

GCE

Mathematics A

H240/02: Pure Mathematics and Statistics

Advanced GCE

Mark Scheme for June 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

| Annotation in scoris | Meaning |
|------------------------|--|
| ✓and × | |
| BOD | Benefit of doubt |
| FT | Follow through |
| ISW | Ignore subsequent working |
| M0, M1 | Method mark awarded 0, 1 |
| A0, A1 | Accuracy mark awarded 0, 1 |
| B0, B1 | Independent mark awarded 0, 1 |
| SC | Special case |
| ^ | Omission sign |
| MR | Misread |
| Highlighting | |
| | |
| Other abbreviations in | Meaning |
| mark scheme | |
| E1 | Mark for explaining a result or establishing a given result |
| dep* | Mark dependent on a previous mark, indicated by * |
| cao | Correct answer only |
| oe | Or equivalent |
| rot | Rounded or truncated |
| soi | Seen or implied |
| www | Without wrong working |
| AG | Answer given |
| awrt | Anything which rounds to |
| BC | By Calculator |
| DR | This question included the instruction: In this question you must show detailed reasoning. |

С

Subject-specific Marking Instructions for A Level Mathematics A

- a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.
- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader. The following types of marks are available.

Μ

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

Α

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

В

Mark for a correct result or statement independent of Method marks.

Е

Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

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|---------|---|--|
| d | When a part of a question has two or more 'method' steps, the M marks are in principle independent unless to otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate the dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a course in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be a constructed. | at a particular mark is andidate has once gone wrong the other hand, when two or |
| e | The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from pre Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate sta various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark please, escalate the question to your Team Leader who will decide on a course of action with the Principal E: Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A n through'. In such cases you must ensure that you refer back to the answer of the previous part question ever image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question | (accuracy) marks are not given age of a solution, there may be k scheme. If this is not the case xaminer. narks will often be 'follow n if this is not shown within the |
| f | We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is the scheme explicitly says so. | usually only penalised where |
| | When a value is given in the paper only accept an answer correct to at least as many significant figures as the | ne given value. |
| | When a value is not given in the paper accept any answer that agrees with the correct value to 3 s.f. unless for another level of accuracy. NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement r | |
| g | Follow through should be used so that only one mark is lost for each distinct accuracy error. Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/s examiners should do as the candidate requests; if there are two or more attempts at a question which have n should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-sp those in the assessor handbook. | ot been crossed out, examiners |
| h | For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the ques according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as there are no other errors. E marks are lost unless, by chance, the given results are established by equival | generally appropriate, though as cao may be awarded as long ent working. 'Fresh starts' will |
| i | not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a m If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for co course, that there is nothing in the wording of the question specifying that analytical methods are required). V there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting meth consult your Team Leader. | rrect answers (provided, of Vhere an answer is wrong but |
| j | If in any case the scheme operates with considerable unfairness consult your Team Leader. | |

Mark Scheme

| | | | FinalVersion | | | | |
|---|---------|------|--|-----------------|--------------------|--|---|
| (|)uestio | n | Answer | Mark | AO | Guidane | |
| | | | NB Answers must be correct to 3 sf, except where | e otherw | ise indi | <u>cated</u> . If correct answer seen (to > 3 sf), is | nore later rounding. |
| 1 | | | | | | Ignore incorrect \int or dx in all parts | |
| 1 | (a) | (i) | $\frac{(2x+1) \times 2x - x^2 \times 2}{(2x+1)^2} \text{ oe}$ $(\text{eg} = \frac{2x^2 + 2x}{(2x+1)^2} \text{ or } \frac{2x(x+1)}{(2x+1)^2} \text{ oe})$ Alternative method $x^2(-2)(2x+1)^{-2} + 2x(2x+1)^{-1} \text{ oe}$ | B1 B1 B1 | 1.1a 1.1 1.1 | $2x(2x + 1) \underline{\text{or}} - 2x^{2} \text{ oe in numerator} \qquad B1$ Correct denominator B1 Correct numerator B1 No need to see this $\pm 2x^{2}(2x + 1)^{-2} \qquad \text{oe} \qquad B1$ | Condone missing brackets 1st B1 Allow correct equivalent forms ISW for further "simplifications" |
| | | | | [3] | | $+2x(2x+1)^{-1}$ oe B1 All correct B1 | Allow correct equivalent forms ISW for further "simplifications" |
| 1 | (a) | (ii) | $(2x-3)\sec^2(x^2-3x)$ oe | B1 B1 [2] | 1.1a 1.1 | B1 for $\sec^2 (x^2 - 3x)$ B1 for all correct | Condone missing brackets 1st B1 Condone $\sec^2 (x^2 - 3x)(2x - 3)$ ISW for further "simplifications" |
| | | | Allow without mod in both parts (b) and (c) | | | | |
| 1 | (b) | | $x = (u+1)^2, \frac{dx}{du} = 2(u+1) \text{ oe}$ or $\frac{du}{dx} = 0.5x^{-0.5} \text{oe}$ | M1 | 1.1 a | EITHER attempt x in terms of u & diff OR attempt $\frac{du}{dx}$ & obtain $k x^{-0.5}$ oe | Allow in form $dx = \dots$ or $du = \dots$ |
| | | | $2\int \frac{(u+1)}{u} du$ or $2\int \left(1+\frac{1}{u}\right) du$ oe | A1 | 2.5 | Allow $k \int \frac{(u+1)}{u} du$ or $k \int \left(1 + \frac{1}{u}\right) du$ | or $\int \frac{(ku+j)}{u} du$ or $\int \left(k+\frac{j}{u}\right) du$ |
| | | | $=2(u+\ln u/) \qquad (+c)$ | A1 | 2.1 | Allow without $+ c$ here | |
| | | | $= 2(\sqrt{x} - 1 + \ln \sqrt{x} - 1) + c$ oe | A1 | 1.1 | | Not penalise + <i>c</i> in both (b) &(c) |
| | | | or $2(\sqrt{x} + \ln \sqrt{x}-1) + c$ oe or $2\sqrt{x} + \ln(\sqrt{x}-1)^2 + c$ oe | | | All correct incl $+ c$ | ISW for further "simplifications" |
| | | | | [4] | | | Integration by parts: Use same scheme. |

