Level 3 Certificate

Certificate Quantitative Problem Solving (MEI)

H867/02: Statistical Problem solving

OCR Level 3 Certificate

Mark Scheme for June 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i Anything in the mark scheme which is in square brackets […] is not required for the mark to be earned, but if present it must be correct.

Question		Answer	Marks	Guidance
1	(i)	Quota	B1	
			[1]	
	(ii)	Almost everyone agrees cheaper fares are important The next most important issue is seen as overcrowding on the trains Cleaner trains is more of an issue for adult women than for the other groups Hardly anybody sees better information at the stations as important	B1 B1 B1	Any three different sensible answers. A statement may be general or particular to one group.
			[3]	
	(iii)	One of them selected 6 adult women and 4 adult men	B1	Award this mark for saying the proportions of men and women are not right; so condone 2 extra women and 2 men too few.
			[1]	

2	(i)	The 4 beaches chosen may not be typical of the others.	B1	Accept too small a sample, however expressed.
			[1]	

2	(ii)	<table><tr><td>Weight, w kg</td><td>$0 \le w < 1$</td><td>$1 \le w < 1.5$</td><td>$1.5 \le w < 2$</td><td>$2 \le w < 2.5$</td><td>$2.5 \le w < 3$</td></tr><tr><td>Frequency</td><td>**1**</td><td>**1**</td><td>**4**</td><td>**5**</td><td>**9**</td></tr></table> <table><tr><td>$3 \le w < 3.5$</td><td>$3.5 \le w < 4$</td><td>$4 \le w < 4.5$</td><td>$4.5 \le w < 5$</td><td>$5 \le w < 6$</td><td>**Total**</td></tr><tr><td>**8**</td><td>**6**</td><td>**3**</td><td>**2**</td><td>**1**</td><td>**40**</td></tr></table> **Table 2.1**	B1 B1	2 correct entries All entries correct
			[2]	

2	(iii)	(*A*) Bell-shaped	B1	Accept symmetrical with central mode or mean, or equivalent
			[1]	
		(*B*) Mean = 3 or 2.75	B1	FT their mean for the rest of the question
			[1]	
		(*C*) 1 = mean - 2 × sd, 5 = mean + 2 × sd	M1	Calculating a z-value. Award if 2 standard deviations seen
		95 % should lie within mean $\pm 2 \times$ sd, 5% outside it	M1	Interpreting z-value as a percentage. Award if 95% or 5% seen
		2 out of 40, so 5%, do indeed lie outside $1 \le w < 5$	A1	38 out of 40 is 95%, or 2 out of 40 is 5%, required
			[3]	

2	(iv)	**There are many ways of answering this question. All use the same general mark scheme, as indicted immediately below. Specific examples follow.**		
		M1 For a complete method that correctly applied would lead to the weight **A1** For the correct weight in any units **B1** For converting their answer into tonnes Allow FT for answers for the mean in part (iii)(*B*) in the range 2 kg to 4 kg inclusive.		
		Special case For a final answer of 6.75 × a power of 10 (eg for 675, 0.675) award **SC2**		
		Method 1 Using number of stretches		
		Number of stretches = $\frac{45000}{20} = 2250 \Rightarrow$ Weight = 2250 × 3	**M1**	
		Weight = 6750 kg	**A1**	FT from their mean in (iii)(*B*)
		6.75 tonnes	**B1**	FT their answer for the weight in kg
			[3]	
		Method 2 Using a proportional argument for the weights		
		3 kg for 20 m $\Rightarrow$ 150 kg for 1 km $\Rightarrow$ Weight = 150 × 45	**M1**	Proportional argument for weights
		Weight = 6750 kg	**A1**	FT from their mean in (iii)(*B*)
		= 6.75 tonnes	**B1**	FT their answer for the weight in kg
			[3]	

