
7896	Candidates are strongly advised to apply for recertification (or certification) for 3895, 7895 and 3896 in the same series as certificating for 7896. If a candidate has certificated or is certificating for A-level Mathematics with a different specification or Awarding Body then a Manual Certification form* must be completed and returned to OCR.
3897	Candidates are strongly advised to apply for recertification (or certification) for 3895, 7895, 3896 and 7896 in the same series as certificating for 3897.
7897	Candidates are strongly advised to apply for recertification (or certification) for 3895, 7895, 3896, 7896 and 3897 in the same series as certificating for 7895.

Report on the Units taken in January 2009

*A copy of the Manual Certification form is available on the GCE Mathematics pages on the OCR website. It may be photocopied as required, and should be returned to:

The Qualification Manager for Mathematics, OCR, 1 Hills Road, Cambridge, CB1 2EU; Fax: 01223 553242.

An electronic copy of the form may be requested by emailing fmathsmancert@ocr.org.uk When completed the form can be returned by email to the same address.

4751 Introduction to Advanced Mathematics (C1)

General Comments

This paper proved to be more accessible than the last two January papers, with many candidates gaining high marks on section A, particularly in the first five questions. In general, the majority of candidates seemed well prepared, with fewer who had little idea of what was required being entered than in past years. For instance, there were fewer poor attempts at using the quadratic formula than there used to be. However, some were thrown by anything that required them to think beyond questions that they had done in practice, with question 7(i) in particular gaining a surprisingly poor response.

Arithmetic with fractions remains a problem, with $\frac{4}{12} = 3$ being quite a common error in question 11(iii) for example, and some candidates failing to simplify results such as $\frac{-1}{-5}$ (where $\frac{1}{5}$ or 0.2 was expected) or $\frac{12}{3}$ or $\sqrt{144}$ (where integer answers were expected).

Candidates need to realise that, in questions where they are required to reach a given result, they must give sufficient indication of their method to show that the result has been obtained independently. In this paper, this particularly applied to questions 11(i) and 13(ii).

Centres are asked to ensure that candidates follow the instructions on the front of the question paper and use black ink, with pencil used only for graphs and diagrams.

Comments on Individual Questions

Section A

- 1) Most candidates knew the facts about negative and zero powers and gained both marks here. 8 or -8 were the common errors in part (i), whilst a few zeros were seen in part (ii).
- 2) The methods for finding the equation of a straight line joining two points were well known. Some inverted the gradient but even they generally managed a mark for a correct method to find the constant. Arithmetic or algebraic errors in expansion sometimes spoiled the final answer.
- 3) Nearly all candidates obtained the 3 marks here. The only real problem occurred with those who obtained $-6x < -9$ and then divided by -6 without reversing the inequality. Those who worked towards $9 < 6x$ were usually successful. A few candidates worked only with an equation instead of an inequality or made errors in collecting the terms.
- 4) The majority knew how to apply the factor theorem apart from those who used $f(-2)$ or failed to equate $f(2)$ to zero. A few candidates attempted long division or equating coefficients, but such attempts were rarely successful.

- 5) In the first part, a number of candidates failed to get both x^3 terms in the expansion, with obtaining $7x^2$ instead of $7x^3$ being a common error. The majority expanded the product fully rather than just looking for the terms which would give x^3 . A few candidates omitted this part, which although testing a basic technique had not been asked in this way on past C1 papers. The second part was done reasonably but a significant number failed to square the 2 whilst a smaller cohort used the wrong number from Pascal's triangle or made an error in finding $\binom{7}{2}$.
- 6) Some candidates had problems here in coping with the fraction, with common wrong first steps being $2x(3x+1) = 4 - 2x$ and $2x(3x+1) = 4 \times 2x$. The vast majority obtained the solution correctly and efficiently.
- 7) Part (i) caused problems for most candidates. The expected $5^3 \times 5^{\frac{1}{2}} = 5^{\frac{3}{2}}$ was not often seen. Some gave $5^3 \times 5^{\frac{3}{2}} = 5^{\frac{1}{2}}$ but many made errors such as $125\sqrt{5} = \sqrt{125} \times \sqrt{5}$ or $\sqrt[125]{5}$ etc and worked with those.
In part (ii), most had no problems, although common errors were $16a^5b^7$ and $4a^6b^{10}$, whilst some thought it was a binomial expansion.
- 8) Some candidates omitted this part or simply attempted trials of various values for k and/or x . Most candidates used $b^2 - 4ac$, but the condition for no real roots was often not clearly stated, with some candidates working with $b^2 - 4ac = 0$ or $b^2 - 4ac > 0$ or with statements such as $\sqrt{k^2 - 144} < 0$. Such statements were often followed by recovery to give a correct answer, although many omitted the condition $k > -12$, simply giving $k < 12$ or sometimes $k < \pm 12$. The arithmetic in working out $4 \times 2 \times 18$ also produced frequent errors.
- 9) How candidates coped with this varied from centre to centre (or group to group within a centre). Some seem well-prepared and knew how to proceed. Some others had little idea how to cope with two y terms and made several attempts, none getting very far before making an error.
- 10) Most candidates knew how to simplify and add the surds and did so correctly, although some thought $\sqrt{75} + \sqrt{48} = \sqrt{123}$ or, more commonly, $25\sqrt{3} + 16\sqrt{3} = 41\sqrt{3}$. Poor arithmetic such as $48 = 18 \times 3$ spoilt some answers. In the second part, most candidates knew they had to multiply numerator and denominator by $3 + \sqrt{2}$. Errors were sometimes made in doing so, with 11 as denominator instead of 7 for instance, or in failing to divide both terms by 7 for the final answer, with wrong 'cancelling' usually seen in such cases.

Section B

- 11) (i) Candidates from some centres showed a good understanding of what is required by 'Show that ...' and obtained all 4 marks. However, many candidates produced incomplete or incorrect answers. A common mistake by those who obtained the length of AB was to say that the square root of 160 is 40. Also frequently seen was subtracting the x and y coordinates to obtain incorrectly (5,2). The words centre and radius were absent from many answers, as was the general formula of a circle.
- (ii) Most candidates attempted to substitute $x = 0$ in the equation of the circle. A common mistake was to omit the 25 which results from squaring $x - 5$. A few candidates omitted $-4y$ when they attempted to square $y - 2$. Some candidates failed to arrange the resulting quadratic so that it could be solved, and their solutions petered out in wrong algebra. Many candidates had a correct method but did not gain the final mark because they were unable to simplify their answer correctly to the form required.
- (iii) Some candidates did this part well after obtaining few marks in parts (i) and (ii). Most knew that the tangent of the circle was perpendicular to the radius and how to obtain a perpendicular gradient. Most know how to substitute this and the coordinates of a point in the equation of a straight line. Those who made error(s) such as $4/12 = 3$ were often able to obtain the final two marks for the intersections with the axes, since follow-through was applied here, but some omitted to attempt this, and some only obtained the point of intersection with one axis.
- 12) (i) Most candidates made a reasonable attempt at the first part of this question. The favoured method was to equate the expressions for y as shown on the mark scheme. Most of these attempts yielded the correct quadratic equation though it was not uncommon to see $3x^2 + 10x - 8 = 0$. Most candidates successfully factorised this quadratic and went on to get the correct values for x . Some candidates used the formula to get these roots. However not many candidates successfully determined both y values because they could not cope with the fraction work and negative signs involved when using $x = -\frac{4}{3}$. A few candidates used the subtraction method successfully to arrive at the quadratic in x . One or two attempts were made at substituting for x , but they rarely coped with the manipulation required to yield the correct quadratic in y .
- (ii) Most candidates realised the need to take out a factor of 3 and so gained the marks for $a = 3$. However only the really good candidates got much further. The value for b was often seen as 2 or 3, but again only the best candidates had much idea about how to determine the value of c . The presence of the factor 3 caused the problem. A few candidates who checked their work by expanding their answer realised their error and were able to correct it.

- (iii) Some candidates did realise that completing the square was a good lead-in to answering this part and clearly identified that there was a minimum at $y = 7$. However many candidates chose to base their argument on the basis of the value of the discriminant of the equation. Most of these showed it to be negative but rarely gave an adequate explanation as to the significance of this. Some concluded that it showed no real roots, sometimes going on to say that the graph would not cross the axis, but hardly any attempts went on to determine whether the graph was above or below the x -axis. There were also some arguments about turning points and some attempts to use a sketch, but these were rarely complete.
- 13) (i) As I have commented in the past, many candidates do not appreciate properly the distinction between plotting and sketching a curve – some have wasted time in past papers by plotting when a sketch was all that was required. This time, when a plot was requested, some did a sketch, calculating the minimum and joining this with a sketch to $(0, 5)$ and $(5, 5)$ and failing to pass correctly through points such as $(1, 1)$ and $(2, -1)$. Some candidates produced poor curves, with feathering or doubling or missing plotted points, with some not helping themselves since they had used pen rather than pencil for drawing. However, many good graphs were seen.
- (ii) This was handled well by a very high proportion of candidates, although some were careless with “= 0”, or did not show an intermediate step before the given answer and therefore lost the second mark. The most common incorrect solution seen was for candidates to simply verify the result by substituting values.
- (iii) A pleasing number of candidates successfully found the correct quadratic factor – usually by inspection or long division, but occasionally by comparing coefficients. Candidates who found the quadratic factor, or those who made a simple error in finding it, most often went on to complete the question. A few candidates who had made an error arrived at complex roots and confused irrational with complex – genuinely (or so it would seem) thinking that their negative discriminant implied irrational roots. Not many candidates realised that it is sufficient to find the value of the discriminant for the quadratic equation to show that the other two roots are irrational, but most did obtain full marks by finding the roots of the quadratic in surd form. A few candidates just stated that the quadratic could not be factorised without justifying their comment at all.

4752 Concepts for Advanced Mathematics (C2)

General Comments

The paper was generally well received, with very few low-scoring scripts. Some high-scoring candidates lost marks by failing to show enough working when producing a given answer. There were some very good scores in section A; nevertheless, section B was generally better received, with a significant minority of candidates scoring full marks.

Comments on Individual Questions

Section A

- 1) Most candidates scored well on this question. Some strong candidates lost an easy mark by omitting “+c”. Weak candidates failed to simplify $\frac{20}{5}$, or were unable to deal with $\frac{6}{-\frac{1}{2}}$. A small number of candidates differentiated instead of integrating.
- 2) This question was generally well done, although some candidates wasted time by calculating the areas of individual trapezia and then summing the areas, instead of using the composite rule. Candidates who omitted the first pair of brackets did not score at all.
- 3) This question was generally well done. Only a few candidates failed to sum the terms, even fewer made arithmetical slips. Some candidates attempted to use formulae for the sums of arithmetic or geometric series.
- 4) This question was not done well. An initial step for most was “ $\sin x = -0.25$ ”, which resulted in no marks. Often the better candidates failed to obtain a complete solution. 15° was often presented as part of the final answer.
- 5) There were many excellent responses to this question. In part (i) some candidates translated 2 units to the left; a few translated 2 up or 2 down. The usual error in part (ii) was to misplace (2, 3), but a small number of candidates stretched by a factor of $\frac{1}{3}$ or misplaced the horizontal line.
- 6) Part (i) was accessible to most. Nearly all candidates obtained the correct answer, with a small minority making arithmetical slips.
There were some excellent answers to part (ii). However, many candidates didn't take the hint from part (i) and calculated $S_{50} - S_{21}$. Some missed the point and calculated $S_{50} - u_{21}$ or even $u_{50} - u_{21}$. Those who did use part (i) often evaluated S_{29} instead of S_{30} .
- 7) The differentiation was often well done, but only the best candidates dealt successfully with $8x - \frac{1}{x^2} = 0$. Some candidates differentiated to obtain $8x - 1$ or even $8x - x$.
- 8) Part (i) was very well done. “Geometric” was the required answer, “oscillating” was condoned; nothing else scored. In part (ii) very few candidates realised the need to quote the condition for convergence. Those that did often didn't relate it to the question. Many obtained the correct answer for the sum. A common error was to calculate $S = \frac{192}{1-\frac{1}{2}}$. A few candidates substituted $r = 2$ or -2 in the correct formula.