| 1 | (c) | | $\ln 2x^2 - 8x - 1 $ or $\ln \frac{1}{2}x^2 - 2x - \frac{1}{4} $ seen | M1 | 1.2 | or $u = 2x^2 - 8x - 1$ and $\ln u $ seen | or $u = x - 2$ and $\ln 2u^2 - 9 $ seen |
|---|-----|---------------|---|------------------|--------------|--|--|
| | | | $\frac{1}{4}\ln 2x^2 - 8x - 1 + c \text{or } \frac{1}{4}\ln \frac{1}{2}x^2 - 2x - \frac{1}{4} + c$ | A1 | 1.1 | All correct including + c Correct answer seen: M1A1 even if eg $(x-2)\frac{\ln 2x^2-8x-1 }{4x-8} = \frac{1}{4}\ln 2x^2-8x-1 $ | Not penalise + <i>c</i> in both (b) &(c) |
| | | | | [2] | | $(x-2)\frac{4x-8}{4x-8} = \frac{4}{4}\ln 2x^2 - 8x - 1 $ | ISW for further "simplifications |
| 2 | (a) | | = -48384 or -48400 | [2] B1 [1] | 1.1 | Allow –48384 <i>x</i> ⁵ | |
| 2 | (b) | (i) | $1 + 0.5 \times 3x + \frac{0.5 \times (-0.5)}{2} \times (3x)^2$ | | | M1 for at least 3 terms correct | |
| | | | + $\frac{0.5 \times (-0.5)(-1.5)}{3!} \times (3x)^3$ | M1 | 1.1a | Condone any missing brackets | SC $1 + \frac{3}{2}x - \frac{3}{8}x^2 + \frac{3}{16}x^3$: M1 |
| | | | $= 1 + \frac{3}{2}x - \frac{9}{8}x^2 + \frac{27}{16}x^3$ or $1 + 1.5x - 1.125x^2 + 1.6875x^3$ | A1 A1 | 1.1 1.1 | A1 for 3 terms correct A1 for all correct | |
| | | | or $1 + 1.5x - 1.125x^2 + 1.0875x^2$ | [3] | | | |
| 2 | (b) | (ii) | $-\frac{1}{3} < x < \frac{1}{3}$ | B 1 | 1.2 | Allow $ x < \frac{1}{3}$ | |
| | | | 5 5 | [1] | | _ | |
| 2 | (b) | (iii) | Sub $x = 0.01$ in their expansion | M1 | 3.1 a | | Other correct methods may be seen, eg subst $x = 0.2 & \sqrt{1.6}$ |
| | | | gives $\sqrt{1.03} = 1.014889$ | A1 | 1.1 | Allow 1.01489 here (5 dps for series) | |
| | | | From series $\sqrt{103} = 10.14889(188)$ From calculator $\sqrt{103} = 10.14889(157)$ | A1 | 2.2b | If no working seen, 10.1488919 or better must be seen as evidence that series has been used. Both these must be seen for A1 | 5dps for $\sqrt{103}$ in both |
| | | | (Hence expansion may be correct) | [3] | | Allow without statement | |
| | | | | | | | |