2	(v)	**Using the number of stretches** Number of stretches = $\frac{45000}{20} = 2250$ Total time taken = $2250 \times 0.25 = 562.5$ hours (33750 mins) Assume each volunteer works 6 hours 562.5 ÷ 6 = 93.75 So 95 to 100 volunteers needed	 **B1** **B1**	 Allow FT for answers for the mean in part (iii)(*B*) in the range 2 kg to 4 kg inclusive. Total time seen in hours or minutes Allow any reasonable number of hours worked per day. Do not allow 24 hours. Number of volunteers consistent with their time Accept any realistic/justifiable estimate that follows from their hours. Notice that 6 hours per day gives 93.75 volunteers, so 95 to 100 8 hours per day gives 70.3125 volunteers so 70 to 75.
			[2]	
		Alternative Using the time taken 3 kg in 15 minutes is the same as 12 kg in 1 hour Or 6750 kg in 562.5 hours 562.5 ÷ 6 = 93.75 So 95 to 100 volunteers needed	 **B1** **B1**	 Allow FT for answers for the mean in part (iii)(*B*) in the range 2 kg to 4 kg inclusive. Total time seen in hours or minutes. Number of volunteers consistent with their time Accept any realistic/justifiable estimate that follows from their hours.

<table>
<tr><td>3</td><td>(i)</td><td>H_0: The proportions of the different types of mouse are independent of the wood

H_1: The proportions of the different types of mouse are not independent of the wood</td><td>

B1</td><td>Allow
H_0: No association between types of mouse and woodlands
H_1: Association between types of mouse and woodlands
Do not allow “correlation”.
Both required</td></tr>
<tr><td></td><td></td><td></td><td>[1]</td><td></td></tr>
<tr><td>3</td><td>(ii)</td><td><table>
<tr><th colspan="2">Expected frequency, f_e</th><th>New mouse</th><th>Wood mouse</th><th>Yellow-necked</th><th>Total</th></tr>
<tr><td colspan="2">Wood A</td><td>19.5</td><td>25.5</td><td>15</td><td>60</td></tr>
<tr><td colspan="2">Wood B</td><td>19.5</td><td>25.5</td><td>15</td><td>60</td></tr>
<tr><td colspan="2">Wood C</td><td>13</td><td>17</td><td>10</td><td>40</td></tr>
<tr><td colspan="2">Total</td><td>52</td><td>68</td><td>40</td><td>160</td></tr>
</table>
Table 3.2</td><td>

B1

B1</td><td>

Wood B row correct

All correct</td></tr>
</table>

		<table><tr><th></th><th>New mouse</th><th>Wood mouse</th><th>Yellow-necked</th></tr><tr><td>Wood A</td><td>$\frac{(16-19.5)^2}{19.5} = 0.62$</td><td>0.7941</td><td>0.0667</td></tr><tr><td>Wood B</td><td>0.6282</td><td>0.2451</td><td>0.0667</td></tr><tr><td>Wood C</td><td>3.7692</td><td>2.8824</td><td>0.0000</td></tr></table> **Table 3.3**	**B1**	All entries correct to at least 3 d.p.
		9.081	**M1**	Attempt to find X^2 from table. Allow answers between 8 and 11. Notice that 9.0672 comes from 0.06 not 0.0667 .
		Degrees of freedom, $\nu = (3-1)\times(3-1) = 4$	**M1**	Attempt to use the formula $v = (r - 1)(c - 1)$ find the critical value
		Critical value for $\nu = 4$ at 5% significance level = 9.488	**A1**	Dependent on previous M mark. Correct critical value
		Since 9.081 < 9.488, the result is not significant	**A1**	Dependent on both M marks. Comparison between X^2 and critical value must be seen. Allow FT for their values.
			[7]	
	(iii)	The test involved changes of location but not changes over time No hypothesis test shows a result "conclusively". The test would be significant at another level. The threat to other species was not the subject of the test.	**B1** **B1**	Any two sensible criticisms
			[2]	

	(iv)	The total numbers of mice are constant The numbers of the New mouse are increasing The numbers of the Yellow-necked mouse are declining The New mice are endangering the Yellow-necked mice The numbers of the Wood mouse are constant	**B1** **B1**	Any two sensible different conjectures. Be prepared to condone statements that are more like conclusions from the data than genuine conjectures.
			[2]	