- 9) Part (i) was very well done. In part (ii) most scored M1 for a correct use of one of the log laws, but a surprisingly high number obtained the answer $\frac{3}{2} \log x$ in various different ways. Part (iii) was well done; $\log 1 - \log x$ and $\log x^{-1}$ did not score.

Section B

- 10) (i) This was done very well indeed, with many candidates scoring full marks. Some candidates set $\frac{dy}{dx} = 0$ and used “ $m = 3.5$ ”, and some found 3 correctly, and then went on to use “ $m = \frac{1}{3}$ (or $- \frac{1}{3}$)”. A few candidates tried to work back from the intercept without finding $y = 4$. This was rarely, if ever, successful.
- (ii) This was done well. Many showed $f(1) = 0$, and then went on to use the Factor Theorem successfully to show that $x = 6$ is the other root. Many factorised successfully. The usual errors were $(x-1)(x-7)$ and $(x-1)(x+6)$.
- (iii) Most candidates scored well on the integration, although dealing with the double negative defeated many. Surprisingly few calculated the area of the correct triangle, and those who attempted a solution by integration were rarely successful. A common error was to take the height of the triangle as 2 units.
- 11) (i) Part A was done well, with only a handful of candidates clearly having no idea what to do. Some wasted time by converting to degrees; of these some then lost accuracy through premature rounding. There were many fully correct answers to part B, but a sizeable minority of candidates calculated $\frac{1}{2} \times 80^2 \times 2.5 - \frac{1}{2} \times 60^2 \times 2.5$, and some candidates stopped at $\frac{1}{2} \times 140^2 \times 2.5$.
- (ii) This was done extremely well, with an overwhelming majority scoring full marks. Some candidates calculated the wrong angle, and a few rounded off before finding the angle, thus losing the accuracy mark.
- (iii) This was generally well done, but many candidates presented convoluted solutions, penalising themselves by wasting time that could have been devoted elsewhere.
- 12) (i) This was usually well done. Occasionally $t \log ab$ or $\log a \times \log b$ were seen.
- (ii) Three decimal places were required – and usually presented – in the table. Some candidates lost a mark through one or more incorrect plots and a few candidates lost the third mark by failing to use a ruler, but generally speaking this was answered very well.
- (iii) Many candidates used the long winded method of solving two equations from their table or their graph simultaneously. More often than not they lost accuracy marks in the process. The expected approach of $\log a = \text{intercept}$ and $\log b = \text{gradient}$ generally yielded full marks, although some candidates were not able to produce a gradient within the expected range. Occasionally “ $\log a = \text{gradient}$ and $\log b = \text{intercept}$ ” was seen; rather less common was “ $t = \text{gradient}$ ”.
- (iv) Many candidates thought they had to start again here, instead of using the value obtained for a , and often lost the mark as a result. Many candidates omitted “million”, and thus didn’t score.

Report on the Units taken in January 2009

- (v) A variety of approaches were seen. Many candidates were successful. In some cases obviously wrong answers (e.g. negative or ridiculously large numbers) were presented, but it did not usually seem to occur to the candidate that anything was amiss.

4753 Concepts in Pure Mathematics (C3) (Written Examination)

General Comments

This paper proved to be a straightforward and accessible test of candidates' abilities, and there were many excellent scripts scoring over 60 marks. Even low-scoring candidates managed to obtain around 30 marks. There was no evidence of time problems.

The standard of presentation was, however, variable – some candidates often took pages of working to process relatively simple algebra. As usual there was evidence of sloppy notation, in particular omission of essential brackets from working, which was penalised in 'E' marks.

It is helpful to the marking process if extra sheets are attached, using treasury tags, to the back of answer booklets, rather than inter-leaved.

Comments on Individual Questions

Section A

- 1) The response of candidates to modulus questions is improving, although there remains evidence of misunderstandings in the notation. For example, some candidates write $|x - 1| < -3$, albeit obtaining the correct limit $x < -2$, and writing $|x| < 4$ scored no marks. Another common error is $x - 1 < -3$. Although some candidates used squaring and a quadratic correctly, the simplest route to the correct answer is to expand the modulus as $-3 < x - 1 < 3 \Rightarrow -2 < x < 4$.
- 2) (i) Nearly all candidates spotted this as a product rule, and many got it right. The most common errors were in the derivative of $\cos 2x$, for example omitting the negative sign or the factor of 2.
(ii) Integration by parts was generally well known, but the usual confusions between integral and derivative results for trigonometric functions led to errors in $v = \frac{1}{2} \sin 2x$. Omitting the arbitrary constant cost quite a few candidates the final 'A' mark.
- 3) This question was very well answered, with only the weakest candidates failing to get 3 marks, either using $f^{-1}(x) = 1/f(x)$, or expanding $\ln(x - 1)$ as $\ln x - \ln 1$.
- 4) Although this is a fairly standard integration by substitution, quite a lot of candidates failed to obtain the correct answer. Many are omitting 'dx' and 'du' altogether, or getting $du = 4 dx$ but failing to introduce a factor of $\frac{1}{4}$ in the integrand. Another quite common error is to (needlessly) convert back to 'x' and use 'u' limits. It would be nice to see an improvement in the use of accurate notation here!

- 5) (i) The ‘period’ has not been asked for in recent papers, so tripped up quite a few candidates, mistaking it for ‘domain’. ‘2’ was a common error here, but we generously condoned $0 < x < 180$.
- (ii) There was the usual plethora of alternatives to ‘translate’ and ‘stretch’ (i.e. ‘move’, ‘squeeze’, ‘contract’, ‘shift’ etc.). We insisted on ‘translate’ (notwithstanding the correct vector being specified) and ‘stretch’.
- (iii) We prefer candidates **not** to use graph paper for a ‘sketch’. Most candidates gave us a sketch with the correct domain, but there were many errors in its shape.
- 6) (i) The success of candidates in this question did not seem to correlate that well with marks in the other questions. Many seemed to ‘discover’ incorrect counter-examples. We allowed $p = 0$ or $q = 0$ for M1 and E1 if accompanied by explanations such as ‘infinite’ or ‘undefined’.
- (ii) We wanted conditions of some generality here, although logically speaking stating it for a single value of p and q can lead to a true statement!
- 7) (i) Implicit differentiation was quite well done, although some differentiated the power incorrectly. The most common error was to start $dy/dx = \dots$, and the crucial zero on the right hand side was sometimes missing. Candidates who start d/dx (LHS) = d/dx (RHS) usually got it right. The algebra required to navigate to the given answer was often incorrect.
- (ii) Many candidates who failed to score in part (i) got 3 easy marks here. Errors were usually associated with evaluating $-(8/1)^{1/3}$ incorrectly, often omitting the negative sign.

Section B

- 8) This question had plenty of accessible marks, thought the final part proved testing for even the best candidates.
- (i) Most candidates got this correct. The most common errors were to getting the gradient fraction the wrong way round, or use the derivative intended for part (ii).
- (ii) The derivative was sometimes written as $2x - 1/8 x$, leading to a fortuitously correct gradient.
- (iii) Equating their derivative to zero gave a very accessible ‘M’ mark However, in the case where the derivative was correct, a surprising number of candidates showed algebraic immaturity by failing to solve the subsequent equation by multiplying through by x (or equivalent). Also, many candidates missed out on the final ‘A’ mark by approximating their y-coordinate.

- (iv) Most spotted the product rule to differentiate $x \ln x$, though some used $u = x \ln x$ and $v = x$, and others forgot to take away the '1' from the derivative of x . This might be caused by a lack of organisation of side working.

Having achieved the $\ln x$ from the first part, some candidates then failed to spot the connection with the $1/8 \ln x$ in the area integral, using integration by parts. Many also failed to put the $x \ln x - x$ in a bracket. Only the 'A' grade candidates negotiated the fractional arithmetic convincingly to derive the given result.

- 9) Again, this question offered plenty of accessible marks.

- (i) Most candidates set $2x - x^2$ to zero and solved to get $a = 2$ (some requiring the quadratic formula!). However, the domain was less well done.
- (ii) About a half of the candidates used a quotient rule rather than a chain rule, which gave more opportunities for error. We withheld the final 'E' mark if there were missing brackets, at any stage, round the ' $2 - 2x$ '.

The second part, finding the turning point, offered a very straightforward 3 marks, but the range was quite often omitted or incorrect (e.g. $y > 1$ or $x \geq 1$).

- (iii) Most candidates offered an algebraic proof of the even-ness of g , though some wrote $1 - (-x^2)$ or $1 - -x^2$. Sometimes the way the argument is written makes the implication unclear. For example, the 'proof' below is incomplete:

g is even if $g(x) = g(-x)$

$$\text{So } \frac{1}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-(-x)^2}}$$

On the other hand, the argument convinces if it is written as follows:

$$g(-x) = \frac{1}{\sqrt{1-(-x)^2}} = \frac{1}{\sqrt{1-x^2}} = g(x)$$

The proof that $g(x - 1) = f(x)$ was generally sound, though some omitted brackets round the $(x - 1)^2$ and lost the 'E1'.

Part (C) required candidates to state the symmetry of g in the y -axis, the translation of g to f one unit to the right, and the new axis of symmetry $x = 1$.

4754 Applications of Advanced Mathematics (C4)

General Comments

This paper was of a similar standard to that set in January 2008. The Section A was slightly more difficult than last year. The Comprehension was well understood and the marks were generally high.

As usual in the January session, a high standard of work was seen. All questions were answered well by some candidates. The paper was accessible to all and a large number of candidates were able to obtain full credit in many areas.

Some candidates failed, once again, to give sufficient working when establishing given results-often losing unnecessary marks. Questions involving 'show that' need to show all stages of working. The work with algebra was also often disappointing-particularly the use of brackets.

It is vital that candidates' scripts for paper A and the Comprehension paper are attached to each other using treasury tags before being sent to the examiner.

Comments on Individual Questions

Paper A

Section A

- 1) The majority of candidates started with the correct partial fractions.
 $3x+2 =A(x^2+1)+Bx+C(x)$ was often seen when multiplying up the fractions although correct work often followed. The commonest mistake was a failure to assemble the final fractions correctly.
$$\frac{2}{x} + \frac{3-2x}{x^2+1}$$
 often incorrectly being given as $\frac{2}{x} - \frac{2x+3}{x^2+1}$.
- 2) The Binomial expansion was usually well understood and used appropriately. The main errors were failing to fully establish the given result and either missing out the validity or including equality signs in it.
- 3) This vector question was usually successful, but occasionally omitted. Most candidates used simultaneous equations to find the values of λ and μ but others found the values by inspection and then checked their results. Many scored full marks.
- 4) The trigonometric proof was more successful than this type of question has been in the past, although some candidates do not offer logical arguments for their proofs - often working with 'both sides' in a confused manner.
- 5) (i) This was usually correct although occasionally the correct answer, 73.2° , was subtracted from 180° or 90° .
(ii) Although it did not affect their marks on this occasion, $r =$ was often omitted before the vectors. Provided that they started correctly, this part was usually completed successfully. $6\lambda = -3$, $\lambda = -2$ was a relatively common error.

