OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4757

Further Applications of Advanced Mathematics (FP3)

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Monday 1 June 2009
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Option 1: Vectors

1 The point A $(-1, 12, 5)$ lies on the plane P with equation $8x - 3y + 10z = 6$. The point B $(6, -2, 9)$ lies on the plane Q with equation $3x - 4y - 2z = 8$. The planes P and Q intersect in the line L.

(i) Find an equation for the line L. **[5]**

(ii) Find the shortest distance between L and the line AB. **[6]**

The lines M and N are both parallel to L, with M passing through A and N passing through B.

(iii) Find the distance between the parallel lines M and N. **[5]**

The point C has coordinates $(k, 0, 2)$, and the line AC intersects the line N at the point D.

(iv) Find the value of k, and the coordinates of D. **[8]**

Option 2: Multi-variable calculus

2 A surface has equation $z = 3x(x + y)^3 - 2x^3 + 24x$.

(i) Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$. **[4]**

(ii) Find the coordinates of the three stationary points on the surface. **[7]**

(iii) Find the equation of the normal line at the point P $(1, -2, 19)$ on the surface. **[3]**

(iv) The point Q $(1 + k, -2 + h, 19 + 3h)$ is on the surface and is close to P. Find an approximate expression for k in terms of h. **[4]**

(v) Show that there is only one point on the surface at which the tangent plane has an equation of the form $27x - z = d$. Find the coordinates of this point and the corresponding value of d. **[6]**

Option 3: Differential geometry

3 A curve has parametric equations $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$, for $0 \leqslant \theta \leqslant \pi$, where a is a positive constant.

(i) Show that the arc length s from the origin to a general point on the curve is given by $s = 4a\sin\frac{1}{2}\theta$. **[6]**

(ii) Find the intrinsic equation of the curve giving s in terms of a and ψ, where $\tan\psi = \dfrac{\mathrm{d}y}{\mathrm{d}x}$. **[4]**

(iii) Hence, or otherwise, show that the radius of curvature at a point on the curve is $4a\cos\frac{1}{2}\theta$. **[3]**

(iv) Find the coordinates of the centre of curvature corresponding to the point on the curve where $\theta = \frac{2}{3}\pi$. **[6]**

(v) Find the area of the surface generated when the curve is rotated through 2π radians about the x-axis. **[5]**

Option 4: Groups

4 The group $G = \{1, 2, 3, 4, 5, 6\}$ has multiplication modulo 7 as its operation. The group $H = \{1, 5, 7, 11, 13, 17\}$ has multiplication modulo 18 as its operation.

(i) Show that the groups G and H are both cyclic. **[4]**

(ii) List all the proper subgroups of G. **[3]**

(iii) Specify an isomorphism between G and H. **[4]**

The group $S = \{\mathrm{a, b, c, d, e, f}\}$ consists of functions with domain $\{1, 2, 3\}$ given by

$$\begin{array}{lll} \mathrm{a}(1) = 2 & \mathrm{a}(2) = 3 & \mathrm{a}(3) = 1 \\ \mathrm{b}(1) = 3 & \mathrm{b}(2) = 1 & \mathrm{b}(3) = 2 \\ \mathrm{c}(1) = 1 & \mathrm{c}(2) = 3 & \mathrm{c}(3) = 2 \\ \mathrm{d}(1) = 3 & \mathrm{d}(2) = 2 & \mathrm{d}(3) = 1 \\ \mathrm{e}(1) = 1 & \mathrm{e}(2) = 2 & \mathrm{e}(3) = 3 \\ \mathrm{f}(1) = 2 & \mathrm{f}(2) = 1 & \mathrm{f}(3) = 3 \end{array}$$

and the group operation is composition of functions.

(iv) Show that $\mathrm{ad} = \mathrm{c}$ and find da. **[4]**

(v) Give a reason why S is not isomorphic to G. **[1]**

(vi) Find the order of each element of S. **[4]**

(vii) List all the proper subgroups of S. **[4]**

[Question 5 is printed overleaf.]

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

5 Each level of a fantasy computer game is set in a single location, Alphaworld, Betaworld, Chiworld or Deltaworld. After completing a level, a player goes on to the next level, which could be set in the same location as the previous level, or in a different location.

In the first version of the game, the initial and transition probabilities are as follows.

Level 1 is set in Alphaworld or Betaworld, with probabilities 0.6, 0.4 respectively.

After a level set in Alphaworld, the next level will be set in Betaworld, Chiworld or Deltaworld, with probabilities 0.7, 0.1, 0.2 respectively.

After a level set in Betaworld, the next level will be set in Alphaworld, Betaworld or Deltaworld, with probabilities 0.1, 0.8, 0.1 respectively.

After a level set in Chiworld, the next level will also be set in Chiworld.

After a level set in Deltaworld, the next level will be set in Alphaworld, Betaworld or Chiworld, with probabilities 0.3, 0.6, 0.1 respectively.

The situation is modelled as a Markov chain with four states.

(i) Write down the transition matrix. **[2]**

(ii) Find the probabilities that level 14 is set in each location. **[3]**

(iii) Find the probability that level 15 is set in the same location as level 14. **[3]**

(iv) Find the level at which the probability of being set in Chiworld first exceeds 0.5. **[3]**

(v) Following a level set in Betaworld, find the expected number of further levels which will be set in Betaworld before changing to a different location. **[3]**

In the second version of the game, the initial probabilities and the transition probabilities after Alphaworld, Betaworld and Deltaworld are all the same as in the first version; but after a level set in Chiworld, the next level will be set in Chiworld or Deltaworld, with probabilities 0.9, 0.1 respectively.

(vi) By considering powers of the new transition matrix, or otherwise, find the equilibrium probabilities for the four locations. **[5]**

In the third version of the game, the initial probabilities and the transition probabilities after Alphaworld, Betaworld and Deltaworld are again all the same as in the first version; but the transition probabilities after Chiworld have changed again. The equilibrium probabilities for Alphaworld, Betaworld, Chiworld and Deltaworld are now 0.11, 0.75, 0.04, 0.1 respectively.

(vii) Find the new transition probabilities after a level set in Chiworld. **[5]**