4	(a)	(i)	Population of China = 1 355 692 576 Population density = $\frac{\text{population}}{\text{land area}} = \frac{1355692576}{9326410} = 145.3\ ...$	B1 B1	
				[2]	
		(ii)	For India, $\frac{\text{population}}{\text{land area}} = \frac{1236344631}{\text{land area}} = 416$ Land area = $\frac{1236344631}{416} = 2971982$, so 2 971 982 km^2	M1 A1	Condone an incorrect value of the population for this mark only
				[2]	
4	(b)	(i)	Malaysia: Population = 30 073 353 GDP per capita = 17 500 US$ Total GDP = 30 073 353 × 17 500 = 5.262 ... × 10^{11} = 5.26 × 10^{11} US$ to 3 s.f.	M1 A1	Attempting to find and multiply population and GDP/person for Malaysia
				[2]	
		(ii)	Total GDP for Singapore is 5 567 301 × 62 400 = 3.473 ... × 10^{11} Combined GDP is 5.262 ... × 10^{11} + 3.473 ... × 10^{11} = 8.736 ... × 10^{11} US$ Combined population is 30 073 353 + 5 567 301 = 35 640 654 Mean = $\frac{8.736... \times 10^{11}}{35640654} = 24500$ (to 3 s.f.)	M1 M1 M1 A1	Condone rounding errors or inappropriate rounding but otherwise CAO. Dependent on all 3 M marks.

				[4]	

5 (i)

Country	GDP	GDP rank, x	LE	LE rank, y	$d = x - y$	d^2
Denmark	37 800	4	79.09	6	-2	4
Estonia	22 400	8	74.07	8	0	0
Faeroe Islands	30 500	6	80.11	4	**2**	**4**
Finland	35 900	5	79.69	5	**0**	**0**
Iceland	40 700	**3**	81.22	**3**	**0**	**0**
Latvia	19 100	**9**	73.44	**9**	**0**	**0**
Lithuania	22 600	7	75.98	7	**0**	**0**
Norway	55 400	1	81.60	2	-1	1
Sweden	40 900	2	81.89	1	1	1
				Total	**0**	**10**

Table 5.1

B1 Latvia correct

B1 All correct. The entry 0 for Σd must be included.

H_0: There is no association between GDP per capita and life expectancy.

H_1: There is association between GDP per capita and life expectancy.

B1 Both H_0 and H_1 required

Accept correlation, rank correlation, relationship, positive association.

Do not accept independent and dependent

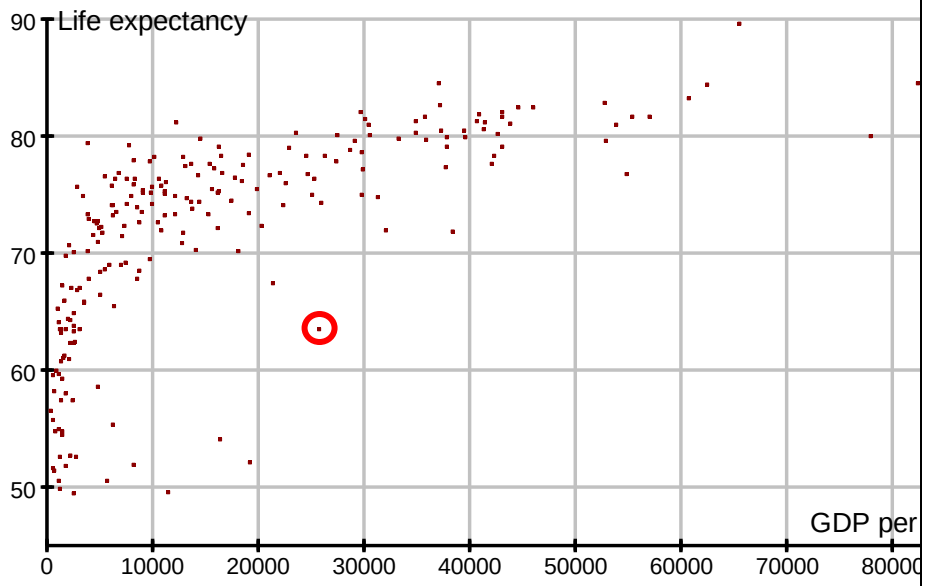
$$r_s = 1 - \frac{6\sum d^2}{n\left(n^2 - 1\right)} = 1 - \frac{6 \times 10}{9 \times \left(81 - 1\right)} = 0.917$$

M1 Attempt to use correct formula. Answer implies method.

