

**ADVANCED SUBSIDIARY GCE**  
**MATHEMATICS (MEI)**  
Numerical Methods

**4776/01**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

**Other Materials Required:**

- Scientific or graphical calculator

**Monday 24 May 2010**  
**Afternoon**

**Duration: 1 hour 30 minutes**



**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

## Section A (36 marks)

- 1 (i) Show that the equation

$$\frac{1}{x} = 3 - x^2 \quad (*)$$

has a root,  $\alpha$ , between  $x = 1$  and  $x = 2$ .

Show that the iteration

$$x_{r+1} = \frac{1}{3 - x_r^2},$$

with  $x_0 = 1.5$ , converges, but not to  $\alpha$ . [5]

- (ii) By rearranging (\*), find another iteration that does converge to  $\alpha$ . You should demonstrate the convergence by carrying out several steps of the iteration. [3]

- 2 A function  $f(x)$  has the values shown in the table.

$x$	2.8	3	3.2
$f(x)$	0.9508	0.9854	0.9996

- (i) Taking the values of  $f(x)$  to be exact, use the forward difference method and the central difference method to find two estimates of  $f'(3)$ . State which of these you would expect to be more accurate. [5]

- (ii) Now suppose that the values of  $f(x)$  have been rounded to the four significant figures shown. Find, for each method used in part (i), the largest possible value it gives for the estimate of  $f'(3)$ . [2]

- 3 (i)  $X$  is an approximation to the number  $x$  such that  $X = x(1 + r)$ . State what  $r$  represents.

Show that, provided  $r$  is small,  $X^n \approx x^n(1 + nr)$ . [4]

- (ii) The number  $G = 0.577$  is an approximation to the number  $g$ .  $G$  is about 0.04% smaller than  $g$ . State, in similar terms, relationships between

(A)  $G^2$  and  $g^2$ ,

(B)  $\sqrt{G}$  and  $\sqrt{g}$ . [3]

- 4 The expression,  $\sin x + \tan x$ , where  $x$  is in radians, can be approximated by  $2x$  for values of  $x$  close to zero.

- (i) Find the absolute and relative errors in this approximation when  $x = 0.2$  and  $x = 0.1$ . [4]

- (ii) A better approximation is  $\sin x + \tan x \approx 2 \left( x + \frac{x^3}{k} \right)$ , where  $k$  is an integer.

Use your results from part (i) to estimate  $k$ . [3]

- 5 A quadratic function,  $f(x)$ , is to be determined from the values shown in the table.

$x$	1	3	6
$f(x)$	-10	-12	30

Explain why Newton's forward difference formula would not be useful in this case.

Use Lagrange's interpolation formula to find  $f(x)$  in the form  $ax^2 + bx + c$ . [7]

### Section B (36 marks)

- 6 The integral

$$I = \int_1^{1.8} \sqrt{x^3 + 1} \, dx$$

is to be estimated numerically. You are given that, correct to 6 decimal places, the mid-point rule estimate with  $h = 0.8$  is 1.547 953 and that the trapezium rule estimate with  $h = 0.8$  is 1.611 209.

- (i) Find the mid-point rule and trapezium rule estimates with  $h = 0.4$  and  $h = 0.2$ .

Hence find three Simpson's rule estimates of  $I$ . [7]

- (ii) Write down, with a reason, the value of  $I$  to the accuracy that appears to be justified. [2]

- (iii) Taking your answer in part (ii) to be exact, show in a table the errors in the mid-point rule and trapezium rule estimates of  $I$ .

Explain what these errors show about

(A) the relative accuracy of the mid-point rule and the trapezium rule,

(B) the rates of convergence of the mid-point rule and the trapezium rule. [8]

- 7 (i) Show that the equation

$$x^5 - 8x + 5 = 0 \quad (*)$$

has a root in the interval  $(0, 1)$ .

Find this root, using the Newton-Raphson method, correct to 6 significant figures.

Show, by considering the differences between successive iterates, that the convergence of the Newton-Raphson iteration is faster than first order. [11]

- (ii) You are now given that equation (\*) has a root in the interval  $(1.4, 1.5)$ . Find this root, correct to 3 significant figures, using the secant method. Determine whether or not the secant method is faster than first order. [8]

**THERE ARE NO QUESTIONS ON THIS PAGE**



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