| 3 | (a) | (i) | $(x-3)^2 + (y+4)^2 = 4\cos^2\theta + 4\sin^2\theta$ | M1 | 3.1a | or $\left(\frac{x-3}{2}\right)^2 + \left(\frac{y+4}{2}\right)^2 = \cos^2\theta + \sin^2\theta$ | Condone sign errors or one arith slip or missing brackets for M1 |
|---|-----|---------------|--|------------|------|--|---|
| | | | $\Rightarrow (x-3)^2 + (y+4)^2 = 4 \text{ oe ISW}$ | A1 | 2.1 | or $\left(\frac{x-3}{2}\right)^2 + \left(\frac{y+4}{2}\right)^2 = 1$ oe | or $y = -4 + 2\sqrt{1 - (\frac{x-3}{2})^2}$ M1A1 or similar with $x =$ |
| | | | | | | or $\cos^{-1}\left(\frac{x-3}{2}\right) = \sin^{-1}\left(\frac{y+4}{2}\right)$ M1A1 | or $y = -4+2\sin(\cos^{-1}\frac{x-3}{2})$ M1A1 or similar with $x =$ |
| | | | | [2] | | ISW for all answers | |
| 3 | (a) | (ii) | Centre (3, -4), radius 2 | [2] B1f | 2.2a | ft their (i) if both consistent with (i) But if absolutely correct, not ft: B1. | |
| 3 | (b) | | DR | [1] | | | |
| 5 | (0) | | NB Allow decimals to 2 sf instead of surds | | | | |
| | | | thoughout, except answer to 3 sf | | | | |
| | | | $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$ | M1 | 3.1a | Attempt diff x & y wrt t & find $\frac{dy}{dt} \div \frac{dx}{dt}$ | |
| | | | $= -\frac{1}{2}\cot t \text{or} -\frac{1}{2}\frac{\cos t}{\sin t}$ | A1 | 1.1 | soi | |
| | | | $t = \frac{\pi}{6}$: $\frac{dy}{dx} = -\frac{1}{2}\cot \frac{\pi}{6}$ or $-\frac{\sqrt{3}}{2}$ or | M1 | 1.1 | Substitute $t = \frac{\pi}{6}$ in their $\frac{dy}{dx}$ | Allow sign error |
| | | | Alternative methods for gradient | | | | |
| | | | $\left(\frac{x}{4}\right)^2 + \left(\frac{y}{2}\right)^2 = 1, \qquad \frac{x}{8} + \frac{y}{2}\frac{dy}{dx} = 0 \qquad M1$ | | | Attempt cartesian equn & differentiation | $\frac{\rm d}{{\rm d}x}(0.5(16-x^2)^{-0.5})$ |
| | | | $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{x}{4y} \qquad \qquad$ | | | soi | or $\frac{dy}{dx} = \frac{1}{4}(16 - x^2)^{-0.5}(-2x)$ oe |
| | | | $t = \frac{\pi}{6}: \ \frac{dy}{dx} = -\frac{4\cos(\frac{\pi}{6})}{8\sin(\frac{\pi}{6})} \text{ or } -\frac{1}{2}\cot\frac{\pi}{6} \text{ or } -\frac{\sqrt{3}}{2} \text{ M1}$ | | | Substitute $t = \frac{\pi}{6}$ in x (and y) & their $\frac{dy}{dx}$ | |
| | | | | | | | |

Mark Scheme

| | | | Equn of <i>L</i> is $y - 2\sin\frac{\pi}{6} = -\frac{\sqrt{3}}{2}(x - 4\cos\frac{\pi}{6})$ oe or $y - 1 = -\frac{\sqrt{3}}{2}(x - 2\sqrt{3})$ oe $0 - 1 = -\frac{\sqrt{3}}{2}x + 3$ oe Cuts at $(\frac{8\sqrt{3}}{3}, 0)$ oe or (4.62, 0) (3 sf) | M1 M1 A1 [6] | 1.1 | or $y=-\frac{\sqrt{3}}{2}x+c$ $\underline{\&}$ subst $(4\cos\frac{\pi}{6}, 2\sin\frac{\pi}{6})$ or $y=-\frac{\sqrt{3}}{2}x+4$ oe or $0=-\frac{\sqrt{3}}{2}x+4$ oe Allow just $\frac{8\sqrt{3}}{3}$ or 4.62 (3 sf) | ft their grad (not -ve reciprocal) Must not involve t This mark may be implied by next mark Subst $y = 0$ in their line equn, not involving t Allow equivalents, eg $\frac{8}{\sqrt{3}}$ |
|---|-----|------|---|-----------------------|------|--|--|
| 4 | | | In all parts ignore nos except 20, & 1020 | | | BOD if describe growth rather than rate in (a) and (b) | Condone muddle between P and growth of P in (a) and (b) |
| 4 | (a) | | A: Growth (rate) increases, then decreases Grows slowly, then quickly, then slowly B: Growth (rate) decreases Grows quickly then slowly Both | B1 [1] | 2.2b | Allow increase, constant, then decrease or "levels off", "tails off", "plateaus" Allow "levels off", "tails off", "plateaus" | NOT " <i>P</i> " decreases, for A or B Ignore "exponentially" |
| 4 | (b) | (i) | A: <i>P</i> (decreases and) tends to 20 or (Decreases and) doesn't go below 20 | B1 [1] | 3.4 | Allow (Decrease and) reach 20, Must mention 20 (as population, not years) | Ignore all else |
| 4 | (b) | (ii) | B: <i>P</i> tends to 1020 oe <i>P</i> doesn't exceed 1020 | B1 [1] | 3.4 | Growth is asymptotic around 1020 Settles at 1020. Saturates at 1020 Converges to 1020. Allow reaches 1020 Plateaus at 1020. Asymptote at 1020 Must mention 1020 | NOT: Pop increases, but slowly Diverges to 1020 Tends to 1020, then down Ignore all else |
| 4 | (c) | (i) | A: Food (almost) runs out, or is used up oe or becomes very low or there will be a shortage oe or begins to run out | B1 | 3.5a | or will only support a population of 20 Won't sustain large nos. Insufficient NB "Limited" allowed in c(ii), not c(i) | NOT: just Limited, Finite NOT: just "Decreases" Ignore all else |
| 4 | (c) | (ii) | B: Food sufficient to support a pop ≈ 1020 Enough to sustain equilibrium (or population) Barely enough, can't support increase in P Food limited so pop can't continue to grow | B1 [1] | 3.5a | Stays stable Sustainable Constant | Must imply at least two of: 1. Food won't run out <u>and</u> 2. Food limited or equilibrium 3. Can't support increase in <i>P</i> Ignore all else |