- 6) (i) The first part was usually successful. Many candidates omitted the R when stating $R\cos\alpha = 1$ or $R \sin\alpha = \sqrt{3}$ but were not penalised as they proceeded correctly. α was usually given correctly in terms of π .
- (ii) This was less successful. The derivative of $\tan \theta$ was often correct - quite often being found from first principles. Much then depended upon candidates realising the connection between parts (i) and (ii). Good candidates gave completely correct solutions although for some the numerical factor changed to 4 or 2 or $\frac{1}{2}$. Weaker candidates often missed this part out.

Section B

- 7) (i) The times were almost always right.
- (ii) Although most candidates separated the variables correctly, the constant was occasionally omitted. When anti-logging, e^{-kt+c} was often seen as $e^{-kt} + c$ or $e^{-kt} + e^c$. On this occasion, the answer was given so they had to be clear about their use of $A = e^c$ (or equivalent) to obtain full marks.
- (iii) $A = 48$ was usually found, but when finding the value of k there were two common errors. These were either using $1.5 = -48k$ (and omitting the $(-)$) or substituting for $t=1$ in $96.5 = 50 + A e^{-kt}$ and not using the differential.
- (iv) This was usually correct provided the values of 89 and 80 were substituted. Some attempted to take the logarithms of negative numbers in their incorrect working.
- (v) Discussions of Newton's law of cooling or that the temperature changed over time were common. Some only referred to one model. They were required to comment on the time difference between the two models becoming increasingly different as the temperature loss became greater.
- 8) (i) The correct method was usually seen. There were some sign and coefficient errors but the answer was often completely correct.
- (ii) Many failed to verify that $dy/dx = 0$ as they did not substitute values for $\cos\pi/3 - \sin\pi/6$. Some solved the equation in (i) to find the value of θ . This was unnecessary, but equally valid. Although the method for finding the co-ordinates was usually correct, often the value of x was omitted and the value of y was not always given in exact form.
- (iii) This section was usually well done although there were some confused arguments. There were occasional long solutions in (C) when squaring $y = 2\cos\theta + \sin 2\theta$ and then substituting back for x using part (B). The main error was squaring term by term.
- (iv) This was well answered. The commonest mistake was losing the π before the final line.

Paper B: The Comprehension

Candidates should be advised to think carefully before entering their answers in the spaces on the Comprehension answer sheet as their working is often confused or crossed out.

- 1) This was usually correct but as the value of d was given, $d=8$ was not enough to establish the result.
- 2) The working was often confused but the answer was usually derived.
- 3) There were some confused solutions but many were right. Common errors included prematurely rounding the value of H and giving the final answer as 20.8 cm^3 rather than litres.
- 4) A fairly common error was using the formula from Question (2) to find the volume even though this was not a cylinder. Others could not find the area of cross section correctly or did not give their final answer in litres as required.
- 5) (i) This was usually correct but some candidates gave a commentary and did not show the algebra as well.
(ii) The second root was usually found. The commonest error was to compare the value found with the capacity of the tank, or not specifically compare it to the height or y . 'Too big' was not explicit enough.
- 6) This was often fully correct but many found y as a different function of x (often $10x$) which caused them problems. A few had variables such as y or H in their integral with respect to x .

4755 Further Concepts for Advanced Mathematics (FP1)

General Comments

Most candidates were well prepared for the examination and were able to score highly. Although Section A was found more difficult by some candidates the questions in Section B were accessible and very well answered by many. Section A revealed some algebraic errors. It did not appear that candidates suffered from lack of time to complete the paper except in a few cases.

Comments on Individual Questions

- 1) (i) This was usually attempted by using the formula, although substituting $a+jb$ and equating real and imaginary parts to zero was also used by several candidates. It was extraordinary how many went from $\sqrt{(-4)/2}$ to $2j$ instead of j .
(ii) The argument of the root $3-j$ was not always given in the range $-\pi$ to $+\pi$. A few candidates forgot to include j in the expressions using r and θ and some believed that having found the modulus and the argument, they had finished. Use of the abbreviation $cjs\theta$ is not to be recommended at this stage.
- 2) Most candidates compared coefficients. Some used substitution. A common error (and a bad one at this stage) was to expand $-B(x - 2)$ to $-Bx - 2B$, and another to use 9 instead of $9A$ to find C .
- 3) This question proved surprisingly difficult for quite a few candidates.
 - (i) This part was less well answered than part (ii); many did not know how to obtain P and several candidates covered a lot of paper in trying to find it.
 - (ii) Several candidates pre-multiplied the matrix by their column vectors, somehow. Which point was which was often left for the examiner to determine.
 - (iii) Enlargement was extensively used instead of stretch to describe the transformation, although the combination of one with the other, provided scale factors were appropriate, was accepted. Sometimes there was seen the curious description of a stretch “about a point”.
- 4) There were many candidates who did not know that this required a statement about the argument of a complex variable. They earned no marks. The argument of a modulus does not exist, but credit was earned for a variable appearing.
- 5) There was a variety of approaches to this question. Finding α using the sum of the roots and thence obtaining the actual roots was successfully employed by many candidates, whether they then proceeded to obtain the factors and multiply out, or continued to use the $\Sigma\alpha\beta$ and $\alpha\beta\gamma$ relationships. Using the latter often led to $q=-12$, confusing signs. Another popular method was to form the factors in terms of α and multiply out. Yet another method used was the substitution of roots in terms of α and the attempted solution of the resulting equations. In the latter case in particular the resulting algebra proved to be too difficult.

- 6) This was probably the most successfully answered question in Section A, with many candidates scoring full marks. A very few failed to remember the correct result for Σr . A few did not convincingly demonstrate the steps leading to the required factorisation, which was given in the question.
- 7) There were very many excellently reasoned answers to this question. Those that kept to the “standard” form of words and were able correctly to form the algebraic argument scored full marks. But there were still many candidates who cannot quite show their full understanding of the method, by lack of attention to details, perhaps trying to go too fast. “Assume $n=k$ ” is not the same as “Assume the result is true when $n=k$ ”. “It is true for $n=k$ and $n=k+1$ ” is not the same as “if it is true for $n=k$ then it is true for $n=k+1$ ”. There were also the careless omissions of summations to spoil the statements presented. As far as the needful algebra was concerned, a few candidates were under the impression that $4x3n=12n$.
- 8) There were many answers that scored full marks, even among the less confident candidates.
- (i) Some candidates were casual about writing down co-ordinates. Some failed to realise that there were three intersections with the two axes to account for.
 - (ii) Most candidates gave the asymptotes correctly.
 - (iii) Only two branches had to be sketched here but many candidates consider that a sketch can be extremely “rough”. The structure of the graph should be shown, with clear approaches to the asymptotes, not a vague indication. Several sent the left-hand branch to infinity on the wrong side of $y=1$. Where it was shown that the asymptote had been crossed there should have been some indication of a minimum point. Less commonly the left-hand branch was shown with $y \rightarrow -\infty$ as $x \rightarrow -2^0$. In just a few cases the right-hand branch was shown crossing $y=1$.
 - (iv) Where candidates had drawn a good graph the inequalities were usually correct. Some were confused between $<$ and $>$ and gave the wrong intervals. Some did not give the right inclusion at $\pm\sqrt{3}$. Those that attempted an algebraic solution were unsuccessful.
- 9) This was also a well answered question by the majority of the candidates, part (i) in particular.
- (ii) This also was well done, failure to give an equation not being penalised here.
 - (iii) Following the hint in (ii) most candidates successfully multiplied two appropriate quadratic expressions. Those who tried using the relationships between the roots sometimes mixed up the signs of the coefficients in the terms. In some instances not enough terms were included in $\Sigma \alpha\beta$. There was a penalty here for failure to include “=0” in presenting the result.
- 10) All parts were well done by most candidates.
- (i) A few candidates lost marks for failing to show sufficient working, the answer was given.
 - (ii) Usually correct, but in (iii) some candidates thought that \mathbf{A}^{-1} was $(1/42) \mathbf{A}$.
 - (iii)
 - (iv) Some did not show the correct sequence of pre-multiplication of the column vector by the inverse matrix. Failure to use (iii) was also a reason for loss of marks here.

4756 Further Methods for Advanced Mathematics (FP2)

General Comments

The candidates generally responded positively to this paper and very many good scripts were seen, with very few scoring fewer than 20 marks: there were also enough challenging questions for the most able. These candidates were able to display their skills by deploying elegant methods, especially in Question 4. This was certainly a challenging question and some candidates who chose it could not complete it, but this was sometimes because they had used very long methods in, for example, the integral in Question 3(a)(ii), or the Maclaurin series in Q1(a)(ii). The “standard” integrals in Q1(b) and the eigenvalues and eigenvectors in Q3(b) were done very well by the vast majority of candidates, and there were many good responses to the whole of Question 2.

As is always the case, some scripts bordered on the illegible, and there were a few candidates who appeared to delight in separating parts of questions and scattering them around the paper. Some candidates drew every sketch graph on a separate piece of graph paper. This is not necessary: sketches on the lined paper were expected, and quite acceptable.

Comments on Individual Questions

- 1) Maclaurin series, integration
The mean mark on this question was about 12 out of 19.
- (a) Most candidates managed to produce the Maclaurin series for $\cos x$ successfully, although there were some sign errors. Some candidates worked from first principles, writing out the series as $a_0 + a_1x + a_2x^2 + \dots$ and repeatedly differentiating.

The second part was less well done. Many candidates explained that $\cos x \times \sec x = 1$, but many others concentrated on the fact that, if x were small, both series tended to 1, and $1 \times 1 = 1$. Then many candidates ignored the product, and resorted to repeated differentiation of $\sec x$. This was often done successfully as far as the second derivative, but correct fourth derivatives, required for the coefficient of x^4 , were extremely rare, and took a great deal of time. Despite the question asking candidates to work from part (i), some credit was given for this method. Those who did multiply the two expansions were often successful, but there were many algebraic slips, and candidates then sometimes produced $b = 0$ by equating the wrong coefficients to zero.

- (b) The standard result in part (i) was often derived correctly, although a substantial number of candidates could not proceed beyond $\sec^2 y \frac{dy}{dx} = 1$. Some candidates used the derivative of $\arctan x$ from the formula book, and included the a via the Chain Rule: this attracted some credit. Others just stated that the result was the reverse of an integral expression to be found in the formula book: no credit was given for this. A few candidates did not treat a as a constant, and differentiated $\frac{x}{a}$ by using the quotient rule.

The integrals in (ii) were done extremely well, and most candidates gained full

marks here. Errors included failing to divide by a , failing to write the expression in (B) in a form suitable for applying the standard result, and failing to evaluate $\arctan 1$. One or two candidates worked in degrees. Those who tried explicit substitutions were usually very successful.

2) Complex numbers

This question was a good source of marks for most candidates: the mean mark was about 13 out of 18.

