OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS
4730

Mechanics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Monday 25 January 2010
Morning

Duration: 1 hour 30 minutes

* 4 7 3 0 *

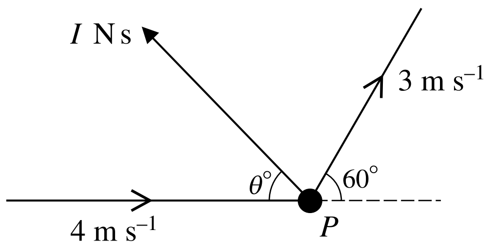
INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

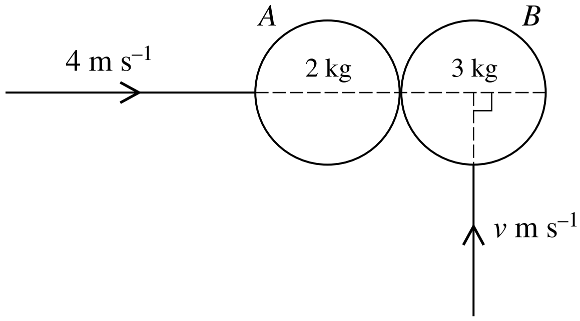
- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1



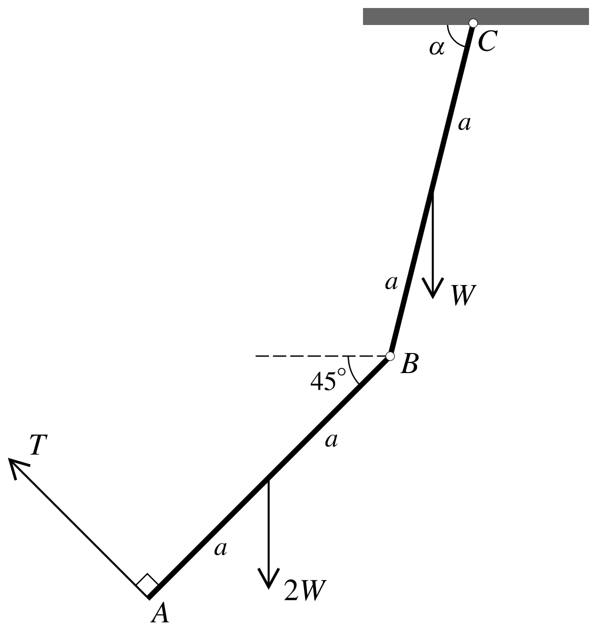
A particle P of mass 0.4 kg is moving horizontally with speed $4\,\text{m}\,\text{s}^{-1}$ when it receives an impulse of magnitude I N s, in a direction which makes an angle $(180 - \theta)^\circ$ with the direction of motion of P. Immediately after the impulse acts P moves horizontally with speed $3\,\text{m}\,\text{s}^{-1}$. The direction of motion of P is turned through an angle of 60° by the impulse (see diagram). Find I and θ. **[7]**

2



Two uniform smooth spheres A and B, of equal radius, have masses 2 kg and 3 kg respectively. They are moving on a horizontal surface when they collide. Immediately before the collision, A has speed $4\,\text{m}\,\text{s}^{-1}$ and is moving along the line of centres, and B has speed $v\,\text{m}\,\text{s}^{-1}$ and is moving perpendicular to the line of centres (see diagram). The coefficient of restitution is 0.6. The direction of motion of B after the collision makes an angle of 45° with the line of centres. Find the value of v. **[7]**

3



Two uniform rods AB and BC, each of length $2a$, have weights $2W$ and W respectively. The rods are freely jointed to each other at B, and BC is freely jointed to a fixed point at C. The rods are held in equilibrium in a vertical plane by a light string attached to A and perpendicular to AB. The rods AB and BC make angles 45° and α, respectively, with the horizontal. The tension in the string is T (see diagram).

