RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE **4731**

Mark Scheme for June 2010

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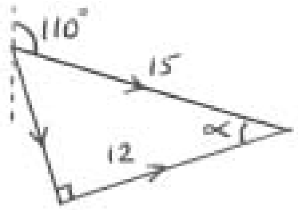
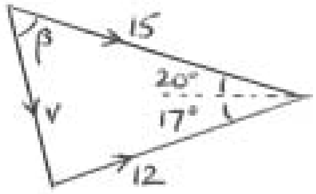
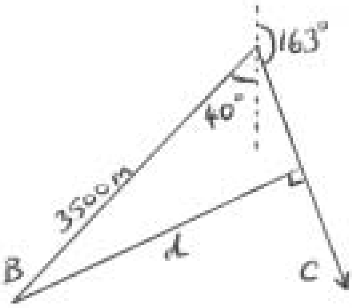
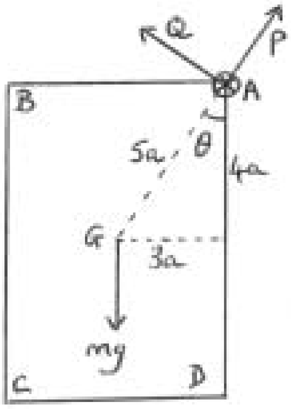
1 (i)	Using $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$, $1020 = 80\times 15 + \frac{1}{2}\alpha\times 15^2$ $\alpha = -1.6$ Angular deceleration is $1.6\ \text{rad s}^{-2}$	M1 A1 **[2]**	
(ii)	Using $\theta = \omega_2 t - \frac{1}{2}\alpha t^2$, $\theta = 0 - \frac{1}{2}\times(-1.6)\times 5^2$ Angle is 20 rad	M1 A1 ft **[2]**	ft is $12.5\|\alpha\|$
(iii)	Using $\omega_2^{\,2} = \omega_1^{\,2} + 2\alpha\theta$, $0 = 80^2 + 2\times(-1.6)\theta$ $\theta = 2000$ Number of revolutions is 318 (3 sf)	M1 A1 ft A1 **[3]**	*Accept* $\dfrac{1000}{\pi}$
2	Area is $\int_0^{\ln 3} \mathrm{e}^{-x}\,\mathrm{d}x$ $= \Big[-\mathrm{e}^{-x}\Big]_0^{\ln 3} \quad (=\frac{2}{3})$ $\int xy\,\mathrm{d}x = \int_0^{\ln 3} x\mathrm{e}^{-x}\,\mathrm{d}x$ $= \Big[-x\mathrm{e}^{-x} - \mathrm{e}^{-x}\Big]_0^{\ln 3} \quad (=\frac{2}{3} - \frac{1}{3}\ln 3)$ $\bar{x} = \dfrac{\frac{2}{3} - \frac{1}{3}\ln 3}{\frac{2}{3}} = 1 - \frac{1}{2}\ln 3$ $\int \frac{1}{2}y^2\,\mathrm{d}x = \int_0^{\ln 3} \frac{1}{2}(\mathrm{e}^{-x})^2\,\mathrm{d}x$ $= \Big[-\frac{1}{4}\mathrm{e}^{-2x}\Big]_0^{\ln 3} \quad (=\frac{2}{9})$ $\bar{y} = \dfrac{\frac{2}{9}}{\frac{2}{3}} = \dfrac{1}{3}$	M1 A1 M1 M1 A1 A1 M1 A1 A1 **[9]**	*Limits not required* For $-\mathrm{e}^{-x}$ *Limits not required* Integration by parts For $-x\mathrm{e}^{-x} - \mathrm{e}^{-x}$ $\int(\mathrm{e}^{-x})^2\,\mathrm{d}x$ or $\int(-\ln y)y\,\mathrm{d}y + (\frac{1}{3}\ln 3)\times\frac{1}{6}$ $-\frac{1}{4}\mathrm{e}^{-2x}$ or $-\frac{1}{2}y^2\ln y + \frac{1}{4}y^2$ *(dep on M1)* *Max penalty of 1 mark for correct answers in an unacceptable form (eg decimals)*
3 (i)	By conservation of angular momentum $I_2\times 15 = 0.9\times 16$ $I_2 = 0.96$ $I_2 = 0.9 + m\times 0.4^2$ Mass is 0.375 kg	M1 A1 M1 A1 **[4]**	Using $I\omega$
(ii)	KE before is $\frac{1}{2}\times 0.9\times 16^2$ KE after is $\frac{1}{2}\times 0.96\times 15^2$ Loss of KE is $115.2 - 108 = 7.2$ J	M1 A1 ft A1 **[3]**	Using $\frac{1}{2}I\omega^2$ Both expressions correct

