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Examiners' report

FURTHER MATHEMATICS A

H235
For first teaching in 2017

Y535/01 Summer 2019 series

Version 1

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Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper Y535 series overview

This represented the second live paper for this component. Entry figures were slightly greater than for its predecessor in 2018, with the outcomes broadly very similar, but with a slightly increased mean mark. There was a modest amount of evidence that candidates – and hence centres – had become slightly more familiar with the Y535 content (https://www.gov.uk/government/publications/investigation-into-the-sawtooth-effect-in-gcses-as-and-a-levels), although the newer topics (in particular, the number theory) will clearly benefit from further acclimatisation. The more routine demands involved in Questions 4, 5 and 7 led to the acquisition of many marks for most of the candidates, while Question 3 – requiring a more conceptual grasp of the vector product, rather than just being a calculational question – was generally found to be challenging. Question 8, which involved working with a *given* modelling scenario, was found very approachable by almost all candidates.

At the lower end of the mark range, there were very few candidates gaining under 20 marks (out of a total of 60). At the higher end of the scale, no candidates scored full marks – largely as a result of the need for explanatory comments at several stages (Questions. 1, 3, 6, 7 and 8 all required comments or reasons at some point) – although there were many high-scoring scripts, with about one in seven of the candidature gaining 50 marks or more overall.

An especially positive, noteworthy point is that far fewer candidates seemed to waste their time in performing (and re-performing) tasks, as happened rather a lot in 2018, getting 'down to business' straightaway for the most part. Candidates should be aware of the specification defined assessment command words used across the OCR A suite of Mathematics and Further Mathematics (H230/H235/H240/H245).

i	**OCR support**	A poster detailing the different command words and what they mean is available here: https://teach.ocr.org.uk/italladdsup

Question 1 (a)

1 In decimal (base 10) form, the number N is 15 260.

(a) Express N in binary (base 2) form. **[1]**

There were two main approaches taken by candidates here: the majority expressed 15260 as a sum of powers of 2, while others seemed to just write the answer down, possibly from a calculator.

Question 1 (b)

(b) Using the binary form of N, show that N is divisible by 7. **[2]**

The key points here were that $7_{10} = 111_2$ and that the binary form of *N* consists of three blocks of "111". Pointing out these two features of *N* in binary was a perfectly adequate means of earning the two marks. A lot of candidates opted instead to do the division and obtain the quotient $100\,010\,000\,100_2$.

Question 2 (a)

2 **(a)** The convergent sequence $\{a_n\}$ is defined by $a_0 = 1$ and $a_{n+1} = \sqrt{a_n} + \dfrac{4}{\sqrt{a_n}}$ for $n \geqslant 0$.

Calculate the limit of the sequence. **[1]**

This is one of those occasions where candidates' thoughts need to be guided by the number of marks available. Even on a basic scientific calculator, the terms of a sequence given by a first-order recurrence relation can be quickly found by setting up an iterative process that involves little more than repeat pressing of the "=" button. This ability to use the available technology is an important one in this module and would have very quickly yielded the required answer, 4. Far too many candidates worked away at this algebraically, with a lot of them getting distracted along the way and ending up wasting valuable time and still not identifying the correct limit.

	AfL	The command word 'Calculate', along with the single mark available should sign post that an answer only response, possibly obtained through efficient use of a calculator, is all that is required.

Exemplar 1

$a_0 = 1 \quad a_{n+1} = \sqrt{a_n} + \frac{4}{\sqrt{a_n}} \quad n \geq 0$

$L = \sqrt{L} + \frac{4}{\sqrt{L}}$ $\quad L^{-\frac{1}{2}} = 0$ or $L^{-\frac{1}{2}} = -\frac{1}{4}$

