

Mathematics

Advanced Subsidiary GCE 4722

Core Mathematics 2

Mark Scheme for June 2010

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Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

1 (i)	$f(2) = 8 + 4a - 2a - 14$ $2a - 6 = 0$ $a = 3$	M1*	Attempt f(2) or equiv, including inspection / long division / coefficient matching	
		M1d*	Equate attempt at f(2), or attempt at remainder, to 0 and attempt to solve	
		A1	Obtain $a = 3$	3
(ii)	$f(-1) = -1 + 3 + 3 - 14$ $= -9$	M1	Attempt f(-1) or equiv, including inspection / long division / coefficient matching	
		A1 ft	Obtain -9 (or $2a - 15$, following their a)	2
5				
2 (i)	$\text{area} \approx \frac{1}{2} \times 3 \times (\sqrt[3]{8} + 2(\sqrt[3]{11} + \sqrt[3]{14}) + \sqrt[3]{17})$ ≈ 20.8	B1	State or imply at least 3 of the 4 correct y -coords, and no others	
		M1	Use correct trapezium rule, any h , to find area between $x = 1$ and $x = 10$	
		M1	Correct h (soi) for their y -values – must be at equal intervals	
A1	Obtain 20.8 (allow 20.7)	4		
(ii)	use more strips / narrower strips	B1	Any mention of increasing n or decreasing h	1
5				
3 (i)	$(1 + \frac{1}{2}x)^{10} = 1 + 5x + 11.25x^2 + 15x^3$	B1	Obtain $1 + 5x$	
		M1	Attempt at least the third (or fourth) term of the binomial expansion, including coeffs	
		A1	Obtain $11.25x^2$	
		A1	Obtain $15x^3$	
4				
(ii)	$\text{coeff of } x^3 = (3 \times 15) + (4 \times 11.25) + (2 \times 5)$ $= 100$	M1	Attempt at least one relevant term, with or without powers of x	
		A1 ft	Obtain correct (unsimplified) terms (not necessarily summed) – either coefficients or still with powers of x involved	
		A1	Obtain 100	3
7				

4 (i)	$u_1 = 6, u_2 = 11, u_3 = 16$	B1	1	State 6, 11, 16
(ii)	$S_{40} = \frac{40}{2} (2 \times 6 + 39 \times 5)$ $= 4140$	M1		Show intention to sum the first 40 terms of a sequence
		M1		Attempt sum of their AP from (i), with $n = 40$, $a =$ their u_1 and $d =$ their $u_2 - u_1$
		A1	3	Obtain 4140
(iii)	$w_3 = 56$ $5p + 1 = 56$ or $6 + (p - 1) \times 5 = 56$ $p = 11$	B1		State or imply $w_3 = 56$
		M1		Attempt to solve $u_p = k$
		A1	3	Obtain $p = 11$
				7
5 (i)	$\frac{\sin \theta}{8} = \frac{\sin 65}{11}$ $\theta = 41.2^\circ$	M1		Attempt use of correct sine rule
		A1	2	Obtain 41.2° , or better
(ii) a	$180 - (2 \times 65) = 50^\circ$ or $65 \times \frac{\pi}{180} = 1.134$ $50 \times \frac{\pi}{180} = 0.873$ A.G. $\pi - (2 \times 1.134) = 0.873$	M1		Use conversion factor of $\frac{\pi}{180}$
		A1	2	Show 0.873 radians convincingly (AG)
(ii) b	area sector = $\frac{1}{2} \times 8^2 \times 0.873 = 27.9$ area triangle = $\frac{1}{2} \times 8^2 \times \sin 0.873 = 24.5$ area segment = $27.9 - 24.5$ $= 3.41$	M1		Attempt area of sector, using $(\frac{1}{2}) r^2 \theta$
		M1		Attempt area of triangle using $(\frac{1}{2}) r^2 \sin \theta$
		M1		Subtract area of triangle from area of sector
		A1	4	Obtain 3.41 or 3.42
				8