- (i) This part caused little trouble to the vast majority of candidates, although some produced the right answers after three-quarters of a page of working, including extensive trigonometric manipulations.
- (ii) There were very many fully correct solutions.
A substantial minority of candidates produced an incorrect diagram, having read the question as “ABB’ is equilateral” where B and B’ were the two possible positions of b . Others seemed to ignore the word “equilateral” and produced triangles in which B and B’ were on the axes. The modulus of a , and hence of b , was frequently given as $\sqrt{2}$.
- (iii) Candidates did not always show that $z_1^6 = 8$, which was given, in sufficient detail: quite a few just produced $\sqrt{2}e^{\frac{j\pi}{3}}$ as one of the sixth roots of 8, without further comment. Because of the appearance of $\frac{\pi}{3}$ in the question, some candidates produced the sixth roots of $8e^{\frac{j\pi}{3}}$ rather than 8, displacing their regular hexagon by $\frac{\pi}{18}$. A few thought the roots did not all have the same modulus, and another group found the fifth roots instead of the sixth roots. But again, there were very many fully correct and efficiently-managed solutions.
- (iv) This was frequently fully correct. However, a substantial number of candidates forgot to plot $1 + j$ anywhere, or plotted it with an obviously different modulus to their roots in (iii): it is just a rotation of z_1 .
- (v) Although many candidates arrived at $8e^{-\frac{j\pi}{2}}$ or equivalent, this was often left as the final answer: evaluation to the simpler $-8j$ was expected. When this was attempted, it was often -8 . A few applied the Binomial Theorem to $(1 + j)^6$, usually with complete success.

3) Polar curves, eigenvalues and eigenvectors

Part (b) of this question was better done than part (a). The mean mark was about 12 out of 17.

- (a) The curve was frequently correct although it was often tiny. Some candidates marked 0 and $\frac{\pi}{3}$ on their horizontal axis, which did not inspire confidence.
The area of the region caused a great deal of trouble, and many candidates spent far too much time here, usually on futile trigonometry. Most could write a correct integral expression for the area although the limits were sometimes wrong. Then comparatively few knew how to integrate $\tan^2 \theta$: they often entered a “comfort

“zone” by writing it as $\frac{\sin^2 \theta}{\cos^2 \theta}$, and then employed double angle formulae and the like, although some hit on the right method by writing $\sin^2 \theta$ as $1 - \cos^2 \theta$ and dividing out. Those who obtained the correct answer were sometimes not sure which region they were working with, as their sketches showed: they subtracted areas of triangles or other integrals, which lost them the final mark. Another group, having obtained $1 - \frac{\pi}{4}$ in their answer, “simplified” it to $\frac{3\pi}{4}$, although this was condoned if the correct answer could be seen and there was no other error.

- (b) The methods here were well known and were often carried out completely correctly. Some candidates, having obtained the correct eigenvalues and reached $0.3x + 0.8y = 0$ to find one of their eigenvectors, wrote this eigenvector down as $\begin{pmatrix} 3 \\ 8 \end{pmatrix}$ or $\begin{pmatrix} 3 \\ -8 \end{pmatrix}$ rather than the correct $\begin{pmatrix} 8 \\ -3 \end{pmatrix}$. Fortunately for those candidates, the other eigenvector was $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$! A number of candidates decided that the decimals were not to their liking, and worked with $10\mathbf{M}$ rather than \mathbf{M} , which produced the correct eigenvectors, but not the correct eigenvalues. The characteristic equation, $\lambda^2 - 0.9\lambda - 0.1 = 0$, caused a few problems: those who used the quadratic formula to solve it sometimes made slips, and produced horrible surd eigenvalues with which they then had to work to obtain the eigenvectors. In part (ii), most knew exactly what they had to do: some went on to recalculate \mathbf{M} by finding the product \mathbf{QDQ}^{-1} , which wasted a lot of time.

4)

Hyperbolic functions

This was the choice of the vast majority of the candidates in Section B, although most found it a stiff test: the mean mark was just under 9 out of 18.

- (a) Most candidates approached part (i) confidently and produced a convincing proof. Slips included losing the $\frac{1}{4}$ and mismanaging the 2s, which occasionally did not appear at all. The responses to part (ii) were very variable, with (B) better done than (A), although many candidates left this part out altogether. Others asserted that sin and sinh, cos and cosh, and tan and tanh were interchangeable “by Osborn’s rule” and produced “proofs” of one or two lines, which attracted no credit. But some very elegant and resourceful work was seen, especially in part (B). A few very able candidates, not wishing to be defeated, proved (B) first, and then used the exponential form of $\tanh x$, the log laws and a great deal of paper to produce the result in (A).
- (b) Most candidates scored full marks in (i): a proof that $\operatorname{artanh} x$ is an odd function was not expected, but some candidates produced one anyway. The partial fractions in (ii) caused more problems: it was worrying, at this level, to see $\frac{1}{1-x^2} = \frac{1}{1} - \frac{1}{x^2}$ asserted so frequently, and very many candidates attempted to proceed without factorising the denominator. Some linked parts (i) and (ii) at this stage, looked up the logarithmic form of $\operatorname{artanh} x$ in the formula book, and gave this as the answer: this attracted some credit. Completely correct answers to part (iii) were rare: candidates often fudged the result, obtaining $2 \operatorname{artanh} \frac{1}{2} = \frac{1}{2} \ln 3$ by only using one limit, and then ignoring the inconvenient 2. A few able candidates used the oddness of $\operatorname{artanh} x$ again here, using limits 0 and $\frac{1}{2}$ in a very elegant solution.

5) Investigations of Curves

About ten candidates attempted this question, and the mean mark was about 6 out of 18. Virtually all candidates produced a correct curve in (i), and then answered several of the parts of (ii) correctly, but none scored more than 1 mark in (iii).

4758 Differential Equations (Written Examination)

General Comments

Many candidates demonstrated a good understanding of the specification and high levels of algebraic competency.

When sketching graphs, candidates are not expected to do any detailed analysis, but they should identify features which have been given or obtained in the question.

Comments on Individual Questions

- 1) (i) This was generally done very well, although some candidates made errors in the other roots of the auxiliary equation.
 - (ii) Most candidates correctly identified the particular integral. Some unnecessarily tried a linear or quadratic expression; this often resulted in a correct answer but was more time consuming and error-prone.
 - (iii) Many candidates stated one of the arbitrary constants is zero without explanation. For a given answer some explanation must be given. Most candidates successfully calculated the other constants.
 - (iv) Many candidates found the coordinates of the stationary point. Some were unable to justify that there are only two roots, often wrongly assuming it was sufficient to substitute the values of x and show $y = 0$.
 - (v) The sketch was often broadly correct, but sometimes did not label the relevant details – the given conditions and the calculated stationary point.
-
- 2) (i) This was often done well, but some candidates made a sign error leading to an integrating factor of $\cos x$ rather than $\sec x$. Candidates are advised to check their integrating factor carefully to avoid such an error.
 - (ii) The sketches were often poor. Detailed curve sketching was not required, but candidates were expected to indicate the initial condition and give some indication of scale on the x -axis.
 - (iii) The numerical solution was generally well done. However some candidates gave a list of incorrect numbers with no evidence of method for which no credit can be given.
 - (iv) Some candidates struggled to make any progress. Many made some progress by using the correct integrating factor. However, few completely correct answers were seen as candidates often failed to deal correctly with the initial condition.

- 3) (i) Many candidates were able to write down a correct equation of motion, but some did not realise that the acceleration can be expressed as $v \frac{dv}{dx}$. Most could solve the differential equation, although a few candidates omitted the constant. It is vital for candidates to realise the importance of including the constant and dealing properly with it when rearranging their solution.
- (ii) Candidates who solved the differential equation correctly almost always completed this calculation correctly.
- (iii) Most candidates were able to write down a correct equation of motion and solve it, although some again omitted the constant of integration.
- (iv) Most candidates integrated the velocity but some omitted the constant of integration or made errors in calculating the constant.
- 4) (i) Almost all candidates correctly calculated the values of x and y .
- (ii) The vast majority of candidates took a correct approach to the elimination of y but some made algebraic errors in the process.
- (iii) Most candidates correctly found the general solution, but some made errors in the roots of the auxiliary equation.
- (iv) It was pleasing to see the vast majority of candidates using their general solution and the first differential equation to find y . To find the *corresponding* solution, candidates should not construct and solve a new differential equation and only a few candidates used this incorrect approach.
- (v) Generally candidates were able to use the given conditions to find the solutions.
- (vi) The sketches were often done well, but some candidates omitted to identify the key features, in particular the initial conditions and the asymptotes.

4761 Mechanics 1

General Comments

There were many perfect solutions to every question and it was pleasing to see most of the candidates making good progress with several questions. There was little evidence that time was a problem to any candidate who tackled most of the questions reasonably efficiently.

Q2, involving the interpretation and use of kinematics graphs, presented great problems to many candidates. As has been the case for questions on this topic in recent sessions, it seems that many candidates do not have sufficient experience with answering this type of question.

Q4 and Q6 were unstructured and some of the weaker candidates failed to organise their ideas sufficiently well to make much progress.

Many of the scripts were well presented and showed a good knowledge of the course and the ability to select appropriate techniques.

Comments on Individual Questions

Section A

1) A constant acceleration problem in two stages

- (i) Most candidates obtained the correct answers but many did not seem to recognise that they were dealing with an expression of the form $v = u + at$. These evaluated v at some time (usually 5 s), assumed constant acceleration and then used $v = u + at$ to deduce a .
- (ii) $v(5)$ was usually found correctly but quite a few candidates made substitution or evaluation errors.
- (iii) There were many correct answers but some candidates failed properly to deal with the second part of the motion being linked to the first but with different acceleration. The most common errors were: to forget to add the 80 m from part (ii); to take the initial velocity to be 6 m s^{-1} from part (i) instead of 26 m s^{-1} from part (ii); to use the acceleration of the first part of the motion; to think that the second part of the motion lasted for 15 s instead of 10 s; to use integration but omit the arbitrary constants.

2) Use of an acceleration-time graph

Many candidates made little progress with this question as they did not seem properly to understand what the graph was telling them. Others wrote down the correct answers.

- (i) Many candidates did not know what to do. Some were clearly not considering the time interval $0 \leq t \leq 2$ but the time $t = 2$ and tried to take account of the acceleration changing at that time. The most common mistake was to neglect the initial velocity and give the *change* in velocity of 2×4 as if it were the velocity when $t = 2$.
- (ii) Again many candidates did not know what to do. The most common mistakes were in identifying the change of velocity as being from the answer to part (i) to -6 m s^{-1} . Commonly the change was given as being from +6 to -6 or from their answer to part (i) to +6. Many candidates who obtained 2.5 s forgot that they had to add 2 s.

3) The resultant of two vectors, Newton's second law applied in 2 dimensions and the direction of a vector

- (i) Apart from the few candidates who did not understand the vector notation, this part was usually done accurately with the most common error being subtracting the forces instead of adding them.
- (ii) Many candidates failed to recognise the direction given and gave the complementary angle to the one required. Quite a few thought they had to subtract $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ from their force before using the standard technique for finding the bearing of a vector.

- 4) The horizontal distance travelled by a projectile before it is descending at a given height above the point of projection

This question was unstructured but most candidates were familiar with problems of this type and knew what to do.

A surprising number of candidates threw away the given expression for the height of the stone and started again.

Most candidates used the obvious method of equating the height of the stone at time t to 3.675, solving the quadratic equation and selecting the larger root as the stone is descending. Very few candidates simplified the expression and tried to factorize; most used the quadratic formula. The chief problems were with signs, either because the sign of the 3.675 term became wrong during reorganisation of the quadratic expression or because there was a mistake during substitution into the formula (especially when the expression was left as $-4.9t^2 + 9.8t - 3.675 = 0$). Most candidates chose the greater root (and quite a few said why) but some gave two answers to the question.

Some candidates took on the longer method of finding the time to the top of the motion and then the time to fall to the required height; many gave up or made a mistake.

Finding the horizontal range from the time taken was usually done accurately.

- 5) Connected particles accelerating upwards in a lift

Quite a few candidates scored no marks on this question; more made progress with part (i) than with part (ii). Of course, many candidates wrote down the answers with accomplished ease.