$L = 0 \quad L^{-1} = \frac{1}{8}$

$L = \sqrt{L} + 4L^{-\frac{1}{2}}$ $\quad \underline{L = 8}$

$L = L^{\frac{1}{2}} + 4L^{-\frac{1}{2}}$

$0 = L^{-\frac{1}{2}} + 4L^{-\frac{3}{2}}$

let $L^{-\frac{1}{2}} = x$

$0 = x + 4x^2 \quad x = 0$ or $x = -\frac{1}{4}$

$0 = x(1+4x)$

Exemplar 2

$a_{n+1} = \sqrt{a_n} + \frac{4}{\sqrt{a_n}} \quad n_0 = 1$

$n_1 = 5$

$\cancel{a\sqrt{a} = a + 4} \quad n_2 = \frac{9\sqrt{5}}{5}$

$\cancel{0 = a\sqrt{a} - a - 4} \quad n_3 = 400019$

$n_4 = 4$

$\sqrt{4} + \frac{4}{\sqrt{4}} = 2 + 2 = \underline{4}$

$\Rightarrow$ ~~converges to 4~~

$\Rightarrow$ limit $= 4$

These scripts illustrate nicely the range of responses: from highly inappropriate (and inadvisable) long-winded algebra through to the more sensible – in light of the number of marks available – approach of using one's calculator to generate the terms of the sequence quickly and effectively, leading to a simple statement of the correct answer. In Exemplar 1, the candidate has gone algebraic right from the start; then got confused amidst an array of strangely incorrect "equations" which they are simply unable to address. In Exemplar 2, this candidate was also, initially, tempted by an algebraic approach, but then abandoned it in favour of using their calculator to see what the terms of the sequence actually were. You will see that the sequence converges very rapidly to the required answer, 4.

Question 2 (b)

(b) The convergent sequence $\{b_n\}$ is defined by $b_0 = 1$ and $b_{n+1} = \sqrt{b_n} + \frac{k}{\sqrt{b_n}}$ for $n \geqslant 0$, where k is a constant.

Determine the value of k for which the limit of the sequence is 9. **[3]**

Those candidates who simply set all the *b* terms equal to 9 found themselves with a very easy equation to solve. Others clearly experimented with a variety of *k*s, effectively taking a 'trial and improvement' approach to the problem using their calculator. Many of these found the correct value for *k* but did not earn all 3 marks. They could/should have noted (and some did) that the command word "Determine" requires of them the presentation of sufficient evidence to show how the answer has been obtained; on this occasion, I would suggest that at least one value of $k < 18$, and one $k > 18$, along with their respective limits, would have counted as satisfactory supporting evidence.

Exemplar 3

$$9 = \sqrt{9} + \frac{k}{\sqrt{9}}$$

$$9 = 3 + \frac{k}{3}$$

$$6 = \frac{k}{3}$$

$$18 = k$$

This exemplar shows how straightforward this question actually is using that fact that having converged, $b_{n+1} = b_n = 9$

Exemplar 4

$\cancel{b_{n+1} = \sqrt{b_n} + \frac{k}{\sqrt{b_n}}}$ $\quad b_{n+1} = \sqrt{b_n} + \frac{k}{\sqrt{b_n}}$

$b_0 = 1$

$\cancel{b_1 = \sqrt{b_0} + \frac{k}{\sqrt{b_0}}}$ $\quad b_1 = 1 + \frac{k}{1}$

$b_1 = \cancel{1} + \frac{k}{1}$ →

$9 = \sqrt{b_n} + \frac{k}{\sqrt{b_n}}$

$\cancel{b_1 = 1 + \frac{k}{1}}$ $\quad a\sqrt{b_n} = b_n + k$

$k = a\sqrt{b_n} - b_n$

$\cancel{b_2 = \sqrt{1 + \frac{k}{1}}} + \frac{k_1}{\sqrt{1+\frac{k}{1}}}$ $\quad k = 9\sqrt{1} - 1$

$k = 8$

Let $k = 10$

$k = 15$

$k = 17 \rightarrow 8.71$ $\quad \therefore k = 18$ for b_{n+1} converging to 9

$k = 18 \rightarrow 9$

$k = 19 \rightarrow 9.28$

Although this candidate has initially attempted an algebraic method, full marks given for the clear use of a systematic trial and improvement method.

Question 3 (a)

3 The non-zero vectors $\mathbf{x}$ and $\mathbf{y}$ are such that $\mathbf{x} \times \mathbf{y} = \mathbf{0}$.

(a) Explain the geometrical significance of this statement. **[2]**

The basic properties of the vector product are included in syllabus items 8.04a to 8.04d, and many candidates seemed to be uncertain about how to use these properties. Here, in part (a), there were two marks indicated, yet the majority of responses offered only the one comment, almost invariably noting that **x** and **y** were parallel. The second mark required of them was for explaining, from the definition of the vector product, that this was because the $\sin\theta$ term must be zero, which would arise from an angle of $\theta = 0$ (or $\theta = \pi$), even though we did not penalise oversight of the *anti-parallel* possibility.