Mark Scheme

| 5 | (a) | | An example of a correct method: | | | Other correct methods may be seen eg $lr = 4$: B1, find <i>h</i> ito <i>l</i> & <i>r</i> : B1, | NB Must be <u>clearly</u> using this alternative method to score any |
|---|-----|----|--|------------|------|---|---|
| | | | | | | Subst h & lr into V: M1, convincing: A1 | marks on this scheme. |
| | | | $l = \frac{4}{r}$ or $l = \frac{4\pi}{r\pi}$ exactly (not $lr = 4$) | B 1 | 3.1a | Express l correctly in terms of r May be implied | Allow $l = \frac{4-r^2}{r}$ oe B1 |
| | | | $(h = \sqrt{l^2 - r^2})$ | | | | $(h^2 = l^2 - r^2)$ |
| | | | $h = \sqrt{\frac{16}{r^2} - r^2}$ or $\frac{\sqrt{16 - r^4}}{r}$ oe | B 1 | 1.1 | Express h (or h^2) correctly in terms of r alone | or $h^2 = \frac{16}{r^2} - r^2$ or $\frac{16 - r^4}{r^2}$ |
| | | | $V = \frac{1}{3}\pi r^2 \sqrt{\frac{16}{r^2} - r^2} \qquad \text{or } \frac{1}{3}\pi r^2 \frac{\sqrt{16 - r^4}}{r} \text{ oe}$ | M1 | 1.1 | Sub their <i>h</i> (in terms of <i>r</i> alone) into $\frac{1}{3}$ | |
| | | | 3° $\sqrt{r^2}$ 3° r | | | $\pi r^2 h$ | |
| | | | $(=\frac{\pi}{3}\sqrt{16r^2-r^6} \ AG)$ | A1 | 2.1 | Must see a correct previous expression in terms of <i>r</i> only, and the answer | |
| | | | | [4] | | | |
| 5 | (b) | DR | $\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{\pi}{3}\sqrt{16r^2-r^6}\right)$ | M1 | 1.1a | Attempt differentiate V or $\frac{V}{\pi}$ or $3V$ | or $\frac{3V}{\pi}$ or $\sqrt{16r^2 - r^6}$ or |
| | | | | | | | $16r^2 - r^6$ |
| | | | $\frac{\pi(32r-6r^5)}{3\times 2\sqrt{16r^2-r^6}} = 0 \text{oe}$ | A1 | 2.1 | Correct derivative of one of the above Condone missing brackets. | All subsequent marks can be scored even if this A1 not scored. |
| | | | (Their derivative = 0) | | | | |
| | | | $r = \frac{2}{\sqrt[4]{3}}$ or $\sqrt[4]{\frac{16}{3}}$ oe or 1.52 (3 sf) Allow 1.5 | A1 | 1.1 | Lose this mark if incorrect values of r also given, eg $r = \pm 2$ obtained from | |
| | | | or $r^2 = \frac{4}{\sqrt{3}}$ | | | $(16r^2 - r^6)^{-\frac{1}{2}} = 0$ | Allow without $r = 0$ |
| | | | $r = -\frac{2}{\frac{4}{\sqrt{3}}}$ or -1.52 invalid OR $r = 0$ invalid or $r > 0$ | B1f | 3.2a | Comment needed, about their negative r (ft) or about $r = 0$ | |
| | | | π. Γ | | | | T & I: 5.20 (3sf) SC B2 |
| | | | $(V_{\rm max} = \frac{\pi}{3}\sqrt{16 \times 1.51967^2 - 1.51967^6})$ | | | | 5.2 (2 sf) SC B1 |
| | | | Max $V = 5.20$ (3 sf) Allow 5.2 or a.r.t. 5.2 | A1 | 1.1 | Condone $V = 5.20 \text{ m}^3$ | |
| | | | | [5] | | | |