- (i) Many candidates did not realize that they had to consider the equation of motion of the parcel not that of the man or man with parcel. The most common mistake from those who did consider only the parcel was to neglect its weight.
- (ii) Those who considered the man with the parcel often managed to get the right answer if they remembered the weight; those who considered only the man usually forgot the 55 N.

- 6) The collision of two particles moving in the same vertical line.

This unstructured question was less familiar than Q4 and fewer candidates managed to find a satisfactory method. Although many candidates struggled to make much progress, many others did well and produced efficient complete answers.

The most commonly used successful method was to use the information given directly in the question and equate the speeds at time t , thereby establishing that the collision took place after 1.5 s; the speed at collision followed as did the initial separation. The most common error made by candidates who adopted this approach was to equate velocities instead of speeds and then find that their term in t disappeared. Most candidates who used the given answer $T = 1.5$ failed to substitute it in expressions for the speeds of *each* of the particles and/or failed to make a statement explaining how they had established the result.

There were many false assumptions made, a common one being that the two particles travelled equal distances before collision; many candidates based their whole argument on this – others correctly found the distance travelled by one particle and incorrectly doubled it to find H . Other candidates based their attempts on assertions that were *true* but gave no reason *why* they must be true; for instance, the collision takes place after half the time it would have taken particle A to reach its greatest height.

A surprising number of candidates gave the value of H as the distance travelled by just one of the particles.

- 7) Static equilibrium and the application of Newton's second law to motion on horizontal and inclined planes.

Many candidates answered most of this question very well. It was especially pleasing to see many correct solutions to part (v) which involves motion on a slope and many correctly finding the normal reaction in part (i).

- (i) Most candidates produced an accurate diagram. The common mistakes were to omit an arrow (usually on the force in the string – perhaps they were drawing the string not the force) or the normal reaction or a label.

With the answer given, most candidates used the correct trigonometric ratio to find the frictional force. Many merely asserted that $121\cos 34$ is 100 (correct to 3 significant figures) without showing any evidence.

Many candidates accurately found the normal reaction. As always, many candidates wrongly believed that the normal reaction is the component of the weight perpendicular to the tangent of contact and so in the question ignored the component of the force in the string.

- (ii) Many candidates correctly stated that the sledge continued to move at the constant speed of 0.5 m s^{-1} but a few argued that it slowed down or even that it speeded up.
- (iii) Many candidates did this well. The common errors were to forget to resolve the tension and/or to omit the frictional force.

- (iv) There were many completely correct solutions to this part. Most candidates found the new acceleration but not all of these found the frictional force. The common errors were to use the wrong trigonometric ratio or use 'false' resolution obtained from the 'wrong' triangle to give the weight component as $\frac{980}{\sin 26}$.

- 8) The kinematics requiring calculus of a toy boat moving in 2 dimensions; the direction of the boat as seen from the origin, the direction of motion of the boat and the path of the boat

Most of the candidates used calculus appropriately in the question. Compared with previous sessions, more candidates correctly recognised when to use the direction of the position vector and when to use the direction of the velocity vector.

Some of the notation used was poor with candidates writing, for example, $\mathbf{i} = 0$ when they meant (and used) $v_x = 0$.

- (i) Most candidates used calculus to find the velocity of the boat in the x -direction and did so accurately. Quite a few then went on to find v_x when $t = 0$ instead of t when $v_x = 0$ but quoted their answer as a time.
- (ii) It was pleasing to see how well this was generally done. Most candidates convincingly obtained the expression $y = t^2 - 4t^2 + 4t + c$ but a few then simply stated that $c = 2$ without any attempt to demonstrate this is true.
- (iii) Many candidates did this completely correctly. Most realized that they needed the direction of the position vector and needed a zero \mathbf{i} component. It was disappointing that many omitted the $t = 0$ solution.
- (iv) This part presented many difficulties. Many candidates realized that they required the direction of the velocity vector but not all of them considered both $v_x = 0$ and $v_y = 0$ and others did not make it clear exactly what they were doing. A common mistake was to go on from the condition that both $v_x = 0$ and $v_y = 0$ to solve one of the equations $v_x \pm v_y = 0$. It was also clear that some candidates thought that they needed either $v_x = 0$ or $v_y = 0$.

Quite a few candidates correctly found the position vector when $t = 2$ but did not, as requested, find the distance of the boat from the origin.

There were many complete answers beautifully displayed and clearly argued.

- (v) There were a few correct answers. A common error was to plot a graph of x against t or y against t or distance against t .

4762 Mechanics 2

General Comments

Many excellent responses to this paper were seen and the majority of the candidates made progress worth marks on every question. There was some evidence that some candidates found the paper long but, on the whole, these were candidates who were using inefficient methods of solution, particularly in Q1 and in Q3. The standard of presentation in many cases was poor; diagrams were either absent or badly drawn and poor notation led to errors. The standard of algebraic manipulation of a significant minority of the candidates was a cause for concern. The majority of candidates appeared to understand the concepts required but did not always make it clear which principle or process was being used; as has happened in previous sessions those parts of the questions that were least well done were usually those requiring candidates to *show* a given answer. A minority of candidates wasted time by attempting to work back from the given answers rather than trying to find and employ the principles required to solve the problem.

Comments on Individual Questions

- 1 The majority of candidates understood the principles required and applied them well. However, as has happened in previous series with this type of question, those candidates who drew a diagram and indicated direction clearly were more successful than those who failed to do so.
 - (i) This part was well done by almost all of the candidates. However, very, very few of the responses included the direction of the force.
 - (ii) Most candidates obtained several marks for this part. They appreciated the need for two equations, obtained them and solved them simultaneously. It was encouraging to see far fewer sign errors in the use of Newton's experimental law this series than have been seen in the past. Those candidates who chose to use an energy equation along with either conservation of linear momentum or Newton's experimental law tended to be less successful than those who used both conservation of momentum and Newton's experimental law. Difficulties were encountered by some candidates when trying to solve a pair of equations one of which was quadratic; algebraic errors were common.
 - (iii) It was pleasing to see that many candidates made real progress with this part of the question. The main errors were usually sign related and again those candidates who drew a diagram were more successful than those who did not. Only those whose answers were all of high standard gave the direction of the impulse on the *barrier*.

- 2 More excellent answers were seen to this question than for similar questions set in previous series. The majority of the candidates appeared to understand the underlying concepts and were able to apply them correctly.
- (i) Almost all of the candidates gained full marks for this part.
 - (ii) A minority of candidates had difficulty with this part. The main problem appeared to be with the trigonometry required to find the distance and an apparent lack of knowledge of the identity $\tan \theta = \frac{\sin \theta}{\cos \theta}$
 - (iii) Very few incorrect solutions were seen to this part.
 - (iv) Correct solutions to this part were common with the majority of candidates opting to use $P = Fv$ rather than $P = \frac{\text{Work Done}}{\text{Time}}$. Of those using $P = Fv$ the most common mistake was to omit the component of the weight.
 - (v) The vast majority of candidates managed to score marks here. The mistakes that did occur were usually sign related.
- 3 This question was not as well done as similar questions in previous series. Candidates did not seem to appreciate that their responses had to show ALL the relevant steps when trying to show a given answer.
- (i) The majority of candidates gained many of the marks for this part. However, the working offered by a minority was muddled and inconsistent and did not show what it purported to show.
 - (ii) Only the strongest candidates gained full marks for this. Most candidates appreciated that, because of the position of the centre of mass, the fish-slice would topple but failed to explain that an unbalanced moment would be produced when placed as shown.
 - (iii) Again this part involved showing a given answer. Many correct solutions were seen but a sizeable minority of the candidates failed to explain exactly what they were doing and how they arrived at the lengths and masses used in working out the new centre of mass.
 - (iv) Many of the diagrams offered in this part were poor. Some were too small to be of any real use and others did not show the information requested. Those candidates who drew a clear diagram were usually successful in obtaining the required angle and gained full marks.

- 4 Many candidates could gain several marks for their solutions to this question. Part a) was usually done better than part b).
- (a) (i) Few candidates had problems with this part.
- (ii) Again well done by the majority of candidates. A small number of them did not appreciate that the reaction at P would be zero and tried setting up simultaneous equations. Others assumed that the values of the reactions would be the same as in the previous part, even though the conditions had changed.
- (b) (i) Most candidates successfully took moments about A and obtained the given answer. However, some employed resolution and stated $R = 2 \times 340\cos\alpha$ without giving any explanation as to why this was so. In order to obtain full marks for this method, candidates had to refer to the symmetry of the system and produce a clear argument.
- (ii) Diagrams rarely obtained full marks often because internal forces were inadequately labelled and/or the forces in the hinge at A were omitted.
- (iii) Most candidates obtained some marks for this part. Almost all appreciated the need to resolve at the pin joints but sign errors were common with inconsistencies between equations and/or the diagram. A small number of candidates unwisely chose to resolve at A but did not appreciate that they had to include the force at the hinge.

4763 Mechanics 3

General Comments

This paper was found to be somewhat harder than the January 2008 paper. Nevertheless, there were many excellent scripts, demonstrating a sound knowledge of all the topics being examined, and about 30% of the candidates scored 60 marks or more (out of 72). Circular motion and simple harmonic motion continue to cause problems for a considerable number of candidates, and it was common for a candidate to lose most of the marks in question 2 or question 3, or both.

Comments on Individual Questions

1 Dimensional analysis

This question was answered very well indeed, with about 70% of the candidates scoring full marks.

- (i) Almost every candidate gave the dimensions of force and density correctly.
- (ii) Almost all candidates knew how to find the dimensions of viscosity, although there were a few slips here.
- (iii) This was also well done. Some candidates did not complete their argument convincingly, for example by failing to mention the dimensions of velocity, or by leaving in the numerical factor $2/9$. Those who had incorrect dimensions for viscosity usually asserted that the dimensions of their right-hand-side simplified to LT^{-1} when they clearly did not; but a few candidates used this part to correct a previous error.
- (iv) The method for finding the indices was well understood, although there were quite a number of slips (usually sign errors) made when forming and solving the simultaneous equations.
- (v) The use of the Reynolds number to calculate the velocity was very well understood, and almost every candidate who had the right formula for R was able to complete this correctly.

2 Circular motion

About one third of the candidates scored full marks on this question, and the average mark was 14 out of 19.

- (a)(i) The tension was very often found correctly, although there were many misunderstandings in this part. Some assumed that the two sections of the string were perpendicular, and some had only one term involving the tension.
- (ii) Those who had answered the first part correctly almost always found the speed correctly in this part. Similarly, any misunderstandings in the first part were usually repeated here.
- (b)(i) Almost all candidates could find the speed at the highest point correctly.
- (ii) Many candidates began this part by writing down the radial equation of motion (which is not required until part (iii)), but most then realised that they needed to apply conservation of energy. The energy equation often contained errors, such

as incorrect signs and sine and cosine muddles, in the potential energy term. Many candidates knew the formula $gsin\theta$ for the tangential component of acceleration and could apply it efficiently, although there were quite a few who tried to do something with the formula $r\dot{\omega}$ obtained from the formula book.

- (iii) Most candidates understood how to find the tension in the string.

3 Simple harmonic motion

Very few candidates scored full marks on this question, and the average mark was about 10 out of 19.

