OCR
Oxford Cambridge and RSA

Monday 13 May 2019 – Afternoon

AS Level Further Mathematics A

Y531/01 Pure Core

Time allowed: 1 hour 15 minutes

7770864034

You must have:
- Printed Answer Booklet
- Formulae AS Level Further Mathematics A

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets **[ ]**.
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **4** pages.

Answer **all** the questions.

1 You are given that $z = 3 - 4\,\mathrm{i}$.

(a) Find

- $|z|$,
- $\arg(z)$,
- z^*. **[3]**

On an Argand diagram the complex number w is represented by the point A and w^* is represented by the point B.

(b) Describe the geometrical relationship between the points A and B. **[2]**

2 Matrices **P** and **Q** are given by $\mathbf{P} = \begin{pmatrix} 1 & k & 0 \\ -2 & 1 & 3 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} (1+k) & -1 \end{pmatrix}$ where k is a constant.

Exactly one of statements A and B is true.

Statement A: **P** and **Q** (in that order) are conformable for multiplication.
Statement B: **Q** and **P** (in that order) are conformable for multiplication.

(a) State, with a reason, which **one** of A and B is true. **[2]**

(b) Find either **PQ** or **QP** in terms of k. **[2]**

3 The position vector of point A is $\mathbf{a} = -9\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$.
The line l passes through A and is perpendicular to **a**.

(a) Determine the shortest distance between the origin, O, and l. **[2]**

l is also perpendicular to the vector **b** where $\mathbf{b} = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$.

(b) Find a vector which is perpendicular to both **a** and **b**. **[1]**

(c) Write down an equation of l in vector form. **[1]**

P is a point on l such that $PA = 2OA$.

(d) Find angle POA giving your answer to 3 significant figures. **[3]**

C is a point whose position vector, **c**, is given by $\mathbf{c} = p\mathbf{a}$ for some constant p. The line m passes through C and has equation $\mathbf{r} = \mathbf{c} + \mu\mathbf{b}$. The point with position vector $9\mathbf{i} + 8\mathbf{j} - 12\mathbf{k}$ lies on m.

(e) Find the value of p. **[3]**

4 In this question you must show detailed reasoning.

You are given that $f(z) = 4z^4 - 12z^3 + 41z^2 - 128z + 185$ and that $2 + i$ is a root of the equation $f(z) = 0$.

(a) Express $f(z)$ as the product of two quadratic factors with integer coefficients. **[5]**

(b) Solve $f(z) = 0$. **[3]**

Two loci on an Argand diagram are defined by $C_1 = \{z : |z| = r_1\}$ and $C_2 = \{z : |z| = r_2\}$ where $r_1 > r_2$. You are given that two of the points representing the roots of $f(z) = 0$ are on C_1 and two are on C_2. R is the region on the Argand diagram between C_1 and C_2.

(c) Find the exact area of R. **[4]**

(d) ω is the sum of all the roots of $f(z) = 0$.

Determine whether or not the point on the Argand diagram which represents ω lies in R. **[2]**

5 In this question you must show detailed reasoning.

You are given that α, β and γ are the roots of the equation $5x^3 - 2x^2 + 3x + 1 = 0$.

(a) Find the value of $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$. **[5]**

(b) Find a cubic equation whose roots are α^2, β^2 and γ^2 giving your answer in the form $ax^3 + bx^2 + cx + d = 0$ where a, b, c and d are integers. **[4]**

6 A transformation T is represented by the matrix $\mathbf{T}$ where $\mathbf{T} = \begin{pmatrix} x^2 + 1 & -4 \\ 3 - 2x^2 & x^2 + 5 \end{pmatrix}$.

A quadrilateral Q, whose area is 12 units, is transformed by T to Q'.

Find the smallest possible value of the area of Q'. **[5]**

Turn over for questions 7 and 8

7 A transformation A is represented by the matrix $\mathbf{A}$ where $\mathbf{A} = \begin{pmatrix} -1 & x & 2 \\ 7-x & -6 & 1 \\ 5 & -5x & 2x \end{pmatrix}$.

The tetrahedron H has vertices at O, P, Q and R. The volume of H is 6 units.

P', Q', R' and H' are the images of P, Q, R and H under A.

(a) In the case where $x = 5$

- find the volume of H',
- determine whether A preserves the orientation of H. **[3]**

(b) Find the values of x for which O, P', Q' and R' are coplanar (i.e. the four points lie in the same plane). **[4]**

8 **In this question you must show detailed reasoning.**

$\mathbf{M}$ is the matrix $\begin{pmatrix} 1 & 6 \\ 0 & 2 \end{pmatrix}$.

Prove that $\mathbf{M}^n = \begin{pmatrix} 1 & 3(2^{n+1}-2) \\ 0 & 2^n \end{pmatrix}$, for any positive integer n. **[6]**

END OF QUESTION PAPER