Question 3 (b)

(b) Use your answer to part **(a)** to explain how the line equation $\mathbf{r} = \mathbf{a} + t\,\mathbf{d}$ can be written in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{d} = \mathbf{0}$. **[2]**

This part also revealed some confusion as to how to explain things appropriately. Successful approaches followed one of two paths. In the first, candidates noted that $\mathbf{r} - \mathbf{a} = t\mathbf{d}$, and since vectors which are a multiple of each other are parallel, the conclusion follows. In the second, "crossing" both sides of $\mathbf{r} - \mathbf{a} = t\mathbf{d}$ with $\mathbf{d}$ and then observing that $\mathbf{d} \times \mathbf{d}$ must be $\mathbf{0}$ (nb, vector zero) also leads to the final answer. For the most part, candidates scored either none or both of the marks.

Question 4

4 The sequence $\{u_n\}$ is defined by $u_1 = 1$ and $u_{n+1} = 2u_n + n^2$ for $n \geqslant 1$.

Determine u_n as a function of n. **[8]**

Apart from occasional slips with the calculations, this routine solution of a first-order recurrence relation was very well handled and meant that more than half of attempts scored 7 or 8 of the 8 marks available, with those scoring 7 dropping a mark due to a minor arithmetic or algebraic slip. The standard approach – finding the General Solution as the sum of a Particular Solution and the Complementary Solution (with quote-able, or easily calculable, forms for both) and then using the initial term to determine the value of the one arbitrary constant involved – was widely, and successfully, deployed. However, a small number of candidates adopted an alternative approach, one which works especially well using the calculator facility to solve algebraic systems of equations (or the corresponding equivalent in terms of matrices): namely, note the general form, work out the first few terms of the sequence (1, 3, 10, 29, …) and used the general form in the first four cases to write four equations that can be solved quite speedily to give all four required constants (one from the CS and three from the quadratic PS).

Question 5

5 The tetrahedron T, shown below, has vertices at $O(0, 0, 0)$, $A(1, 2, 2)$, $B(2, 1, 2)$ and $C(2, 2, 1)$.

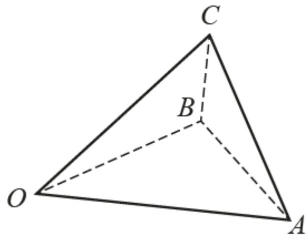

Diagram not drawn to scale

Show that the surface area of T is $\frac{1}{2}\sqrt{3}\left(1+\sqrt{51}\right)$. **[8]**

This was another popular and high-scoring question, with around two-thirds of the candidates gaining 6, 7 or 8 marks on it. Although the question does not explicitly state that the vector product (VP) *must* be used (and the question can, of course, be answered without it) most candidates rightly used the VP as intended. With a given answer, it is always important to make sure that one's working matches the eventual answer, not only in terms of sufficiency of detail but also in respect of the visible accuracy of the working. Although marks were high, many 1s and 2s were lost due to carelessness in the calculation of various VPs where, quite often, incorrect signs appeared in the working.

Another interesting feature of this question, and the choice of the numbers involved in the given initial vectors, is that the carefully chosen position vectors of *A*, *B* and *C* reduce the amount of working that needs to be done: so, for instance, having calculated the area of ΔOAB, it is obvious that the areas of ΔOAC and ΔOBC *must* be the same. Very, very few candidates spotted this obvious "assist" and there was, as a result, a great deal of unnecessary working presented by almost all candidates, with the obvious additional opportunities to get a sign wrong in the extra working and thus lose a mark or two unnecessarily.

Question 6 (a)

6 **(a)** Determine all values of x for which $16x \equiv 5 \pmod{101}$. **[4]**

Marks for Question 6, as a whole, were found to be uniformly distributed across all 9 possible mark totals available. This seems to indicate that candidates had the full range of grasps of the subject matter. The stronger responses tended to follow one of two approaches. The principal expected approach is the basic one of adding 101s to the RHS of the equation until an equivalent of 5 (mod 101) is found with a factor in common with the 16 on the LHS; at this point, "cancelling" can be done *provided that one considers the necessary condition for doing so*. This "cancelling" can either be done in stages or after arriving at a multiple of 16 on the RHS, at which point the initial statement can be reduced to $x \equiv \ldots$ (mod 101). The proviso mentioned above requires the justification of this apparent division process (which is not something that candidates should be presenting in their working): the required condition is that 16 and 101 are co-prime and candidates must get into the habit of noting this. A lot of candidates **did** note that hcf(16, 101) = 1 but then used it to justify that the solution is unique modulo 101 (which is true but not pertinent to the 'divisibility' issue at hand).