Mark Scheme

| 1 | | | | | | |
|---|-----|---|-----------|--------------|--|---|
| 6 | | $2^{2k} - 1$ or $4^k - 1$ (where k is an integer > 1) | M1 | 3.1a | or $2^{2k+2}-1$ or $2^{2k+4}-1$ Allow $2^{2n}-1$ | Induction: |
| | | $= (2^{k})^{2} - 1$ = $(2^{k} - 1)(2^{k} + 1)$ | A1 | 2.1 | $=(2^{k+1}-1)(2^{k+1}+1)$ oe | Assume $2^k - 1$ is \div by 3 (k even) M1 Let $2^k - 1 = 3p$ (p integer) |
| | | = (2 - 1)(2 + 1) | AI | 2.1 | $= (2^{k-1})(2^{k+1}) \text{ of } (2^{k+2}-1)(2^{k+2}+1) \text{ of } (2^{k+2}-1)(2^{k+2$ | Let $2 - 1 = 5p$ (p integer) |
| | | $(2^{k} + 1) > 1$ and $k > 1$, hence $(2^{k} - 1) > 1$ | M1 | 1.1 | | $2^{k+2} - 1 = 4 \times 2^k - 1 $ M1 |
| | | Hence $(2^k - 1)(2^k + 1)$ is the product of two | | | | $=4\times 2^{k}-4+3$ |
| | | integers, both > 1 , and hence $2^n - 1$ is not prime | A1 | 2.2a | Both statements needed | $=4(2^{k}-1)+3$ |
| | | | | | | $= 4 \times 3p + 3 \qquad A1$ which is \div by 3 |
| | | | | | | When $k = 2$: $2^2 - 1 = 3$ so \div by 3 A1 |
| | | | [4] | | | Hence true for all even <i>n</i> . Claim true |
| 7 | | DR $u^2 = 36x^4 + 12x^3 + x^2$ | 2.61 | 2.1 | | |
| | | $u^{2} = 36x^{4} + 12x^{3} + x^{2}$ So $36x^{4} + 12x^{3} + 7x^{2} + x - 2 = u^{2} + 6x^{2} + x - 2$ | M1 | 3.1 a | $ (36x^4 + 12x^3 + 7x^2 + x - 2) \div (6x^2 + x) $ = 6x ² + x + 1 rem -2 | x) M1 for attempt $(6x^2 + x)^2$ or attempt \div LHS by $(6x^2 + x)$, |
| | | $50\ 50x\ +\ 12x\ +\ 7x\ +\ x\ -\ 2\ -\ u\ +\ 0x\ +\ x\ -\ 2$ | | | = 0x + x + 1 rem -2 | at least 2 terms correct |
| | | | | | ((6x2 + x) (6x2 + x + 1) = 2) | or obtain any correct equn in |
| | | | | | | terms of <i>x</i> and <i>u</i> |
| | | Equn reduces to $u^2 + u - 2 = 0$ u = -2 or 1 | A1 A1 | 2.1 1.1 | u(u+1) = 2 BC | |
| | | u = -2 or 1 | AI | 1.1 | _ | -1+./47i |
| | | $6x^2 + x = -2$ has no roots because $\Delta = 1 - 48 < 0$ | B1 | 3.2a | Must see correct calc'n for Δ and "< for their quadratic equation | 0" or $x = \frac{-1 \pm \sqrt{47i}}{12}$ given instead of |
| | | | | | for men quadratic equation | "no roots" etc |
| | | $6x^2 + x = 1$ has roots $x = \frac{1}{3}$ or $-\frac{1}{2}$ | A1 | 1.1 | BC Ignore any answers from $u = -2$ | 2 |
| | | 5 2 | | | SC If M1 gained but incorrect or | Otherwise correct ans without |
| | | | | | inadequate method & correct answer | |
| | | | [5] | | M1A0A0B0A1 | (Because DR) |
| 8 | (a) | 65 | [5] B1 | 1.1 | | |
| | () | Quartiles are 76 and 61 | M1 | 1.1 | Allow (75 to 77) and (59 to 63) | |
| | | = 15 | A1 | 1.1 | Must come from 76 – 61 | |
| | | | [2] | | SC Misread $5 \mid 6 = 5.6$: lose B1 onl | ly |
| 8 | (b) | Mean = 69 | [3] B1 | 1.1 | Allow 6.9 | |
| 0 | | sd = 10.5 (3 sf) | B1 B1 | 1.1 | Allow 1.05 (3 sf) | |