6 a	$\int_3^5 (x^2 + 4x) dx = \left[\frac{1}{3}x^3 + 2x^2 \right]_3^5$ $= \left(\frac{125}{3} + 50 \right) - (9 + 18)$ $= 64 \frac{2}{3}$	M1	Attempt integration
		A1	Obtain $\frac{1}{3}x^3 + 2x^2$
		M1	Use limits $x = 3, 5$ – correct order & subtraction
		A1	4 Obtain $64 \frac{2}{3}$ or any exact equiv
b	$\int (2 - 6\sqrt{y}) dy = 2y - 4y^{\frac{3}{2}} + c$	B1	State $2y$
		M1	Obtain $ky^{\frac{3}{2}}$
		A1	3 Obtain $-4y^{\frac{3}{2}}$ (condone absence of $+c$)
c	$\int_1^{\infty} 8x^{-3} dx = \left[\frac{-4}{x^2} \right]_1^{\infty}$ $= (0) - (-4)$ $= 4$	B1	State or imply $\frac{1}{x^3} = x^{-3}$
		M1	Attempt integration of kx^n
		A1	Obtain correct $-4x^{-2}$ ($+c$)
		A1 ft	4 Obtain 4 (or $-k$ following their kx^{-2})
11			
7 (i)	$\frac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} = \frac{\sin^2 x - \cos^2 x}{\cos^2 x}$ $= \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x}$ $= \tan^2 x - 1$	M1	Use either $\sin^2 x + \cos^2 x = 1$, or $\tan x = \frac{\sin x}{\cos x}$
		A1	2 Use other identity to obtain given answer convincingly.
(ii)	$\tan^2 x - 1 = 5 - \tan x$ $\tan^2 x + \tan x - 6 = 0$ $(\tan x - 2)(\tan x + 3) = 0$ $\tan x = 2, \tan x = -3$ $x = 63.4^\circ, 243^\circ \quad x = 108^\circ, 288^\circ$	B1	State correct equation
		M1	Attempt to solve three term quadratic in $\tan x$
		A1	Obtain 2 and -3 as roots of their quadratic
		M1	Attempt to solve $\tan x = k$ (at least one root)
		A1ft	Obtain at least 2 correct roots
		A1	6 Obtain all 4 correct roots
8			

<p>8 a $\log 5^{3w-1} = \log 4^{250}$</p> <p>$(3w-1)\log 5 = 250 \log 4$</p> <p>$3w-1 = \frac{250\log 4}{\log 5}$</p> <p>$w = 72.1$</p>	<p>M1*</p> <p>M1*</p> <p>A1</p> <p>M1d*</p> <p>A1</p>	<p>Introduce logarithms throughout</p> <p>Use $\log a^b = b \log a$ at least once</p> <p>Obtain $(3w-1)\log 5 = 250 \log 4$ or equiv</p> <p>Attempt solution of linear equation</p> <p>Obtain 72.1, or better</p>
<p>b $\log_x \frac{5y+1}{3} = 4$</p> <p>$\frac{5y+1}{3} = x^4$</p> <p>$5y+1 = 3x^4$</p> <p>$y = \frac{3x^4-1}{5}$</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p>	<p>Use $\log a - \log b = \log \frac{a}{b}$ or equiv</p> <p>Use $f(y) = x^4$ as inverse of $\log_x f(y) = 4$</p> <p>Attempt to make y the subject of $f(y) = x^4$</p> <p>Obtain $y = \frac{3x^4-1}{5}$, or equiv</p>
<p>9 (i) $ar = a + d, ar^3 = a + 2d$</p> <p>$2ar - ar^3 = a$</p> <p>$ar^3 - 2ar + a = 0$</p> <p>$r^3 - 2r + 1 = 0$ A.G.</p>	<p>M1</p> <p>M1</p> <p>A1</p>	<p>Attempt to link terms of AP and GP, implicitly or explicitly.</p> <p>Attempt to eliminate d, implicitly or explicitly, to show given equation.</p> <p>Show $r^3 - 2r + 1 = 0$ convincingly</p>
<p>(ii) $f(r) = (r-1)(r^2 + r - 1)$</p> <p>$r = \frac{-1 \pm \sqrt{5}}{2}$</p> <p>Hence $r = \frac{-1 + \sqrt{5}}{2}$</p>	<p>B1</p> <p>M1*</p> <p>A1</p> <p>M1d*</p> <p>A1</p>	<p>Identify $(r-1)$ as factor or $r=1$ as root</p> <p>Attempt to find quadratic factor</p> <p>Obtain $r^2 + r - 1$</p> <p>Attempt to solve quadratic</p> <p>Obtain $r = \frac{-1 + \sqrt{5}}{2}$ only</p>
<p>(iii) $\frac{a}{1-r} = 3 + \sqrt{5}$</p> <p>$a = \left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)(3 + \sqrt{5})$</p> <p>$a = 9/2 - 5/2$</p> <p>$a = 2$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>Equate S_∞ to $3 + \sqrt{5}$</p> <p>Obtain $\frac{a}{1 - \left(\frac{-1 + \sqrt{5}}{2}\right)} = 3 + \sqrt{5}$</p> <p>Attempt to find a</p> <p>Obtain $a = 2$</p>

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

14 – 19 Qualifications (General)

Telephone: 01223 553998

Facsimile: 01223 552627

Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

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