4 **(i)**	 $\cos\alpha = \frac{12}{15}$ $\alpha = 36.87°$ (4 sf) Bearing of $\mathbf{v}_B$ is $110 - 36.87 = 073.13$ $= 073°$ (nearest degree)	M1 A1 M1 A1 ag **[4]**	Velocity triangle with $90°$ opposite $\mathbf{v}_C$ Correct velocity triangle Finding a relevant angle
(ii)	Magnitude is $\sqrt{15^2 - 12^2} = 9 \text{ m s}^{-1}$ Direction is $90°$ from $\mathbf{v}_B$ Bearing is $73.13 + 90 = 163°$ (nearest degree)	B1 M1 A1 **[3]**	Accept 8.95 to 9.05
	Alternative for **(ii)** (*using given answer in* **(i)**)  $v^2 = 12^2 + 15^2 - 2 \times 12 \times 15 \cos 37°$ $v = 9$ $\frac{\sin\beta}{12} = \frac{\sin 37°}{v}$ $\beta = 53°$ Bearing is $110 + 53 = 163°$	B1 M1 A1	or Relative velocity is $\begin{pmatrix} v\sin\theta \\ v\cos\theta \end{pmatrix} = \begin{pmatrix} 15\sin 110 \\ 15\cos 110 \end{pmatrix} - \begin{pmatrix} 12\sin 73 \\ 12\cos 73 \end{pmatrix} \approx \begin{pmatrix} 2.6 \\ -8.6 \end{pmatrix}$ or $v^2 = (2.6...)^2 + (-8.6...)^2$ Accept 8.95 to 9.05 Finding a relevant angle or $\tan\theta = \frac{2.6...}{-8.6...}$
(iii)	As viewed from *B*  $d = 3500 \sin 56.87°$ Shortest distance is 2930 m (3 sf)	M1 M1 A1 **[3]**	Diagram indicating initial displacement and relative velocity *May be implied* Accept 2910 to 2950
	Alternative for **(iii)** $d^2 = (3500\sin 40° + 2.6...t)^2$ $+(3500\cos 40° - 8.6...t)^2$ Minimum when $-34432 + 162t = 0$ $t = 213$ Shortest distance is 2930 m (3 sf)	M1 M1 A1	Differentiating or completing the square Accept 2910 to 2950

5 (i)	$I = \int_{-a}^{5a} \frac{m}{6a} x^2 \, dx$ or $\int_{-a}^{5a} \rho x^2 \, dx$ $= \left[\frac{m}{18a} x^3 \right]_{-a}^{5a} = \frac{m}{18a}(125a^3 + a^3)$ or $42\rho a^3$ $= \frac{126ma^3}{18a} = 7ma^2$	M1 M1 A1 M1 A1 ag [5]	$(\delta m)x^2$ or $(\rho \delta x)x^2$ or integrating x^2 Using $\delta m = \frac{m \delta x}{6a}$ or $\rho = \frac{m}{6a}$ Correct integral expression for *I* eg $I = \int_0^{5a} ... + \int_0^a ...$ $I = \int_{-3a}^{3a} ... + m(2a)^2$, $I = 2\int_0^{3a} ... + m(2a)^2$ $I = \int_0^{6a} ... - m(3a)^2 + m(2a)^2$ Evaluating definite integral *Dependent on integrating* x^2
(ii)	WD by couple is $\frac{6mga}{\pi} \times 3\pi$ $(= 18mga)$ Gain of PE is $mg(4a)$ $18mga = 4mga + \frac{1}{2}(7ma^2)\omega^2$ Angular speed is $\sqrt{\frac{4g}{a}}$	M1 A1 B1 M1 A1 ft A1 [6]	Using $C\theta$ Equation involving WD, PE and $\frac{1}{2} I \omega^2$