| | | | | [2] | | | |
|---|-----|-------|--|----------------------|--------------|---|--|
| 8 | (c) | | Less (or not) affected by the outlier or anoma Mean (more) affected by the outlier of 99 | aly 99 B1 [1] | 1.1 | oe, but must mention 99 Allow "Median is less skewed by the 99" | Ignore all else, eg "more accurate" |
| 9 | (a) | (i) | 0.761 or 0.762 (3 sf) | B1 [1] | 1.1 | BC Allow 0.76 | |
| 9 | (a) | (ii) | 62.0 (3 sf) | B1 [1] | 1.1 | BC Allow 62 or 61.9 | Allow $m \ge 62.0$ |
| 9 | (a) | (iii) | Use of \overline{X} eg " \overline{X} " or "mean" or $\frac{18}{10}$ or $\sqrt{\frac{18}{10}}$ | | 1.1 a | $\mu = 550$ seen or implied | |
| | | | $\overline{X} \sim N(55, \frac{18}{10})$ | M1 | 3.3 | $\Sigma X \sim N(550, 180)$ Correct | May be implied |
| | | | $P(\bar{X} < \frac{530}{10}) \ dep \ \sigma^2 = \frac{18}{10}$ | M1 | 3.4 | P(Σ <i>X</i> < 530) dep σ^2 = 180 | Stated or implied |
| | | | = 0.0680 (3 sf) | | 1.1 | = 0.0680 (3 sf) Allow 0.068 | Correct answer from limited (or no) working: M1M1M1A1 |
| 9 | (b) | | $P(Y < 72) = 0.75$ $P(Y < 62) = 0.25$ $\Phi^{-1}(0.75)$ or 0.674 $\Phi^{-1}(0.25)$ or -0.674 | | 3.1b 2.4 | oe May be implied, eg on diagram ±0.674 implies M1M1 Allow 0.67 | NB P($62 < Y < 72$) = 0.5 no mks yet |
| | | | $\frac{72-67}{\sigma} \qquad \qquad \frac{62-67}{\sigma}$ | M1 | 2.1 | | M1M1M1 may be implied by A1 |
| | | | $\frac{72-67}{\sigma} = 0.674$ $\frac{62-67}{\sigma} = -0.67$ | 4 A1 | 1.1 | oe, eg 5 = 0.674σ A1 for correct equn, allow 0.67 | Must be seen |
| | | | $\sigma = 7.41 \text{ or } 7.42 (3 \text{ sf})$ | A1 | 1.1 | SC correct answer with no working or irrelevant working: SC B3 (because "determine" rather than "find") | or SC B2 if correct to 2 sf |
| | | | Trial and Improvement $\Phi^{-1}(0.75)$ or 0.674 or $\Phi^{-1}(0.25)$ or -0.674 eg $\sigma = 8$: 67 - 8×0.674 = 61.6 $\sigma = 7$: 67 - 7×0.674 = 62.3 | M2 M1 A1 | | May be implied At least one correct trial Trials leading to values either side of 62 | |
| | | | $\sigma = 7.41: \ 67 - 7.41 \times 0.674 = 62.0 \implies \sigma$ or $\sigma = 7.42: \ 67 - 7.42 \times 0.674 = 62.0 \implies \sigma$ | | | Correct trial using $\sigma = 7.41$ or 7.42 and conclusion $\sigma = 7.41$ or 7.42 | |

Mark Scheme

| | | [5] | | | |
|----|---|----------|------------|--|--|
| 10 | See the exemplars at the end of the MS Allow 2 sf throughout | | | | If 1-tail test: |
| | H ₀ : $\mu = 0.034$ H ₁ : $\mu \neq 0.034$ where $\mu = (pop)$ mean pollutant level Allow any letter (except X and \overline{X} : B0B0) | B1 B1 | 1.1 2.5 | Subtract B1 for each error eg:1-tailB1B0undefined μ B1B0not in terms of parameterB1B0 μ = sample mean impliedB1B0Not include value 0.034B0B0eg H_0 = 0.034 etc:B0B0 | H ₀ : $\mu = 0.034$, defined μ , B1 H ₁ : $\mu < 0.034$ B0 |
| | N(0.034, $\frac{0.0000409}{50}$)& $\overline{X} < 0.0325$ (condone >, =) or $\frac{a - 0.034}{0.00640 \div \sqrt{50}} = -1.96$ | M1 | 3.3 | Stated or implied eg by 0.0486 or 0.951 or 0.322 (2 sf) even if within incorrect statement eg $P(X = 0.0325) = 0.0486$ | As LH column M1 |
| | P($\overline{X} < 0.0325$) = 0.0486 or CV is 0.0322 or acceptance region is 0.0322 to 0.0358 or P($\overline{X} > 0.0325$) = 0.951 | A1* | 3.4 | BC or $\frac{0.0325 - 0.034}{0.00640 \div \sqrt{50}} = -1.66$ | P($\overline{X} < 0.0325$) = 0.0486 or CV is 0.0325(1) A1 |
| | 0.0486 > 0.025 or 0.0325 > 0.0322 or 0.0325 is in AR or 0.951 < 0.975 A1A1 | A1 | 1.1 | or 1.66 < 1.96 or -1.66 > -1.96 dep A1* Must be seen, allow on diag | Comp 0.05 or ±1.645 A1 or 0.0325 < 0.0325(1) or 0.0325 is within CR No more marks |
| | Do not reject H_0 Or Insufficient (or No) evidence to reject H_0 Allow Accept H_0 | M1 | 1.1 | Dep 0.0486 or 0.0322 or -1.66 <u>or</u> 0.951 < 0.975 seen, <u>or</u> P(\overline{X} < 0.0325) stated or implied (possibly with wrong prob leading to opposite conclusion**) but 0 if.951 > 0.025 M0 | May be implied by conclusion Condone Reject H ₁ |
| | Insufficient (or No) evidence that (mean) pollutant level has changed oe or eg "It is unlikely that level has changed" | A1f | 2.2b | In context. Context may be implied by eg "level" or "pollutant" | Not "There is evidence that mean level has not changed" A0 Not definite, eg |

