

Mathematics

Advanced GCE

Unit **4725**: Further Pure Mathematics 1

Mark Scheme for January 2011

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1 (i)	$(7 \ 9)$	B1B1 2	Each element correct SC (7,9) scores B1
<hr style="border-top: 1px dashed black;"/>			
(ii)	(18)	B1* depB1 2	Obtain correct value Clearly given as a matrix
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(iii)	$\begin{pmatrix} 12 & -4 \\ 6 & -2 \end{pmatrix}$	M1 A1 A1 3 $\boxed{7}$	Obtain 2×2 matrix Obtain 2 correct elements Obtain other 2 correct elements

2. (i)	$-12 + 13i$	B1B1 2	Real and imaginary parts correct
<hr style="border-top: 1px dashed black;"/>			
(ii)	$\frac{27}{37} - \frac{14}{37}i$	B1 M1 A1 A1 4 $\boxed{6}$	z^* seen Multiply by w^* Obtain correct real part or numerator Obtain correct imaginary part or denom. Sufficient working must be shown

3		B1* M1* A1* depA1 4 $\boxed{4}$	Establish result true for $n = 1$ or 2 Use given result in recurrence relation in a relevant way Obtain $2^n + 1$ correctly Specific statement of induction conclusion
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4	<i>Either</i>	B1 M1 A1 M1 A1 A1 6 M1 A1 A1 M1 A1 A1 $\boxed{6}$	Correct value for $\sum r$ stated or used Express as sum of two series Obtain correct unsimplified answer Compare coefficients or substitute values for n Obtain correct answers Use 2 values for n Obtain correct equations Solve simultaneous equations Obtain correct answers
	$\frac{a}{4}n^2(n+1)^2 + \frac{bn}{2}(n+1)$		
	$a = 4 \quad b = -4$		
	<i>Or</i>		
	$a + b = 0 \quad 4a + b = 12$		
	$a = 4 \quad b = -4$		

5	\mathbf{A}^2	B1 M1 A1cao 3 $\boxed{3}$	$(\mathbf{A}^{-1})^{-1} = \mathbf{A}$ seen or implied Use product inverse correctly Obtain correct answer
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6 (i) (a)	B1*	Vertical line
(b)	depB1 2	Clearly through (4, 0)
	B1	Sloping line with +ve slope
	B1	Through (0, -2)
	B1ft 3	Half line starting on y-axis 45° shown convincingly

(ii)	B1ft	Shaded to left of their (i) (a)
	B1ft	Shaded below their (i) (b) must be +ve slope
	B1ft 3	Shaded above horizontal through their (0, -2)

NB These 3 marks are independent, but 3/3 only for fully correct answer.

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7 (i) $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$	B1 B1 2	Each column correct
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(ii)	B1*	Enlargement or stretch in <i>x</i> and <i>y</i> axes
	depB1 2	Scale factor $\sqrt{3}$

(iii) (a)	B1	(2,0), (6,2) indicated
	B1	(8, 2) seen
	B1 3	Accurate diagram, including unit square

(b) $\det C = 4$	B1	Correct value found
	B1 2	Scale factor for area

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8 (i) <i>Either</i>		
$\alpha + \beta = \frac{1}{2}, \alpha\beta = \frac{3}{2}$	B1	State or use both correct results in (i) or (ii)
$\alpha + \beta + \frac{\alpha + \beta}{\alpha\beta}$ or $\alpha + \beta + \frac{2}{3}(\alpha + \beta)$	M1	Express sum of new roots in terms of $\alpha + \beta$ and $\alpha\beta$
	M1	Substitute their values into their expression
$p = \frac{5}{6}$	A1 4	Obtain given answer correctly
<i>Or</i>		
$3u^2 - u + 2(= 0)$	B1	Substitute $x = \frac{1}{u}$ and obtain correct quadratic (equation)
	M1	Use sum of roots of new equation
	M1	Substitute their values into their expression
$p = \frac{5}{6}$	A1	Obtain given answer correctly

$$(ii) \alpha' \beta' = \alpha\beta + \frac{1}{\alpha\beta} + \frac{\beta}{\alpha} + \frac{\alpha}{\beta}$$

$$\frac{\beta}{\alpha} + \frac{\alpha}{\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$q = \frac{1}{3}$$

B1	Correct expansion
M1	Show how to deal with $\alpha^2 + \beta^2$
A1	Obtain correct expression
M1	Substitute their values into $\alpha'\beta'$
A1	Obtain correct answer a.e.f.
5	

9 (i)

$$\det \mathbf{M} = a^2 - 7a + 6$$

M1	Show correct expansion process for 3 x 3
M1	Correct evaluation of any 2 x 2
A1	correct answer
3	

(ii)

$$a = 1 \text{ or } 6$$

M1	Solve $\det \mathbf{M} = 0$
A1A1	Obtain correct answer, ft their (i)
3	

(iii)

M1	Attempt to eliminate one variable
A1	Obtain 2 correct equations in 2 unknowns
A1	Justify infinite number of solutions
3	SC 3/3 if unique solution conclusion consistent with their (i) or (ii)

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10 (i)

M1	Use correct denominator
A1	Obtain given answer correctly
2	

(ii)

$$\frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2}$$

M1	Express terms as differences using (i)
M1	Do this for at least 3 terms
A1	First 3 terms all correct
A1	Last 2 terms all correct
M1	Show relevant cancelling
A1	Obtain correct answer a.e.f.
6	

(iii)

$$\frac{1}{2}$$

$$\frac{1}{n+1} - \frac{1}{n+2}$$

B1ft	S_{∞} stated or start at $n+1$ as in (ii)
M1	S_{∞} - their (ii) or show correct cancelling

$$\frac{1}{(n+1)(n+2)}$$

A1	Obtain given answer correctly
3	

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