6 (i)	$\frac{\mathrm{d}V}{\mathrm{d}\theta} = mga(3\cos\theta + 4\sin\theta - 3)$ When $\theta = 0$, $\frac{\mathrm{d}V}{\mathrm{d}\theta} = mga(3+0-3) = 0$ so $\theta = 0$ is a position of equilibrium $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = mga(-3\sin\theta + 4\cos\theta)$ When $\theta = 0$, $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = 4mga > 0$ hence the equilibrium is stable	B1 M1 A1 ag M1 A1 ag **[5]**	Considering $\frac{\mathrm{d}V}{\mathrm{d}\theta} = 0$ Correctly shown Considering $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2}$ (or other method) $V'' = 4mga \Rightarrow \text{Stable}$ M1A0 $V'' = 4mga \Rightarrow \text{Minimum} \Rightarrow \text{Stable}$ M1A1
(ii)	Speed of *P* and *Q* is $a\dot{\theta}$ KE is $\frac{1}{2}(5m)(a\dot{\theta})^2 + \frac{1}{2}(3m)(a\dot{\theta})^2$ or $\frac{1}{2}(8m)(a\dot{\theta})^2$ $= \frac{5}{2}ma^2\dot{\theta}^2 + \frac{3}{2}ma^2\dot{\theta}^2$ $= 4ma^2\dot{\theta}^2$	M1 A1 ag **[2]**	Or moment of inertia of *P* is $5ma^2$ $\frac{5}{2}ma^2\dot{\theta}^2 + \frac{3}{2}ma^2\dot{\theta}^2$ M1A1 $\frac{1}{2}(5ma^2)\dot{\theta}^2 + \frac{1}{2}(3ma^2)\dot{\theta}^2$ M1A0 $\frac{1}{2}(8ma^2)\dot{\theta}^2$ M1A0
(iii)	$V + 4ma^2\dot{\theta}^2 = K$ $\frac{\mathrm{d}V}{\mathrm{d}\theta}\dot{\theta} + 8ma^2\dot{\theta}\ddot{\theta} = 0$ $mga(3\cos\theta + 4\sin\theta - 3)\dot{\theta} + 8ma^2\dot{\theta}\ddot{\theta} = 0$ For small θ, $\sin\theta \approx \theta$, $\cos\theta \approx 1$ $mga(3 + 4\theta - 3) + 8ma^2\ddot{\theta} \approx 0$ $\ddot{\theta} \approx -\frac{g}{2a}\theta$ Approximate period is $2\pi\sqrt{\frac{2a}{g}}$	M1 A1 M1 A1 ft A1 **[5]**	$= 0$ is required for A1 *(may be implied by later work)* Linear approximation (ft is dep on M1M1)

7 (i)	$I=\frac{1}{3}m\{(3a)^2+(4a)^2\}+m(5a)^2$ $=\frac{100ma^2}{3}$	M1 A1 A1 [**3**]	Using parallel (or perpendicular) axes rule or $I=\frac{4}{3}m(3a)^2+\frac{4}{3}m(4a)^2$
(ii)	 By conservation of energy, $\frac{1}{2}(\frac{100}{3}ma^2)\omega^2=mg(4a-3a)$ $\frac{50}{3}ma^2\omega^2=mga$ Angular speed is $\sqrt{\frac{3g}{50a}}$ $-mg(3a)=(\frac{100}{3}ma^2)\alpha$ Angular acceleration is $(-)\frac{9g}{100a}$	M1 A1 ft A1 ag M1 A1 [**5**]	Equation involving KE and PE Using $C=I\alpha$
(iii)	$P-mg\cos\theta=m(5a)\omega^2$ $P-\frac{4}{5}mg=m(5a)\left(\frac{3g}{50a}\right)$ $P=\frac{11}{10}mg$ $Q-mg\sin\theta=m(5a)\alpha$ $Q-\frac{3}{5}mg=-m(5a)\left(\frac{9g}{100a}\right)$ $Q=\frac{3}{20}mg$ $F=\sqrt{P^2+Q^2}=\frac{1}{20}mg\sqrt{22^2+3^2}$ $=\frac{\sqrt{493}}{20}mg$	M1 A2 M1 A2 ft M1 A1 ag [**8**]	Equation involving *P* and $r\omega^2$ Give A1 if correct apart from sign(s) *(Allow $\frac{3}{5}H+\frac{4}{5}V$ in place of P)* Equation involving *Q* and $r\alpha$ Give A1 if correct apart from sign(s) ft for wrong value of α ft for wrong value of *r* in second equation *(Allow $\frac{3}{5}V-\frac{4}{5}H$ in place of Q)* *Dependent on previous M1M1*
	Alternative for **(iii)** $H=m(5a)\omega^2\sin\theta-m(5a)\alpha\cos\theta$ $H=m(5a)\left(\frac{3g}{50a}\right)\left(\frac{3}{5}\right)+m(5a)\left(\frac{9g}{100a}\right)\left(\frac{4}{5}\right)$ $V-mg=m(5a)\omega^2\cos\theta+m(5a)\alpha\sin\theta$ $V-mg=m(5a)\left(\frac{3g}{50a}\right)\left(\frac{4}{5}\right)-m(5a)\left(\frac{9g}{100a}\right)\left(\frac{3}{5}\right)$ $H=\frac{27}{50}mg,\ V=\frac{97}{100}mg$	M1 A2 ft M1 A2 ft	Equation involving *H*, $r\omega^2$ and $r\alpha$ Give A1 if correct apart from sign(s) Equation involving *V*, $r\omega^2$ and $r\alpha$ Give A1 if correct apart from sign(s)

	$F=\sqrt{H^2+V^2}=\frac{1}{100}mg\sqrt{54^2+97^2}$ $=\frac{\sqrt{12325}}{100}mg=\frac{\sqrt{493}}{20}mg$	M1 A1 ag	*Dependent on previous M1M1*

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