| [| | | "We can assume that level hasn't changed" | Τ | Ι | ft only ** above | "Mean level has not changed "A0 |
|----|-----|------|---|-----------------------------|--------------|--|--|
| | | | <u> </u> | [7] | | | <u> </u> |
| 11 | (a) | | k > 1.4 	(allow k > 1.1 to 1.6) k < 0.25 	(allow k < 0.2 to 0.3) | B1 B1 | 2.2b 2.2b | Allow \geq and \leq SC: 0.25 < k < 1.4: B1B0 (ranges as on left) | Allow "x" |
| | | | or $1.4 < k < \dots$ B1 $\dots < k < 0.25$ (ranges as above) B1 | [2] | | | |
| 11 | (b) | (i) | 0.797 > 0.5577 or -0.797 < -0.5577 or -0.797 > 0.5577 | B2 [2] | 3.1b 3.2a | $\begin{array}{ccc} 0.797 > 0.6055 \text{ or } -0.797 < -0.6055 \text{ B1} \\ \pm 0.5577 & \text{B1} \end{array}$ | Allow \geq or \leq |
| 11 | (b) | (ii) | There are clusters (or groups etc.) Apparent good correlation caused by clusters or Two clusters with no –ve corr'n within them | B1* | 2.3 | | NOT Too scattered Not represent whole pop Small sample |
| | | | or a comment similar to one of the above. AND Conclusion unreliable or Value of <i>r</i> is misleading oe | B1 dep B1* [2] | 3.5b | or Not bivariate normal distribution B1 so use of tables for <i>r</i> not valid B1 | Clusters not on reg line B1B0 |
| 11 | (c) | | High prop of 65+or Low prop of 18-24Prop of young very similar, or ≈ 0.06 Proportion of senior to young is high | [1] | 2.2b | If consider only <u>one</u> age-group, must be proportion not number If consider <u>both</u> age-groups, allow eg Higher number of seniors than young or Many seniors, few young | NOT: Similar proportions of 65+ Population is elderly |
| 11 | (d) | | Top left points contain high prop of 18-24s. (So these LAs may be areas where there are universities or where they can recruit) | B1 | 2.2b | Shows places where large nos of 18-24s Shows where to focus recruiting. So universities can recruit. 18-24s are their target group. No need to specify "Top left group" | Allow "students" or "young" instead of "18-24s" Any implication that diagram enables you to see information about location of young people |
| 12 | (a) | | k(1+2+3+4+5) (=1) | M1 | 3.3 | Allow $15k (= 1)$ | |
| | | | $k = \frac{1}{15}$ | A1 | 1.1 | May be implied | |

Mark Scheme

| | | | $P(X=3) = 3 \times \frac{1}{15} \text{ or } \frac{3}{15} (=0.2 \text{ AG})$ | A1 | 2.1 | Must see $3 \times \frac{1}{15}$ or $\frac{3}{15}$ and answer 0.2 | | | | | |
|----------|------------|---------------|---|----------|--------------|--|--|------------|---|---------------------------|--|
| | | | 15 15 | [3] | | 15 15 | | | | | |
| 12 | (b) | | $ \begin{array}{cccccccccccccccccccccccccccccccccccc$ | M1 A1 | 1.2 1.1 | M1 for \geq 3 probs correct, ft their <i>k</i> cao. Allow decimals (2 dp) SC: Table with all five probs = 0.2: M1 | Allow $X = 0$ or $X = 6$ or $X = 6+$ if prob shown as 0 | | = 6+ | | |
| | | | | [2] | | | | | | | |
| 12 12 | (c) (c) | (i) | Both parts. Allow mixture of methods | | | | | Specia | 00505 | | |
| 14 | (C) | (1) | | | | | 2-way tab | | All prob |)S | |
| | | | $\frac{3}{15} \times \frac{4}{15} + \frac{2}{15} \times \frac{5}{15}$ oe | | | | Count 4 pa | | = 0.2 $0.2^{2}+0.2$ | | |
| | | | 15 15 15 15 | M1 | 3.4 | Correct products added, ft their table | But if (b) correct: | But if (b) | | But if (b) correct: M0 | |
| | | | $\times 2$ | M1 | 3.1 a | 2×(Sum of two products of probs) | ÷ 25 | M1 | $2 \times (0.2^2 + \text{Allow wit})$ M1 | | |
| | | | $=\frac{44}{225}$ or 0.196 (3 sf) | A1 | 1.1 | cao | = 0.16 | A0 | = 0.16 | A0 | |
| | | | | [3] | | | <u>C</u> 2 | • | | | |
| 12 | (c) | (ii) | P(one value is 2 & $T = 7$) = $2 \times \frac{2}{15} \times \frac{5}{15}$ | M1 | 3.4 | Allow without " $2\times$ ", ft their table | Count 2 pa | urs M1 | 2×0.2^2 | M1 | |
| | | | $=\frac{4}{45}$ | A1f | 1.1 | ft their table (except if all probs $= 0.2$) | | A0 | = 0.08 | A0 | |
| | | | $\frac{P(\text{one value is } 2 \& T = 7)}{P(T = 7)} \qquad (\frac{\frac{4}{45}}{\frac{44}{225}} \text{ or } = \frac{0.0889}{0.196})$ | M1 | 2.1 | $\frac{\text{Allow any probability}}{\text{Their (c)(i) or their } P(T = 7)}$ | ÷4 | M1 | $\frac{0.08}{0.16}$ | M1 | |
| | | | $=\frac{5}{11}$ or 0.455 (3 sf) | A1 | 1.1 | cao NB not 0.454 | = 0.5 | A0 | = 0.5 | A0 | |
| | | | | [4] | | Eg: If (i) $\frac{22}{225}$, $\frac{2}{45} \div \frac{22}{225} = \frac{5}{11}$ M1A0M1A0 | | | | | |