The second main approach used was to find the appropriate inverse (or reciprocal) mod 101. In the case of part (a), this is 19. Then, multiplying throughout by 19 leads straight to $x \equiv 5$ (mod 101). Strictly speaking, the existence of an inverse also should be justified but we decided that it would be silly to expect candidates to justify the existence of something they'd just found, so no further explanation was required. This wasn't the anticipated approach, due to the extra difficulties involved in finding inverses/ reciprocals in modular arithmetic … unless one has a calculator capable of churning out the answer; many candidates clearly had one such calculator, and there is no reason why it should not be used.

One final warning regarding the presentation of the final answer in questions such as this one: candidates will be penalised for an answer such as $x = 95$ that appears to indicate the answer is unique (as opposed to an answer representative of an infinitely large set).

Exemplar 5

$101 = 6\times16 + 5$ $\quad$ $1 = 16 - 3\times5$

$16 = 3\times5 + 1$ $\quad$ $1 = 16 - 3\times(101 - 6\times16)$

$5 = 5\times1 + 0$ $\quad$ $1 = 16 - 3\times101 + 18\times16$

$1 = 19\times16 - 3\times101$

$16^{-1} \equiv 19 \pmod{101}$

$16x \equiv 5 \pmod{101}$

$\times19$ $\quad 304x \equiv 95 \pmod{101}$

$x \equiv 95 \pmod{101}$

Exemplar 5 is really quite high-powered, using an approach (the *Euclidean algorithm*) which is a very fundamental one in number theory *but NOT on this course*. It is a systematic method which has been used by this candidate to calculate the inverse/reciprocal of 16 (mod 101); in this case, 19. Then multiplying by 19 throughout the given modulus "equation" gives the correct answer immediately.

Exemplar 6

$16x \equiv 5 \bmod 101$

$112x \equiv 35 \bmod 101$

$11x \equiv 35 \bmod 101$

$99x \equiv 315 \bmod 101$

$-2x \equiv 12 \bmod 101$

$2x \equiv 89 \bmod 101$

$102x \equiv 4539 \bmod 101$

$x \equiv 95 \bmod 101$

$x \equiv 95 \bmod 101$

Exemplar 6 approach involves multiplying both sides of the present "equation" by some suitably chosen factor and then reducing the numbers modulo 101. This leads very nicely to the answer.

Exemplar 7

$16x \equiv 5 \pmod{101}$ 106

207

308

409

$16x = 5 + 101K$ 510

611

$\frac{16x-5}{101} = k \pmod{101}$ $5 \pmod{101} =$

$16x = -96 \pmod{101}$

$\underline{x = 6} \pmod{101}$

$\frac{16x-5}{101} = k$

In Exemplar 7, the candidate (at the top, on the right hand side of the page) has simply added 101s until a useful equivalent (mod 101) of 5 has been reached: on this occasion, they have arrived at 611 and then noticed that this is $\equiv -96 \pmod{101}$, which is a multiple of 16, enabling them to divided by 16. Unfortunately, they have forgotten to carry forward the negative sign at the final stage. (They have also not justified the "division" process, which is always a dodgy thing to do in modular arithmetic and should be 'handled with care'.)

Question 6 (b) (i)

(b) Solve

(i) $95x \equiv 6 \pmod{101}$, **[2]**

Those who could find inverses/reciprocals had little difficulty here but, in fact, it is very straightforward even without this facility. The ability to switch to negative equivalents (modulo anything) is not a demanding one and the fact that $6 \equiv -95 \pmod{101}$ leads immediately to the solution $x \equiv -1 \pmod{101}$, or $x \equiv 100 \pmod{101}$, with either form found equally acceptable (and appearing in scripts in approximately equal measure).

Question 6 (b) (ii)

(ii) $95x \equiv 5 \pmod{101}$. **[2]**

This part can also be approached via either of the two main methods previously discussed above, but actually only requires the ability to spot that one has already solved this (the other way around) in (a). Around half of all candidates made little or no attempt here after struggling with either or both of the previous parts.

Question 7 (a)

7 You are given the set $S = \{1, 5, 7, 11, 13, 17\}$ together with $\times_{18}$, the operation of multiplication modulo 18.

(a) Complete the Cayley table for $(S, \times_{18})$ given in the Printed Answer Booklet. **[4]**

The group theory question represented another very popular and well understood part of the Y535 module as candidates generally handled it very competently indeed. The Cayley table was correctly completed by almost all candidates.