- (i) Almost every candidate found the extension correctly with the rock in equilibrium.
- (ii) Many candidates omitted this part, and some just quoted a formula such as $\omega^2 = \frac{1}{I_m}$. Candidates were expected to apply Newton's second law, showing the weight and the tension as separate terms; even when this was done, sign errors occurred quite frequently.
- (iii) The simplest approach was to use $v^2 = \omega^2(A^2 - x^2)$ to find the amplitude, but very few candidates did this. Most used energy, often successfully, but there were many errors such as using x in the gravitational or elastic terms where it should be $(1.25+x)$; sometimes one form of energy was completely omitted.
- (iv) Most candidates understood how to find the maximum speed, usually from the formula $A\omega$ (although some used energy again). Those with an incorrect amplitude often obtained an answer which was less than 8.4, but did not appear to realise that this must be wrong.
- (v) This part was found very difficult. The time can be found by solving the equation $3.25\cos 2.8t = -1.25$, but most of the successful candidates used much more complicated strategies. The most common response was to give one quarter of the period.
- (vi) The expected responses were: the rock is a particle; the rope is light; there is no air resistance; the rope obeys Hooke's law. However, few candidates could give more than two of these.

4 Centres of mass

About 15% of the candidates scored full marks on this question, and the average mark was 12 out of 18.

- (a) Most candidates obtained the centre of mass of the lamina correctly.
- (b)(i) Most candidates integrated correctly to obtain $\left(\frac{1}{3}\pi r^2 h\right)\bar{x} = \frac{1}{4}\pi m^2 h^4$. However, many did not see that $r=mh$, and so were unable to complete the proof.
- (ii) The method was well understood, but here a very common error was to take the distance from V of the centre of mass of the cone removed to be $\frac{3}{4} \times 1.1$ instead of $1.3 + \frac{3}{4} \times 1.1$. There was also some confusion about which end of the cone the $\frac{3}{4}h$ was measured from.
- (iii) This part was reasonably well answered, although quite a few put the right-angle in the triangle VQG at G instead of Q.

4766 Statistics 1 (G241 Z1)

General Comments

The level of difficulty of the paper appeared to be entirely appropriate for the candidates with a good range of marks obtained. It was very pleasing to note the performance of the more able candidates who scored highly on all questions. The presentation of work was good in the majority of cases.

Most candidates supported their numerical answers with appropriate explanations and working although some rounding errors were noted. The possible exception was in question 7 where the procedure for distinguishing between hypotheses was not always clear and where the construction of the critical region was occasionally sketchy. There was not much evidence of the efficient use of statistical calculations on a calculator with most candidates (even the most able) preferring to commit all the stages of the calculation to paper.

Weaker candidates often scored a significant proportion of their marks from the calculation of $E(X)$ and $\text{Var}(X)$ in question 3 and from the use of the cumulative frequency curve in question 6. Particularly amongst lower scoring candidates there was evidence of the use of point probabilities in question 7, possibly more so than in very recent papers.

Comments on Individual Questions

- 1 Few candidates scored full marks on this question. Many found the mean as 0.75 but omitted the units. A small number of candidates divided by 1000 not 10000 whilst a few found $\sum fx$ as 5110 (the value of $\sum x$). Many struggled to find the standard deviation correctly with errors including the use of $\sum fx^2$ as 70000, 25010000 or 29250000, or division by 10000 instead of 9999 although this error was less frequent than in the summer. There were a lot of answers around 50.2 from obviously incorrect working.

Fully correct answers to part (ii) were rare. There were many answers involving

$$\frac{50}{10000} \times \frac{50}{10000} + \frac{20}{10000} \times \frac{20}{10000} \quad (\text{with replacement}) \text{ instead of the correct}$$

$$\frac{50}{10000} \times \frac{49}{9999} + \frac{20}{10000} \times \frac{19}{9999} \quad (\text{without replacement}) \text{ whilst others wrote down the}$$

correct probability terms for two £10 prizes and for two £100 prizes but then failed to perform the necessary addition in order to gain the full marks. A small number attempted to use $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ or similar with a value for $P(A \text{ and } B)$.

- 2 Part (i) was often answered well although some candidates gave $(1/6)^6$ as the answer whilst others calculated $6! = 720$ but failed to convert it into the correct probability. Part (ii) did not produce the same success with wrong answers including $6/20$, $1/120$, $20/120$. Others found 6C_3 as 20 but then failed to use it correctly sometimes even using it as part of a binomial expression. Those using $\frac{1}{6} \times \frac{1}{5} \times \frac{1}{4}$ often forgot this could be arranged in $3!$ ways.

- 3 There were many excellent answers to parts (i) and (ii) even from the weaker candidates. The main error was to omit the subtraction of 1.352 in attempting to find the variance. Some of the weaker candidates squared the probabilities instead of r .

Candidates found part (iii) much more taxing with a substantial number not obtaining 0.35; of those that did, few went on to reach 0.35^2 . Some candidates made very heavy weather of this often failing to realise that they could just add the probabilities of 2, 3 and 4 to give the 0.35, for each occasion. Those who did often left it as this answer and failed to square it. Some calculated $1 - 0.65^2$ instead of $(1 - 0.65)^2$. Some considered the individual outcomes but apart from one or two they did not have all nine terms. Generally they had $0.052 + 0.052 + 0.252$. The other common wrong answer was $(0.6875)^2$. Some candidates multiplied by 2 instead of squaring 0.35.

Some tried to tackle the problem by complements, believing that $p(X \geq 2 \text{ on both occasions}) = 1 - p\{0, 0 \text{ or } 1, 0 \text{ or } 1, 1\}$. Very few realised that if they went down this protracted route then what was required was $1 - p\{0, 0 \text{ or } 0, 1 \text{ or } 0, 2 \text{ or } 0, 3 \text{ or } 0, 4 \text{ or } 1, 0 \text{ or } 1, 1 \text{ or } 1, 2 \text{ or } 1, 3 \text{ or } 1, 4 \text{ or } 2, 0 \text{ or } 2, 1 \text{ or } 3, 0 \text{ or } 3, 1 \text{ or } 4, 0 \text{ or } 4, 1\}$

- 4 The stronger candidates regularly scored full marks on this question. Otherwise the main errors in part (i) were the omission of ${}^{50}C_1$ or a miscalculation of a correct binomial expression. Attempts at part (ii) were less successful with a number of answers given as $1 - P(X = 0)$ or as $1 - P(X = 1)$ instead of $1 - P(X \leq 1)$. Most candidates gave the expectation correctly as $240 \times P(X = 1)$ although some still insisted in rounding their answer to an integer. There was the very occasional use of 50 or 12000 instead of 240.

- 5 Although a number of candidates scored full marks, there were some very mixed responses to this question. In part (i) the stronger candidates gave clear and precise reasons as to why the events were not independent either from comparing $P(R|L)$ with $P(R)$, or by comparing $P(R \text{ and } L)$ with $P(R) \times P(L)$. Others did not make the comparison clear, or compared $P(R|L)$ with $P(L)$, or having found that $P(R \text{ and } L)$ was not equal to $P(R) \times P(L)$ said that the events were independent.

The Venn diagram in part (ii) was often poorly answered with probabilities of 0.36, 0.2, 0.25 and 0.19 for the four regions common instead of the correct 0.16, 0.2, 0.05 and 0.59. Another less common error was to replace the correct probability of 0.59 with 0.39 or even 0.41.

Part (iii) produced many correct answers alongside errors such as 0.2/0.25 and 0.25/0.36. Most candidates understood that the expression represented a conditional probability but some failed to give an explanation in context.

6 There were many very good answers to this question with most candidates scoring a good proportion of the marks. It was decided that it would be fairer to candidates to award one extra mark in part (i) and one fewer in (iii). Virtually all used a correct method in part (i) to find the median. A common mistake was to write 4.7 for 4.07 and IQR with the occasional misread from the diagram. There was less success with part (ii) with answers often involving the median or a multiple of the IQR other than 1.5. Not all candidates appreciated the fact one of the boundaries for the outliers (3.05 and 5.05) lay within the data range and the other outside it.

In part (iii) only a few candidates stated that the outlier could be a valid data item but other sensible explanations were seen. The frequency table was often completed correctly and most candidates attempted to use the interval midpoints to estimate the mean with varying degrees of accuracy. The estimate of the mean in degrees Fahrenheit was well answered but the addition of 32 was a common error in attempts to find the standard deviation. Some started all over again causing them to waste time and effort by changing all the mid points to Fahrenheit. Invariably, errors occurred along the way.

7 There were some superb answers to this question with explanations showing a clear understanding of the methods involved. Many candidates, however, struggled with the hypothesis testing and critical region with some scoring marks (if any) only for the initial probabilities.

In part (i) (A) the probability that exactly 8 orders were delivered was usually tackled sensibly either by use of tables or from a binomial expression. The main errors were the use of $1 - 0.6242$ or the omission of ${}^{10}C_8$. Answers to part (B) were less successful with the omission of $P(X = 8)$ in summing probabilities, $1 - P(X = 8)$ or $1 - P(X \leq 8)$ being common mistakes.

In part (ii) many candidates did not define p correctly or omitted it; there also remain errors in the notation used such as $H_0 = 0.8$ or $H_0: P(X) = 0.8$. The use of point probabilities was the major error in the hypothesis test; other mistakes included the sole use of $P(X \leq 11) = 0.0513$ in attempting to distinguish between the two hypotheses and the lack of a conclusion in context.

Attempts at finding the critical region in part (iii) were spoilt by a variety of errors. These included a frequent use of point probabilities, a comparison with 0.05 instead of 0.025, not stating any comparison, a lower critical region omitting 0 and an upper critical region including 17. Some candidates thought they were still testing 12 packets but using a two-tailed test.

Throughout parts (ii) and (iii) many candidates were not precise with their notation by not distinguishing clearly between $<$, \leq and $=$, for example it was fairly common to see $P(X = 12) = 0.1329$ instead of $P(X \leq 12) = 0.1329$ which was then clarified by a written explanation or a diagram. Candidates who tried to answer the hypothesis test using line diagrams or bar charts were often imprecise in their statistical arguments. It is important that they back up their diagrams with clear references to tail probabilities and make it 100% clear which values are in the critical region.

4767 Statistics 2

General Comments

For the majority of candidates this proved to be a straightforward paper, with many high marks achieved. Most candidates demonstrated a good ability to carry out statistical tests and interpret results using appropriate language. It is pleasing to see candidates providing conclusions to their hypothesis tests which are not ‘too assertive’; this is a requirement in Statistics 3 but, at the moment, some flexibility is allowed in Statistics 2. On the whole, candidates scored well on all questions, but question 2 provided the toughest challenge.

Comments on Individual Questions

Section A

- 1) (i) Candidates were required to find the value of Spearman’s rank correlation coefficient from raw data. This produced full marks for most candidates. In addition to numerical mistakes, common errors included incorrect application of the formula - omitting 6 from $6 \times \sum d^2$ and failing to use ‘1 - ...’ were often seen. Very few candidates failed to attempt to rank the data.
- (ii) Most candidates scored well. On the whole, hypotheses were stated correctly, using the appropriate form of null hypothesis – H_0 : No association. Most candidates obtained the correct critical value, sensibly compared their test statistic from part (i) and made an appropriate conclusion. In keeping with previous sessions, the most common reason for losing marks involved failing to carefully specify the hypotheses, in context, to make it clear that the test was for association between city population size and average walking speed of pedestrians in the population.
- (iii) This part was well answered, with many candidates awarded full marks. In some cases, marks were lost through inaccurate working – e.g. giving the value of the gradient of the regression line correct only to one significant figure. Several candidates used x and y instead of w and t. Those candidates who obtained a positive value for the gradient of the regression line (which was clearly shown on the question paper as having a negative gradient) were more heavily penalised than those making minor errors in their calculation of the gradient.
- (iv) (A) Well answered. Most candidates gained both marks and were able to make a sensible comment in part (B).
- (iv) (B) Most candidates realised that using the regression line to estimate the maximum walking speed of a 10-year-old male constituted ‘extrapolation’ – some went on to provide comments that, in this case, it was not sensible due to physical development issues. Those candidates who provided a statistically based comment together with a pertinent contextual comment generally picked up both available marks.