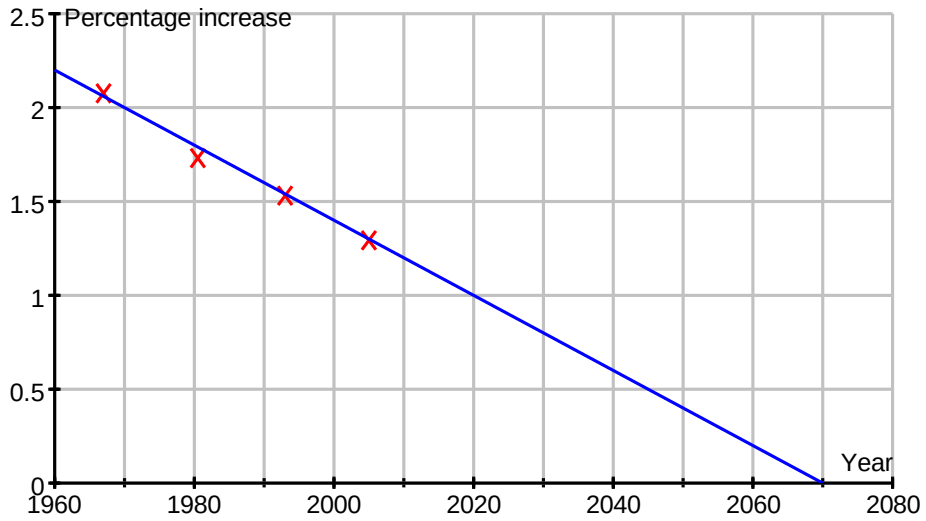
A1 No FT

Critical value for 2-tail test is 0.7000

B1 FT from their H_1. Notice that for a 1-tail test the critical value is 0.6000

		0.917 > 0.7000 so there is evidence of association	**B1**	Comparison must be seen. FT providing a correct procedure has been used.
			[7]	
5	**(ii)**	 **Figure 5.2**	**B1**	
			[1]	
5	**(iii)**	There is positive association/correlation (The relationship is not linear so) it is not correlation There is weak correlation	**B1** **B1**	Any two sensible comments that are different and do not contradict each other Allow comments based on the numbers, eg "There are no countries with life expectancy below 45". Allow "weak positive correlation" for both marks.

		There seem to be two different relationships, one for GDP per capita up to about 10 000 US$, and the other for higher values of the GDP.		
			[2]	

Question		Answer	Marks	Guidance
6	(i)	$1.00565^{123} = 1.999\,693 \ldots = 2.00$ to 3 significant figures 1.00565 is the multiplying factor corresponding to the increase of 0.565% p.a. for the 123 years from 1804 to 1927. In that time the population increased by a factor of 2.	**B1** **B1**	Rounding must be evident Must be related to the data given in the question
			[2]	
6	(ii)	 2070	**M1** **A1**	Straight line drawn Consistent with their graph but must be between 2055 and 2080 inclusive

		It is based on extrapolation beyond the known data	**B1**	Accept statements about the uncertainty of what will happen.
			[3]	
6	**(iii)**	$32 + 0.5 \times 60$ 62	**M1** **A1**	Attempting to use the frequencies for both the first two bars in Fig 6.3. Answer implies method. Allow 32 ± 1
			[2]	
6	**(iv)**	4 (Faroe Islands, Gibraltar, Iceland and Ireland) have birth rates above 12.5	**B1**	The countries need not be named.
			[1]	
6	**(v)**	Table 6.4 illustrates what seems to be a world-wide trend of falling birth rates So, it is reasonable to expect the world population to stop growing at some time but the date of 2070 can only be an estimate.	**B1** **B1**	Two sensible and different comments. At least one of the comments must relate to the world population. Allow comment that highlights that population increase and birth rates are different things
			[2]	

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