OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI) 4776/01

Numerical Methods

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Monday 24 May 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

1 **(i)** Show that the equation

$$\frac{1}{x} = 3 - x^2 \qquad (*)$$

has a root, α, between $x = 1$ and $x = 2$.

Show that the iteration

$$x_{r+1} = \frac{1}{3 - x_r^2},$$

with $x_0 = 1.5$, converges, but not to α. **[5]**

(ii) By rearranging (*), find another iteration that does converge to α. You should demonstrate the convergence by carrying out several steps of the iteration. **[3]**

2 A function f(x) has the values shown in the table.

x	2.8	3	3.2
f(x)	0.9508	0.9854	0.9996

(i) Taking the values of f(x) to be exact, use the forward difference method and the central difference method to find two estimates of $f'(3)$. State which of these you would expect to be more accurate. **[5]**

(ii) Now suppose that the values of f(x) have been rounded to the four significant figures shown. Find, for each method used in part **(i)**, the largest possible value it gives for the estimate of $f'(3)$. **[2]**

3 **(i)** X is an approximation to the number x such that $X = x(1 + r)$. State what r represents.

Show that, provided r is small, $X^n \approx x^n(1 + nr)$. **[4]**

(ii) The number $G = 0.577$ is an approximation to the number g. G is about 0.04% smaller than g. State, in similar terms, relationships between

(A) G^2 and g^2,

(B) $\sqrt{G}$ and $\sqrt{g}$. **[3]**

4 The expression, $\sin x + \tan x$, where x is in radians, can be approximated by $2x$ for values of x close to zero.

(i) Find the absolute and relative errors in this approximation when $x = 0.2$ and $x = 0.1$. **[4]**

(ii) A better approximation is $\sin x + \tan x \approx 2\left(x + \frac{x^3}{k}\right)$, where k is an integer.

Use your results from part **(i)** to estimate k. **[3]**

5 A quadratic function, f(x), is to be determined from the values shown in the table.

x	1	3	6
f(x)	–10	–12	30

Explain why Newton's forward difference formula would not be useful in this case.

Use Lagrange's interpolation formula to find f(x) in the form $ax^2 + bx + c$. **[7]**

Section B (36 marks)

6 The integral

$$I = \int_1^{1.8} \sqrt{x^3 + 1}\ \mathrm{d}x$$

is to be estimated numerically. You are given that, correct to 6 decimal places, the mid-point rule estimate with $h = 0.8$ is 1.547 953 and that the trapezium rule estimate with $h = 0.8$ is 1.611 209.

(i) Find the mid-point rule and trapezium rule estimates with $h = 0.4$ and $h = 0.2$.

Hence find three Simpson's rule estimates of I. **[7]**

(ii) Write down, with a reason, the value of I to the accuracy that appears to be justified. **[2]**

(iii) Taking your answer in part **(ii)** to be exact, show in a table the errors in the mid-point rule and trapezium rule estimates of I.

Explain what these errors show about

(A) the relative accuracy of the mid-point rule and the trapezium rule,

(B) the rates of convergence of the mid-point rule and the trapezium rule. **[8]**

7 **(i)** Show that the equation

$$x^5 - 8x + 5 = 0 \qquad (*)$$

has a root in the interval (0, 1).

Find this root, using the Newton-Raphson method, correct to 6 significant figures.

Show, by considering the differences between successive iterates, that the convergence of the Newton-Raphson iteration is faster than first order. **[11]**

(ii) You are now given that equation (*) has a root in the interval (1.4, 1.5). Find this root, correct to 3 significant figures, using the secant method. Determine whether or not the secant method is faster than first order. **[8]**

THERE ARE NO QUESTIONS ON THIS PAGE