- 2 (i) Well answered. Most candidates gained both marks. Some candidates jumped ahead, stating that the distribution was Poisson, making it difficult to ‘explain why X may be approximated by a Poisson distribution in part (ii).
- (ii) Well answered. Most candidates were awarded all three marks. Some candidates covered all bases, providing general comments to justify use of a Poisson distribution in its own right in addition to those supporting the Poisson approximation to the Binomial distribution. Numerical mistakes were rare.
- (iii) Fully correct answers were plentiful. Few candidates found $P(X = 0)$ rather than $1 - P(X = 0)$. Some used $1 - P(X = 1)$, which scored no marks.
- (iv) Many candidates scored full marks. Use of $X \sim Po(10)$ was seen regularly, leading to 0 marks for this part of the question.
- (v) Well answered. Many candidates felt the need to provide an integer answer; this was condoned provided that 7.87 was seen.
- (vi) Most candidates correctly obtained 0.55 for the mean of the data provided. Attempts to calculate the variance were poor with many failing to use the $(n - 1)$ divisor as required.
- (vii) Many candidates lost marks through failing to provide comments relating to the answer to part (v). Candidates were required to compare the expected number of samples containing at least one four-leaf clover with the observed number and to provide numerical values to show that they were comparing appropriate values. Very few good answers were seen. Many scored a mark for a sensible comment relating to their values for the mean and variance found in part (vi), although some compared mean with standard deviation.
- 3 (i) (A) Most candidates obtained full marks. Very few lost a mark by failing to work with the sufficient accuracy (i.e. making use of the ‘difference’ column in the Normal tables), even fewer failed to standardise correctly, commonly dividing by $\sqrt{\sigma}$ or σ^2 . However, many attempts at continuity corrections were seen.
- (i) (B) Many fully correct answers seen. Most managed to correctly standardise the ends of the given inequality, but many candidates made mistakes with the structure of the required probability calculation. Those applying continuity corrections lost at least the mark for accuracy.
- (i) (C) Well answered by many. Many candidates used a positive z value leading to a value of k greater than the mean, leading to a maximum of 1 mark out of 3. Those failing to use inverse Normal tables (i.e. those using probabilities in place of z values) were awarded no marks.
- (ii) Many candidates scored full marks. However, use of +1.036 instead of -1.036 was common and lead to a negative value for σ ; despite this, most candidates did not spot their error. Attempts to solve simultaneous equations were, on the whole, good; however, those failing to use inverse Normal tables scored no marks.

- (iii) This part caused problems for many, not least identifying suitable z values. Many candidates struggled to use their z value(s) in appropriate equations. However, many correct answers were seen (including non-symmetrical intervals).
- 4 (i) Most candidates scored a mark for providing correct hypotheses, although some failed to write the hypotheses in context. Despite being asked specifically, many candidates failed to ‘include a table of the contributions of each cell to the test statistic’, making it difficult to award marks for accurate working as often only the final X^2 value was given. Most candidates identified the correct number of degrees of freedom and the corresponding critical value, then went on to make an appropriate conclusion to the test. As mentioned in the general comments, it is encouraging to see phrases such as ‘the evidence suggests that’, rather than ‘this proves that’, appearing with increasing regularity.
- (ii) Well answered. In previous years, many candidates have failed to use the sampling distribution of means in their calculation of the test statistic when tackling questions such as this. This year, the vast majority of candidates scored well in this part. Some lost a mark for stating an incorrect critical value. Others lost marks for inappropriate comparisons (typically, comparing a z value with a probability). On the whole, the wording used in conclusions to hypothesis tests has vastly improved compared to previous years, although there are still some candidates who do not use any context in their conclusions and are penalised.

4768 Statistics 3

General Comments

There were 291 candidates from 40 centres (January 2008: 232 from 31) for this sitting of the paper. Overall the general standard of many of the scripts seen was about as good as in recent sessions. However, the work of possibly more candidates showed considerable carelessness. For example, time and again candidates would select the wrong critical value for a hypothesis test or state the final conclusions badly. As in the past, the quality of the comments, interpretations and explanations was patchy, and usually less good than the rest of the work.

Invariably all four questions were attempted. Marks for Questions 1 and 2 were found to be somewhat higher on average than Questions 3 and 4. There was no evidence to suggest that candidates were unable to complete the paper although they may have needed to rush at the end.

Comments on Individual Questions

- 1) Continuous random variables; Wilcoxon single sample test; times for Godfrey to complete his daily puzzle.
 - (a) Most candidates made a decent attempt at this half of the question and many fully correct solutions were seen. Those relatively few candidates who were not successful were usually struggling from the outset.
 - (i) It was pleasing to see the correct integral set up, including limits, and equated to 1. Almost always the correct expression for c was found.
 - (ii) Again the work seen here was usually correct and competent.
 - (iii) In this part it was very pleasing to find that candidates were able to obtain the given expression for the variance in a thorough, careful manner with sufficient working shown for it to be "convincing".
- (b) The Wilcoxon test was found by the vast majority to be straightforward and easily achieved. Only a handful of candidates tried to test the median using a t test.
- 2) Combinations of Normal distributions; t test for a population mean; paperweights.
 - (i) This part was answered correctly by almost everyone.
 - (ii) Except for a very occasional problem with the variance, this part, too, was almost always correct.
 - (iii) In this part most candidates coped very well, finding the variance of the total mass correctly and going on to interpret the requirement correctly. However, some (but not as many as in the past) did forget to square all parts fully when working out the variance of the mass. Also there were a number of candidates who interpreted $P(200 < X < 220)$ as $P(X < 220)$ and $P(X > 200)$, which were then combined in a variety of ways.

- (iv) Although the hypotheses for this test were not requested, many candidates followed good practice and stated them nonetheless. On this occasion it was particularly helpful to be able to see what they thought the question was asking. Much of the time the alternative hypothesis seen was " $H_1: \mu < 200$ ". However the work that followed was either mostly correct (up to deciding to "reject H_0 ") but giving the wrong conclusion in context or it broke down for some other reason (e.g. looking up the wrong critical value).

As for the conclusion in context, candidates might reasonably have reflected that a sample mean as high as 205.6 could not be taken to suggest that the intended reduction had been achieved. In other words, in a one-tail test significance at one end of a distribution does not imply significance at the other end. Even when the sense of the conclusion was correct, it was often flawed in the way it was expressed (e.g. being too assertive or omitting "mean").

- 3) Paired t test for the population mean reduction in hormone concentration; confidence interval for the true population mean.

Throughout this question marks were lost through carelessness and/or a lack of thoroughness.

- (i) This part was very poorly answered. There were only a very small handful of candidates who managed to say anything meaningful at all about why a paired test was being used. Usually the answer given was "because the data occur in pairs", with no thought as to why that was so.
- (ii) Answers to this part of the question were also disappointing. The hypotheses were badly expressed; words like "mean" and "population" were frequently missing. There seemed to be a widespread reluctance to use " μ " or " μ_D " for the population mean reduction/difference.

Similarly, the assumptions were poorly stated. For a paired test it is the population of differences that needs to be Normally distributed, not the "before" and "after" measurements, and, in order to avoid misunderstanding, candidates must be prepared to make clear which population they are referring to. Also, very many candidates overlooked the need for a random sample as one of the assumptions.

- (iii) There was much careless work at the start of this part of the question with candidates making mistakes finding the differences in concentration and/or the mean and standard deviation of the sample of differences. Then, as for the test in Question 2, there were errors finding the correct critical value from the tables and in expressing the final conclusion.
- (iv) Most candidates showed that they were familiar with the structure of a confidence interval. However, time after time, having found the correct percentage point and even related it to the correct entry in the tables for t_{14} they left their answers as "p = 5%", forgetting that the confidence interval would normally be described as "95%".

- 4) Sampling; Chi-squared test of goodness of fit of a given model; numbers of people asked to take part in a survey.
- (i) Most candidates could give a passable explanation of “opportunity sampling” and could suggest a disadvantage of using it. Far fewer were able to provide a convincing explanation of why one might end up using it; the tendency was to focus on “when” rather than “why”.
 - (ii) It was very disappointing to find that hardly any of the candidates appeared able to recognise, let alone write down the sum of, a geometric progression. A very common alternative approach was to substitute $(1 - p)$ for q , expand the binomial expressions and collect all the terms. Almost all attempts at this fell apart fairly quickly as candidates seemed unable to manage the expansions (beyond about $(1 - p)^2$) and the use of Pascal’s triangle. A third possible approach was to replace p by $(1 - q)$ and see all the resulting terms in q cancel out. This worked well and easily for the relatively few who tried it.
 - (iii) For the most part the expected frequencies and the value of the test statistic were calculated correctly, and only occasional inconsistencies in rounding results were noticed. Usually, but not always, the correct number of degrees of freedom and critical value followed. The final conclusion of the test was not as carefully expressed as it should have been. Some candidates thought they were testing a binomial model, others thought that they were testing “ $p = 0.25$ ” rather than a model in which $p = 0.25$.
 - (iv) Although many candidates came to the correct conclusion for the revised test, there were also many who did not think to adjust the critical value to allow for the loss of one degree of freedom. Hardly any candidates explained the change in outcome, i.e. the improved fit of the model, as a consequence of estimating a parameter from the data.

4771 Decision Mathematics 1

General Comments

This unit was marked on-screen for the first time. The process was facilitated by the answer book, which imposes a structure on candidates' work. Most candidates regard this as a support rather than as an imposition.

Comments on Individual Questions

1) Graphs

Most candidates scored well on the bipartite graph, but some failed to obtain full marks. Many of those "some" were unable to sort out the number of hands shaken by David.

In part (ii) there were many who scored on one of the deductions (e.g. If one person shakes 3 hands then nobody shakes zero), but not the converse (e.g. If one person shakes no hands then nobody shakes 3).

Few were able to make any useful progress with part (iii). Many comments were seen relating to the total of the node orders being even. That is proved by the handshaking lemma, but it has nothing to do with this question.

2) Algorithms

Most were able to run the algorithm.

Many gave incomplete justifications in part (ii), often failing to show the computation after substituting 5 for x . Decimal approximations were often seen, but exact computations were required. (Computations involving recurring decimals were accepted.)

A few candidates were unable to identify the cubic complexity in part (iii).

3) Networks

Most candidates were able to apply Dijkstra's algorithm, but there were too many who simply calculated assorted shortest paths. Those latter candidates did not gain credit. Many failed to answer the question in part (ii), often criticising the menu rather than the mathematical modelling.

4) Simulation

This question was answered well.

The simulations asked both for simulated individual weights and for the simulated gondola loading. Some candidates failed to provide one or the other.

The phrasing in part (v) ("the pattern of loading") required the candidates to identify the issue of the grouping of users, rather than the variability of weights.

5) CPA

Candidates found the activity network easy to construct. They also did well on the forward and backward passes. These looked easy, but were intended to deliver extra difficulty through multiple critical paths – hence the care with which reference is always made to "critical activities". Either the candidates did not notice this, or they just took it in their stride.

The "crashing" in part (iii) was much more difficult, but a sizeable minority of candidates managed it well.

Report on the Units taken in January 2009

6) LP

As is traditional, many candidates were unable to identify the variables in part (i), "months" being the favourite answer.

