

**Mathematics**

Advanced GCE 4725

Further Pure Mathematics 1

**Mark Scheme for June 2010**

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<b>1</b>	B1 Establish result true for $n = 1$ or $n = 2$ M1 Add next term to given sum formula M1 Attempt to factorise or expand and simplify to correct expression A1 Correct expression obtained A1 <b>5</b> Specific statement of induction conclusion
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<b>2 (i)</b>	(-7)  BA = $\begin{pmatrix} 5 & -20 \\ 3 & -12 \end{pmatrix}$  $\begin{pmatrix} -7 & -20 \\ 11 & -20 \end{pmatrix}$	M1 Obtain a single value A1 <b>2</b> Obtain correct answer as a matrix
<b>(ii)</b>	Obtain a $2 \times 2$ matrix  All elements correct  4C seen or implied by correct answer  Obtain correct answer, ft for a slip in BA	M1 A1 B1 B1ft <b>4</b>

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<b>3</b>	Either  $\frac{2}{3}n(n+1)(2n+1) - 2n(n+1) + n$  $\frac{1}{3}n(2n-1)(2n+1)$ <b>Or</b> $\sum_{r=1}^{2n} r^2 - 4 \sum_{r=1}^n r^2$  $\frac{1}{6} \times 2n(2n+1)(4n+1) - 4 \times \frac{1}{6}n(n+1)(2n+1)$  $\frac{1}{3}n(2n-1)(2n+1)$	M1 Express as a sum of 3 terms M1 Use standard sum results  A1 Correct unsimplified answer  M1 Attempt to factorise A1 Obtain at least factor of $n$ and a quadratic A1 <b>6</b> Obtain correct answer a.e.f.  M1 Express as difference of 2 $\sum r^2$ series M1 Use standard result  A1 Correct unsimplified answer M1 Attempt to factorise A1 Obtain at least factor of $n$  A1 Obtain correct answer
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- 4 (i)  $5 + 12i$  B1B1 Correct real and imaginary parts  
 $13$  B1ft Correct modulus  
 $67.4^\circ$  or  $1.18$  B1ft 4 Correct argument

(ii)

- M1 Multiply by conjugate  
 A1 Obtain correct numerator

$$-\frac{11}{85} - \frac{27}{85}i$$

- A1 3 Obtain correct denominator

7
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- 5 (a)  $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$  B1B12 Each column correct  
**SC B2 use correct matrix from MF1**  
**Can be trig form**

(b) (i)  
 (ii)

- B1B12 Stretch, in  $x$ -direction sf 5  
 B1B12 Rotation,  $60^\circ$  clockwise

6
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- 6 (i) (a) B1B12 Circle centre  $(3, -4)$ , through origin  
 (b) B1B12 Vertical line, clearly  $x = 3$

(ii)

- B1ft Inside their circle  
 B1ft 2 And to right of their line, if vertical

6
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7	<p><i>Either</i>  <math>\alpha + \beta = -2k \quad \alpha\beta = k</math></p> <p><math>y^2 - 4ky + 4k = 0</math></p> <p><b>Or</b>  <math>\alpha + \beta = -2k</math>  <math>\frac{-2k}{\alpha}</math>  <math>y = \frac{-2k}{x}</math></p> <p><math>y^2 - 4ky + 4k = 0</math></p> <p><b>Or</b></p> <p><math>-k \pm \sqrt{k^2 - k}</math>  <math>\frac{\alpha + \beta}{\alpha} = \frac{2k}{k + \sqrt{k^2 - k}}, \frac{\alpha + \beta}{\beta} = \frac{2k}{k - \sqrt{k^2 - k}}</math></p> <p><math>y^2 - 4ky + 4k = 0</math></p>	<p>B1B1 State or use correct results  M1 Attempt to find sum of new roots  A1 Obtain <math>4k</math>  M1 Attempt to find product of new roots  A1 Obtain <math>4k</math>  B1ft 7 Correct quadratic equation a.e.f.</p> <p>B1 State or use correct result  B1 State or imply form of new roots</p> <p>B1 State correct substitution  M1 Rearrange and substitute for <math>x</math>  A1 Correct unsimplified equation  M1 Attempt to clear fractions  A1 Correct quadratic equation a.e.f.</p> <p>B1 Find roots of original equation</p> <p>B1 Express both new roots in terms of <math>k</math></p> <p>M1 Attempt to find sum of new roots  A1 Obtain <math>4k</math>  M1 Attempt to find product of new roots  A1 Obtain <math>4k</math>  B1ft Correct quadratic equation a.e.f.</p>
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8	(i)	M1 A1	Attempt to rationalise denominator or cross multiply 2 Obtain <b>given</b> answer correctly
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	(ii)	M1 M1 A1 A1 M1 A1	Express terms as differences using (i) Attempt this for at least 1 <sup>st</sup> three terms 1 <sup>st</sup> three terms all correct Last two terms all correct Show pairs cancelling 6 Obtain correct answer, in terms of $n$
			$\frac{1}{2}(\sqrt{n+2} + \sqrt{n+1} - \sqrt{2} - 1)$
<hr/>			
	(iii)	B1	1 <b>9</b> Sensible statement for divergence
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9	(i)	M1 M1 A1	Show correct expansion process for 3 x 3 Correct evaluation of any 2 x 2 3 Obtain correct answer
			$\det \mathbf{A} = a^2 - a$
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	(ii)	M1 A1 M1 A1 B1 B1	Find a pair of inconsistent equations State inconsistent or no solutions Find a repeated equation State non unique solutions State that $\det \mathbf{A}$ is non-zero or find correct solution 6 State unique solution <b>SC if <math>\det \mathbf{A}</math> incorrect, can score 2 marks for correct deduction of a unique solution, but only once</b>
	(a)		
	(b)		
	(c)		
<hr/>			
10	(i)	M1 A1 M1 M1 A1	Attempt to equate real and imaginary parts Obtain both results Eliminate to obtain quadratic in $x^2$ or $y^2$ Solve to obtain $x$ or $y$ value 5 Obtain correct answer as a complex no.
			$x^2 - y^2 = 3 \quad xy = 2$ $z = 2 + i$
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	(ii)	B1	1 Obtain <b>given</b> answer correctly
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	(iii)	M1 A1 M1 M1 A1	Attempt to solve quadratic equation Obtain correct answers Choose negative sign Relate required value to conjugate of (i) 5 Obtain correct answer
			$w^3 = 2 \pm 11i$ $w = 2 - i$
			<b>11</b>

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