

Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4776**: Numerical Methods

Mark Scheme for January 2011

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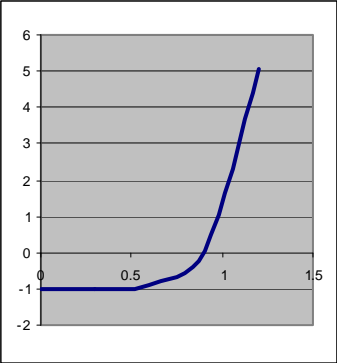
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Question		Answer	Marks	Guidance												
1	(i)	$\begin{array}{rcccl} x & \text{LHS} & & \text{RHS} & \\ 1 & 2 & > & 1.557408 & \\ 1.2 & 2.2 & < & 2.572152 & \end{array}$	M1 A1 [2]	no explicit explanation required												
	(ii)	$\begin{array}{rcccccc} r & 0 & 1 & 2 & 3 & 4 \\ x_r & 1.1 & 0.96476 & 0.442927 & -0.52564 & -1.58007 \end{array}$	M1 A1 [2]	$r = 3$ required												
	(iii)	e.g. re-arrange to $x = \arctan(1 + x)$ $\begin{array}{rcccccc} r & 0 & 1 & 2 & 3 & 4 & 5 \\ x_r & 1.1 & 1.126377 & 1.131203 & 1.132076 & 1.132233 & 1.132261 \\ & & & & & & 1.132 \end{array}$	B1 M1 A1 A1 [4]													
2		<table border="1"> <thead> <tr> <th>h</th> <th>M</th> <th>T</th> </tr> </thead> <tbody> <tr> <td>2</td> <td>1.987467</td> <td>1.354440</td> </tr> <tr> <td>1</td> <td>1.830595</td> <td>1.670954</td> </tr> <tr> <td>0.5</td> <td></td> <td>1.750774</td> </tr> </tbody> </table> <p>Simpson's rule $(2M + T) / 3$ 1.776458 1.777381</p> <p>Reference to justification/accuracy : 1.777 or 1.78</p>	h	M	T	2	1.987467	1.354440	1	1.830595	1.670954	0.5		1.750774	T: M1A1A1 S: M1A1A1 E1 A1 [8]	Lose 1 for any additional 'answer'(s) but do not penalise extrapolation
h	M	T														
2	1.987467	1.354440														
1	1.830595	1.670954														
0.5		1.750774														
3	(i)	$h = 1 \quad g'(0) = (2.0100 - 1.4509)/1 = 0.5591$ $h = 0.5 \quad g'(0) = (1.6799 - 1.4509)/0.5 = 0.458$ Estimate with smaller h (0.458) likely to be more accurate: smaller h is more accurate (provided there is no great loss of significant figures)	B1 B1 B1 E1 [4]													
	(ii)	$h = 0.5 \quad g'(0.5) = (2.0100 - 1.4509)/1 = 0.5591$ This estimate, central diff, likely to be more accurate than either of the forward diffs	M1 E1 [2]													

