

**ADVANCED SUBSIDIARY GCE**  
**MATHEMATICS**  
Further Pure Mathematics 1

**4725**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- List of Formulae (MF1)

**Other Materials Required:**

- Scientific or graphical calculator

**Friday 11 June 2010**  
**Morning**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

- 1 Prove by induction that, for  $n \geq 1$ ,  $\sum_{r=1}^n r(r+1) = \frac{1}{3}n(n+1)(n+2)$ . [5]
- 2 The matrices **A**, **B** and **C** are given by  $\mathbf{A} = \begin{pmatrix} 1 & -4 \end{pmatrix}$ ,  $\mathbf{B} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$  and  $\mathbf{C} = \begin{pmatrix} 3 & 0 \\ -2 & 2 \end{pmatrix}$ . Find
- (i)  $\mathbf{AB}$ , [2]
- (ii)  $\mathbf{BA} - 4\mathbf{C}$ . [4]
- 3 Find  $\sum_{r=1}^n (2r-1)^2$ , expressing your answer in a fully factorised form. [6]
- 4 The complex numbers  $a$  and  $b$  are given by  $a = 7 + 6i$  and  $b = 1 - 3i$ . Showing clearly how you obtain your answers, find
- (i)  $|a - 2b|$  and  $\arg(a - 2b)$ , [4]
- (ii)  $\frac{b}{a}$ , giving your answer in the form  $x + iy$ . [3]
- 5 (a) Write down the matrix that represents a reflection in the line  $y = x$ . [2]
- (b) Describe fully the geometrical transformation represented by each of the following matrices:
- (i)  $\begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix}$ , [2]
- (ii)  $\begin{pmatrix} \frac{1}{2} & \frac{1}{2}\sqrt{3} \\ -\frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}$ . [2]
- 6 (i) Sketch on a single Argand diagram the loci given by
- (a)  $|z - 3 + 4i| = 5$ , [2]
- (b)  $|z| = |z - 6|$ . [2]
- (ii) Indicate, by shading, the region of the Argand diagram for which
- $$|z - 3 + 4i| \leq 5 \quad \text{and} \quad |z| \geq |z - 6|. \quad [2]$$
- 7 The quadratic equation  $x^2 + 2kx + k = 0$ , where  $k$  is a non-zero constant, has roots  $\alpha$  and  $\beta$ . Find a quadratic equation with roots  $\frac{\alpha + \beta}{\alpha}$  and  $\frac{\alpha + \beta}{\beta}$ . [7]

8 (i) Show that  $\frac{1}{\sqrt{r+2} + \sqrt{r}} \equiv \frac{\sqrt{r+2} - \sqrt{r}}{2}$ . [2]

(ii) Hence find an expression, in terms of  $n$ , for

$$\sum_{r=1}^n \frac{1}{\sqrt{r+2} + \sqrt{r}}. \quad [6]$$

(iii) State, giving a brief reason, whether the series  $\sum_{r=1}^{\infty} \frac{1}{\sqrt{r+2} + \sqrt{r}}$  converges. [1]

9 The matrix  $\mathbf{A}$  is given by  $\mathbf{A} = \begin{pmatrix} a & a & -1 \\ 0 & a & 2 \\ 1 & 2 & 1 \end{pmatrix}$ .

(i) Find, in terms of  $a$ , the determinant of  $\mathbf{A}$ . [3]

(ii) Three simultaneous equations are shown below.

$$\begin{aligned} ax + ay - z &= -1 \\ ay + 2z &= 2a \\ x + 2y + z &= 1 \end{aligned}$$

For each of the following values of  $a$ , determine whether the equations are consistent or inconsistent. If the equations are consistent, determine whether or not there is a unique solution.

(a)  $a = 0$

(b)  $a = 1$

(c)  $a = 2$

[6]

10 The complex number  $z$ , where  $0 < \arg z < \frac{1}{2}\pi$ , is such that  $z^2 = 3 + 4i$ .

(i) Use an algebraic method to find  $z$ . [5]

(ii) Show that  $z^3 = 2 + 11i$ . [1]

The complex number  $w$  is the root of the equation

$$w^6 - 4w^3 + 125 = 0$$

for which  $-\frac{1}{2}\pi < \arg w < 0$ .

(iii) Find  $w$ . [5]

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