| H240/02 | | Mark Scheme | | | June 2019 |
|---------|--|----------------------------|---|---|--|
| H240/02 | N(5000×0.26, 5000×0.26×0.74) may be implied = N(1300, 962) 1300-2× $\sqrt{"962"}$ or 1300 - 1.96× $\sqrt{"962"}$ (= 1238) (= 1239) P(X < 1239) or P(X ≤ 1238) or P(X < 1240) or P(X ≤ 1239) P(X ≤ 1238) = 0.0233 OR P(X ≤ 1239) = 0.0251 P(X ≤ 1238) = 0.0233 AND P(X ≤ 1239) = 0.0251 AND Largest n is 1239 or n ≤ 1239 Example of incorrect method: 1300 - 2× $\sqrt{"962"}$ (= 1238) P(X < 1239) = 0.0251 | Ma M1 M1 A1 A1 | 3.1a 3.3 3.4 1.1 2.2a | or $\phi^{-1}(0.025)$ May be implied or -1.96 or 1239(.2) seen One of these attempted, by Binomial or Normal BC Correct P($X \le 1238$) &P($X \le 1239$) and conclusion. Example of inadequate method: $1300 - 1.96 \times \sqrt{962"}$ (= 1239) | or, using Binomial not Normal, B(5000, 0.26) soiM1Attempt $P(X < n)$ for $1230 \le n \le 1250$ or inverse Bin (0.025) (= 1239)M1Allow 0.0232 instead of 0.0233seenIf use normal to find these probs (0.0265 & 0.0246) A0A0NB If two methods used, mark the better one. |
| | P(X < 1238) = 0.0233 Largest <i>n</i> is 1239 SC 4 | [5] | | or inv Bin(0.025) = 1239 Largest <i>n</i> is 1239 SC 4 | NB: SC 1239, no working or incorrect or inadequate working, SC: 4 marks out of 5 |

Exemplars for Q10

Hypotheses

| А | H ₀ : $\mu = 0.034$ H ₁ : $\mu \neq 0.034$ where $\mu = (pop)$ mean pollutant level | B1B1 | | | | | |
|----------------------------|---|------------------------|--|--|--|--|--|
| В | H ₀ : $\mu = 0.034$ H ₁ : $\mu \neq 0.034$ | B1B0 | | | | | |
| C | H₀: The (pop) mean pollutant level is 0.034 H₁: The (pop) mean pollutant level is not 0.034 See Specimen paper q10 MS "Must be in terms of parameter values" | B1B0 | | | | | |
| D | $\begin{array}{l} H_0 {=} 0.034 \\ H_0 {\neq} 0.034 \end{array}$ | B0B0 | | | | | |
| E | H ₀ : $\mu = 0.034$ H ₁ : $\mu = 0.0325$ where $\mu = (pop)$ mean pollutant level | B1B0 | | | | | |
| Probability and conclusion | | | | | | | |
| | | | | | | | |
| F | No statement of distribution P($\overline{X} = 0.0325$) = 0.0486 0.0486 > 0.025 Don't reject H ₀ Likely that mean level of pollutant hasn't changed | M1A1 A1 M1 A1 | | | | | |
| F G | $P(\overline{X} = 0.0325) = 0.0486$ 0.0486 > 0.025 Don't reject H ₀ | A1 M1 | | | | | |

| I J | P($\overline{X} > 0.0325$) = 0.951 0.951 > 0.025 Sufficient evidence that mean poll't level has changed $\overline{X} \sim N(0.034, 0.000000818)$ | | M1A1 A0 M0A0 | |
|------------|---|-----|------------------------|--|
| U | P($\overline{X} < 0.0325$) = 0.013 0.013 < 0.025 Sufficient evidence that level has changed | | M1A0 A0 M1A1 | |
| K | $\mu \pm 1.96\sigma = 0.0322$ to 0.0358 0.0325 lies within this range Reject H ₁ Insufficient evidence that level of poll't has decreased | BOD | M1A1 A1 M1 A0 | |
| L | $\label{eq:cv} \begin{split} CV &= 0.0322\\ 0.0325 > 0.0322\\ \text{Reject } H_0. \text{ Evidence that level of poll't has changed.} \end{split}$ | | M1A1 A1 M0A0 | |
| М | $(0.0322 - 0.034) \div \sqrt{0.0000409/50} = -1.66$ 1.66 < 1.96 Don't reject H ₀ . Level of poll't hasn't changed. | | M1A1 A1 M1A0 | |
| <u>1-t</u> | ail | | | |
| N | H ₀ : $\mu = 0.034$ H ₁ : $\mu < 0.034$ where $\mu = (pop)$ mean pollutant level | | B1B0 | |
| 0 | H ₀ : $\mu = 0.034$ H ₁ : $\mu < 0.034$ | | B0B0 | |
| Р | H_0 : The (pop) mean pollutant level is 0.034 | | | |

P H_0 : The (pop) mean pollutant level is 0.034 H_1 : The (pop) mean pollutant level is less than 0.034B0B0

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