Only a very few candidates were able to score anything in part (ii). However, the structure of the question allowed candidates to continue into part (iii), and many did so – with varying degrees of success.

A sizeable minority failed to draw the $y = x$ line. Another sizeable minority drew the $y = x$ line at 45 degrees when the scales on their axes were not equal. Others only drew the two regions with negatively sloped boundaries.

Many gave no indication at all of how they obtained the optimum from their graph, and so gained no credit for obtaining (6, 6). Mirroring the difficulties with part (ii), few were able to interpret their solution to part (ii) in terms of the original three variables.

4776 Numerical Methods (Written Examination)

General Comments

There was a lot of good work seen on this occasion, with few candidates appearing to be unready for the examination. Routine numerical calculations were generally carried out accurately, though it disappointing that so many candidates still set their work out so badly. An unsystematic layout is difficult to follow for the examiner and the candidate.

It was quite common to find that candidates could not interpret correctly what they were doing; in many cases, no attempt at all was made at parts of questions requiring interpretation.

Comments on individual questions

1) Difference table for a quadratic

The difference table was constructed correctly by the majority, though some were inconsistent in their signs. The quadratic is best found using Newton's formula, but some chose to use Lagrange. The majority of attempts were successful, but the algebra to simplify the result defeated quite a few.

2) Difference of square roots

The algebra in part (i) proved tricky to some, though many saw it immediately as the difference of squares formula. Some candidates failed to see that the required result could be obtained as the reciprocal of $\sqrt{50001} + \sqrt{50000}$. The comment expected in part (iii) was that a more accurate result can be obtained by avoiding subtracting nearly equal quantities. Despite the fact that this idea has been tested before it proved to be beyond many.

3) Numerical integration

The numerical values were generally found accurately and efficiently. Bizarrely, it was common for a final answer of 0.77669 to be rounded to 0.77.

4) Approximation to $\cos x$

This question attracted many completely correct solutions. In particular, candidates seemed confident in their attempts at part (ii).

5) Fixed point iteration

This question was the most striking example of candidates carrying out the numerical calculations correctly but with little apparent understanding of what was going on. The iterations show that $x = 3$ is a root but that iterations starting near to $x = 3$ do not converge. The evaluation of the derivative shows that, at $x = 3$, the gradient of the function does not lie in the interval $[-1, 1]$. The link between these facts eluded the majority.

6) Numerical solution of an equation

The bisection method in part (i) was generally done well, though some candidates carried out too many or too few iterations. Others failed to state some of the required answers. The secant method in part (ii) was usually handled well also.

Part (iii), however, defeated many. They were asked to find the values of $x_r - \alpha$, but some chose to work with differences instead. Only a small minority seemed to know that, for second order convergence, $e_{r+1} \approx k e_r^2$.

7) Numerical differentiation

Once again, the numerical work was handled well in this question, with many candidates getting full marks in parts (i) and (ii). The algebra in part (iii) was seen to be very easy by some, but others spent a page more getting nowhere – or resorted to algebraic sleight of hand. The final part attracted quite a few good solutions, even from those who had not been able to do the algebra. However, it was quite common to see 0.500668 and 0.500695 rounded to 0.5006.

Coursework

Coursework

Administration

The Authentication form CCS160 remains a problem in a small number of centres – failure to deal with the completion of this form when the marks are submitted has caused considerable difficulties for the moderators and for the centres which has contributed to an unnecessary extra burden of time. It also helps the process considerably to have the paperwork for the Moderator complete. This includes the filling in of the cover sheets and especially candidate numbers. Most centres adhered to the deadline set by OCR very well and if the first despatch was only the MS1 then they responded rapidly to the sample request. A small minority, however, cause problems with the process by being late with the coursework despatch. We would ask that all centres heed the deadlines published by the Board and organise their own processes of assessment, internal moderation and administration to enable these deadlines to be met. The new on-line procedures established by OCR have meant that the process of sample selection and request via email has considerably improved the process and we are grateful to examination officers for their ability to deal with this new way of working.

The marks of the majority of centres were appropriate and acknowledgement is made of the amount of work that this involves to mark and internally moderate. The unit specific comments are offered for the sake of centres that have had their marks adjusted for some reason.

Assessors are asked to ensure that they record the criteria marks in such a way that the final mark on the cover sheet agrees with the submitted mark on the MS1 and is the sum of criteria marks. Some assessors, however, give only domain marks. This might be fine if the candidate deserves full marks (or zero!) for a domain, but it makes it very difficult for external moderators to understand the marking if a mark has been withheld – in this case we do not know which of the criteria have in the opinion of the assessor not been met adequately.

It is of concern to us that a few assessors used an incorrect cover sheet for the C3 coursework. OCR corrected the errors within weeks of the first publication of the current specification. Yet, after a number of years they are still being used. This implies that some centres do not note subsequent communications from OCR. Worse, it implies that there are assessors who do not take note of the report that is written and published after every series. Centres have been asked many times to destroy these incorrect sheets so it is distressing to find that some have not heeded this request. This report, however, is more than about correct or incorrect cover sheets, and it is hoped that teachers will note the content for the benefit of their own students. These reports should provide a valuable aid to the teaching of the topics and the marking process and we would urge all Heads of Departments to ensure that these reports are read by all those involved in the assessment of coursework.

Core 3 – 4753/02

The marking scheme for this unit is very prescriptive and often there is more than one point for assessment in a single criterion with one mark. It is therefore appropriate to deduct half a mark for each error or omission. One or two of these cause no difficulty, but there are many centres where so many of the points outlined below are not being penalised appropriately that the mark submitted is too generous.

The following points should typically be penalised by half a mark – failure to penalise four or more results in a mark outside tolerance.

Change of Sign

- Graphs of the function being used do not constitute an illustration of the method. Graphs should therefore be annotated in some way, or more than one “zooming in” to the root should be done.
- It is expected that a root be given and error bounds stated – often it is only the latter that is given.
- Use of trivial equations to demonstrate failure.
- Tables of values which actually find the root.
- Graphs which candidates claim cross the axis or just touch but do not.

Newton Raphson

- If a candidate uses an equation with only one root then the second mark is not accessible. Assessors should not give this mark when there is no work done!
- Simply a statement of the root with no iterates given.
- No work done by the candidate – just a print out from “Autograph”.
- As with the change of sign method, a graph of the function with one tangent drawn by “Autograph” is not a satisfactory illustration of the method.
- Error bounds not established by a change of sign.
- Failures lacking iterates.
- Starting values too far away from the root or too artificial.

Rearrangement

- Graphs not explained.
- We have found this session an increasing number of candidates simply differentiating $g(x)$ and quoting the criterion for convergence of $-1 < g'(x) < 1$. It is expected that the gradient will be related to the illustration of the curve $y = g(x)$ and the line $y = x$. This criterion can be met without any differentiation.

Comparison

- Discussions which are very thin, and do not relate to the software actually being used.
- Different starting values.
- Sometimes different roots are found.
- Different degrees of accuracy.
- Not quoting number of steps to reach given accuracy.

Notation

- Equations, functions, expressions still cause confusion to candidates and teachers! Candidates who assert that they are going to solve $y = x^3 + 2x + 3$ or that they are going to solve $x^3 + 2x + 3 = 0$ should be penalised.

Oral

The specification asks for a written report.

Differential Equations – 4758/02

As is the usual pattern of entry, only a small number of centres submitted work this series. Therefore any generalisations may be a little misleading.

The essential function of the coursework element of this module is to test the candidate's ability to follow the modelling cycle. That is, setting up a model, testing it and then modifying the assumptions to improve the original model. If two or three models are suggested at the outset and tested, more or less simultaneously, and the best chosen, then the modelling cycle has not been followed. Likewise, if a second model is proposed with no review of the assumptions then the process becomes a curve fitting exercise instead of a modelling cycle.

Too often the assumptions are simply listed with little or no discussion as to their relevance and importance. These are an essential part of the modelling process. In fact, a lack of understanding of their importance often leads to the task becoming a curve fitting exercise.

In Domain 5, for instance and in particular for 'Cascades', there is, sometimes, little justification for the modified model.

For 'Aeroplane Landing', (still the most popular task) marks often seem to be automatically allocated for Domain 3 (Collection of data) when there is little discussion of the source or potential accuracy of the data. It is expected also that the first model will be worked for the whole of the motion. Quite often candidates only work on the first part of the motion, then improve the model before dealing with the second part.

It should be ensured that the axes on all graphs are labelled and it is made clear whether the graph represents the predicted or experimental data or both.

Numerical Methods – 4776/02

We still find cases where incorrect work has been ticked. Assessors are requested not to tick work unless it has been checked thoroughly.

The most popular task is to find the value of an integral numerically. The following comments are offered on this particular task.

Domain 1.

Not all candidates fulfil the basic requirement of a formal statement of the problem.

Domain 2

Part of the criteria for this domain is to describe what method is to be used and why. Many candidates often omit this second part of the requirement, but are given credit.

Domain 3

Finding numerical values for the mid-point rule (for instance) up to M_{64} is deemed to be substantial.

Domain 4

A clear description of how the algorithm has been implemented is required, usually by presenting an annotated spreadsheet printout.

Domain 5

One of the problems of not taking the algorithms to, say, M_{64} is that the pattern of the ratio of differences is not established rigorously enough. Using a value to which the ratios are assumed to be converging with the earlier results will produce answers that are not accurate. Furthermore, it is not always the case that the theoretical values are being achieved so it is important to take the process far enough to be confident of the value to which the ratios are converging. Those that make assumptions without finding the ratio of differences should not, of course, be credited with full marks in this domain.

Domain 6

Most of the marks in this domain are dependent on satisfactory work in the error analysis domain and so often a rather generous assessment of that domain led also to a rather generous assessment here as well.

Grade Thresholds

Advanced GCE MEI Mathematics 3895-8/7895-8
January 2009 Examination Series

Unit Threshold Marks

Unit		Maximum Mark	A	B	C	D	E	U
All units	UMS	100	80	70	60	50	40	0
4751	Raw	72	61	53	45	37	30	0
4752	Raw	72	60	53	46	40	34	0
4753/01	Raw	72	61	54	47	40	32	0
4753/02	Raw	18	15	13	11	9	8	0
4754	Raw	90	75	66	57	49	41	0
4755	Raw	72	57	49	41	33	26	0
4756	Raw	72	53	47	42	37	32	0
4758/01	Raw	72	61	53	45	37	29	0
4758/02	Raw	18	15	13	11	9	8	0
4761	Raw	72	58	50	42	34	27	0
4762	Raw	72	57	49	41	33	26	0
4763	Raw	72	53	46	39	32	25	0
4766/G241	Raw	72	57	48	40	32	24	0
4767	Raw	72	60	52	45	38	31	0
4768	Raw	72	53	46	39	33	27	0
4771	Raw	72	57	51	45	39	33	0
4776/01	Raw	72	56	49	43	37	30	0
4776/02	Raw	18	14	12	10	8	7	0

Specification Aggregation Results

Overall threshold marks in UMS (ie after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
3895-3898	300	240	210	180	150	120	0
7895-7898	600	480	420	360	300	240	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
3895	18.3	43.5	65.4	83.8	96.0	100.0	640
3896	39.2	58.8	78.4	86.3	96.1	100.0	94
3897	100.0	100.0	100.0	100.0	100.0	100.0	1
7895	22.2	57.6	81.7	93.0	98.1	100.0	186
7896	18.8	56.3	87.5	87.5	93.8	100.0	16

For a description of how UMS marks are calculated see:

http://www.ocr.org.uk/learners/ums_results.html

Statistics are correct at the time of publication.

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)

Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553