(i) By taking moments about B for AB, show that $W = \sqrt{2}T$. **[3]**

(ii) Find the value of $\tan\alpha$. **[6]**

4 A particle P of mass 0.2 kg travels in a straight line on a horizontal surface. It passes through a point O on the surface with speed $2\,\text{m s}^{-1}$. A resistive force of magnitude $0.2(v + v^2)$ N acts on P in the direction opposite to its motion, where $v\,\text{m s}^{-1}$ is the speed of P when it is at a distance x m from O.

(i) Show that $\dfrac{1}{1+v}\dfrac{dv}{dx} = -1$. **[3]**

(ii) By solving the differential equation in part **(i)** show that $\dfrac{-e^x}{3 - e^x}\dfrac{dx}{dt} = -1$, where t s is the time taken for P to travel x m from O. **[5]**

(iii) Hence find the value of t when $x = 1$. **[3]**

5 A light elastic string of natural length 1.6 m has modulus of elasticity 120 N. One end of the string is attached to a fixed point O and the other end is attached to a particle P of weight 1.5 N. The particle is released from rest at the point A, which is 2.1 m vertically below O. It comes instantaneously to rest at B, which is vertically above O.

(i) Verify that the distance AB is 4 m. **[4]**

(ii) Find the maximum speed of P during its upward motion from A to B. **[7]**

Turn over

6

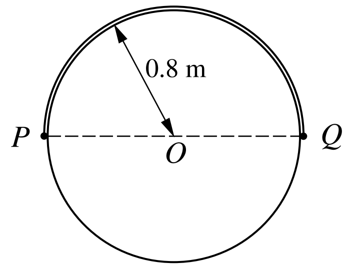


Fig. 1

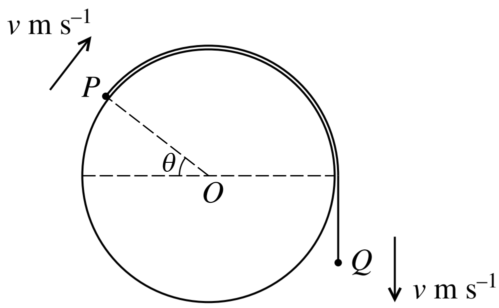


Fig. 2

A light inextensible string of length 0.8π m has particles P and Q, of masses 0.4 kg and 0.58 kg respectively, attached to its ends. The string passes over a smooth horizontal cylinder of radius 0.8 m, which is fixed with its axis horizontal and passing through a fixed point O. The string is held at rest in a vertical plane perpendicular to the axis of the cylinder, with P and Q at opposite ends of the horizontal diameter of the cylinder through O (see Fig. 1). The string is released and Q begins to descend. When OP has rotated through θ radians, with P remaining in contact with the cylinder, the speed of each particle is $v \text{ m s}^{-1}$ (see Fig. 2).

(i) By considering the total energy of the system, obtain an expression for v^2 in terms of θ. **[5]**

(ii) Show that the magnitude of the force exerted on P by the cylinder is $(7.12 \sin\theta - 4.64\theta)$ N. **[4]**

(iii) Given that P leaves the surface of the cylinder when $\theta = \alpha$, show that $1.53 < \alpha < 1.54$. **[4]**

7 A particle P of mass 0.5 kg is attached to one end of each of two identical light elastic strings of natural length 1.6 m and modulus of elasticity 19.6 N. The other ends of the strings are attached to fixed points A and B on a line of greatest slope of a smooth plane inclined at 30° to the horizontal. The distance AB is 4.8 m and A is higher than B.

(i) Find the distance AP for which P is in equilibrium on the line AB. **[5]**

P is released from rest at a point on AB where both strings are taut. The strings remain taut during the subsequent motion of P and t seconds after release the distance AP is $(2.5 + x)$ m.

(ii) Use Newton's second law to obtain an equation of the form $\frac{d^2x}{dt^2} = kx$. State the property of the constant k for which the equation indicates that P's motion is simple harmonic, and find the period of this motion. **[5]**

(iii) Given that $x = 0.5$ when $t = 0$, find the values of x for which the speed of P is 2.8 m s^{-1}. **[4]**