Question		Answer	Marks	Guidance																																								
4	(i)	Max poss loss: 365 (or 366) times 0.01 pence: = 3.65 (or 3.66) pence Arises if each daily amount would round up but gets chopped down Average loss 1.825 (or 1.83) pence, because average is half of max.	B1 E1 B1 E1 [4]																																									
	(ii)	£150 000 divided by 1.825 pence: about 8.2 million (8 million) accounts	M1 A1 [2]																																									
5		<table style="display: inline-table; border: none;"> <thead> <tr> <th><i>x</i></th> <th>P(x)</th> <th>ΔP(x)</th> <th>Δ²P(x)</th> <th>Δ³P(x)</th> </tr> </thead> <tbody> <tr> <td><i>-1</i></td> <td><i>-11</i></td> <td></td> <td></td> <td></td> </tr> <tr> <td>1</td> <td>-10</td> <td><i>1</i></td> <td></td> <td></td> </tr> <tr> <td>3</td> <td>3</td> <td>13</td> <td><i>12</i></td> <td></td> </tr> <tr> <td>5</td> <td>44</td> <td>41</td> <td>28</td> <td><i>16</i></td> </tr> <tr> <td>7</td> <td>129</td> <td>85</td> <td>44</td> <td>16</td> </tr> <tr> <td>9</td> <td>274</td> <td>145</td> <td>60</td> <td>16</td> </tr> <tr> <td><i>11</i></td> <td><i>495</i></td> <td><i>221</i></td> <td><i>76</i></td> <td><i>16</i></td> </tr> </tbody> </table>	<i>x</i>	P(x)	ΔP(x)	Δ²P(x)	Δ³P(x)	<i>-1</i>	<i>-11</i>				1	-10	<i>1</i>			3	3	13	<i>12</i>		5	44	41	28	<i>16</i>	7	129	85	44	16	9	274	145	60	16	<i>11</i>	<i>495</i>	<i>221</i>	<i>76</i>	<i>16</i>	<p>(i) bold: Diff table 3rd diffs constant so cubic</p> <p>(ii) italic: working forwards working backwards</p>	M1 A1 E1 B1 M1 A1 M1 A1 [4] + [4]
<i>x</i>	P(x)	ΔP(x)	Δ²P(x)	Δ³P(x)																																								
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6	(i)	<table style="display: inline-table; border: none;"> <thead> <tr> <th><i>x</i></th> <th><i>f</i></th> <th><i>g</i></th> <th><i>h</i></th> <th>abs err <i>g</i></th> <th>rel err <i>g</i></th> <th>abs err <i>h</i></th> <th>rel err <i>h</i></th> </tr> </thead> <tbody> <tr> <td>0.2</td> <td>0.013351</td> <td>0.013333</td> <td>0.013423</td> <td>0.0000179</td> <td>-0.0013424</td> <td>0.0000716</td> <td>0.0053600</td> </tr> <tr> <td>0.1</td> <td>0.003334</td> <td>0.003333</td> <td>0.003339</td> <td>0.0000011</td> <td>-0.0003339</td> <td>0.0000045</td> <td>0.0013350</td> </tr> <tr> <td></td> <td>A1</td> <td>A1</td> <td>A1</td> <td>A1</td> <td>A1</td> <td>A1</td> <td>A1</td> </tr> </tbody> </table>	<i>x</i>	<i>f</i>	<i>g</i>	<i>h</i>	abs err <i>g</i>	rel err <i>g</i>	abs err <i>h</i>	rel err <i>h</i>	0.2	0.013351	0.013333	0.013423	0.0000179	-0.0013424	0.0000716	0.0053600	0.1	0.003334	0.003333	0.003339	0.0000011	-0.0003339	0.0000045	0.0013350		A1	A1	A1	A1	A1	A1	A1	abs M1 rel M1 [9]	f, g, h values may be implied								
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	(ii)	<p>Errors in g and h are of opposite sign; g is about 4 times as accurate as h.</p> <table style="display: inline-table; border: none;"> <thead> <tr> <th><i>x</i></th> <th><i>f</i></th> <th>(4<i>g</i> + <i>h</i>)/5</th> <th>abs err</th> <th>rel err</th> </tr> </thead> <tbody> <tr> <td>0.2</td> <td>0.013351</td> <td>0.013351</td> <td>-2.5E-08</td> <td>-1.9E-06</td> </tr> <tr> <td>0.1</td> <td>0.003334</td> <td>0.003334</td> <td>-4E-10</td> <td>-1.2E-07</td> </tr> <tr> <td></td> <td></td> <td>A1</td> <td>A1</td> <td>A1</td> </tr> </tbody> </table>	<i>x</i>	<i>f</i>	(4 <i>g</i> + <i>h</i>)/5	abs err	rel err	0.2	0.013351	0.013351	-2.5E-08	-1.9E-06	0.1	0.003334	0.003334	-4E-10	-1.2E-07			A1	A1	A1	E1 E1 M1 [6]																					
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		A1	A1	A1																																								
	(iii)	$x / \sin x \approx 1.000\ 000\ 002 \approx 1$ $g(10^{-4}) = 3.33 \times 10^{-9}$ Subtraction of nearly equal quantities	B1 B1 E1 [3]																																									

Question	Answer	Marks	Guidance																
7 (i)	<p>$f(0) = -1$ $f(1) = 1$ (hence root) $f'(x) = 7x^6 + 5x^4$ which is zero only at $x = 0$. Convincing argument that this is not a turning point No turning points implies no other roots.</p> 	B1 M1 A1 B1 E1 G2 [7]																	
(ii)	<p>NR iteration: $x_{r+1} = x_r - (x_r^7 + x_r^5 - 1) / (7x_r^6 + 5x_r^4)$</p> <table border="0" data-bbox="412 850 837 906"> <tr> <td>r</td> <td>0</td> <td>1</td> <td>2</td> </tr> <tr> <td>x_r</td> <td>0.6</td> <td>1.51756</td> <td>1.289164</td> </tr> </table> <p>On graph: tangent at 0.6, intersection at 1.5, ordinate & tangent, intersection at 1.3</p>	r	0	1	2	x_r	0.6	1.51756	1.289164	B1 A1 A1 G4 [7]									
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