Question 7 (b)

(b) Prove that $(S, \times_{18})$ is a group. (You may assume that $\times_{18}$ is associative.) **[3]**

The basic principle of establishing whether some given object is, or is not, a group seems to be very well grasped. However, many candidates still lost marks by failing to be entirely convincing with their details, and this applies to each of parts (b) through (e). Here, for instance, it is insufficient simply to say "Closure follows *from the table*" without saying *how* this is so. To gain the explanation mark, one should state that the table contains only the elements of the given set; or, equivalently, that there are no "new" elements within the table.

	AfL	It is a good idea to illustrate this particular notion, early on in students' introduction to this new type of algebraic construct, by having them build some Cayley tables in which the given set and operation do not yield a closed table; one simple example would be the set {1, 2, 3, 4, 5} under multiplication modulo 6 … where the product $2 \times_6 3 = 0$, which is **not** an element of the original set. Constructing Cayley tables with certain properties, but not others, is (as suggested above) a very valuable thing to do early on in students' experience of "groups" and requires only a little bit of imagination. Of course, the operations chosen have to be some arbitrary background (and hence unspecified) means of combining elements, but this can still be done in general terms.

?	**Misconception**	Marks were also often lost when candidates made statements regarding inverses such as, "the identity appears in each row and column" so every element has an inverse. Of course, the full definition of an inverse element for a (say) is that there exists an element b (say) in the set for which a □ b □a = must be the same. This is far too tricky a matter to explore fully from a table of almost any non-trivial size so, for groups of small order, the simplest thing to do is to state each element and its inverse. This can be simplified further by noting inverse-pairs (5, 11, and 7, 13 are two inverse-pairs in this question) along with any element which is self-inverse. The identity itself does not, strictly speaking, have an inverse – see comments below, in (c).

Exemplar 8

The only elements that appear in the Cayley table are
1, 5, 7, 11, 13, and 17 ⇒ $(S, \times_{18})$ is closed

Exemplar 8 illustrates a concise, minimal, yet comprehensive – statement regarding closure.

Exemplar 9

As seen in the table above $(S \times_{18})$ is closed.

Exemplar 9 effectively says "look at the table" but, significantly, fails to say what it is that is to be seen. A clarification as to *how* the group table demonstrates the property is needed.

Exemplar 10

Closed ⇒ all elements are in
table

Exemplar 10 *almost* conveys the right message but fails to be specific about what these "elements" are: saying that all the elements are there is not quite the same as noting that there are no new ones as well, and this is the key point regarding closure.

Question 7 (c)

(c) Write down the order of each element of the group. **[2]**

The majority of candidates knew exactly what to do here and gained both marks with little fuss.

	AfL	The identity is always the cause of some disagreement among teachers – and there are contrasting treatments it receives within standard books in the topic. To many, the identity has order 1, as one doesn't need to raise this element to any additional power in order to obtain it. (So, for instance, it is clear that a self-inverse element has order 2.) However, the definition of an inverse, as described above in (b), allows for the perfectly reasonable argument that, since e $\square$e = most constructors of this sort of question, at this level, tend to worry only about the non-identity elements, treating the identity as if it doesn't require an inverse at all; we are inclined to this particular view and, as a result, ignore almost anything claimed by candidates about the identity in such cases.

Question 7 (d)

(d) Show that $(S, \times_{18})$ is a cyclic group. **[1]**

Apart from a few candidates who gave no answer at all, almost everyone knew what was being asked for here, even though some lost the one mark available by – yet again – being careless with the detail. Simply stating that the group has a generator is insufficient to earn the mark; it is a straightforward matter to say what it is (and there are two possible generators to choose from here). It is also acceptable to observe that the group has (at least) one element of order 6 (that is, equal to the order of the group) and this should have been clearly sign-posted to all as the expected response after part (c)'s demand.

Question 7 (e) (i)

(e) **(i)** Give an example of a non-cyclic group of order 6. **[1]**

Some knowledge of the groups of small order – and, up to isomorphism, there really aren't too many of order less than or equal to 7 – is required by the syllabus and there were a number of very shrewd answers offered. Popular choices were D_3, the group of symmetries of the (equilateral) triangle, and S_3, the group of the six permutations of three objects. For one mark, no additional details were required.

Question 7 (e) (ii)

(ii) Give one reason why your example is structurally different to $(S, \times_{18})$. **[1]**

The thrust of the question so far should have indicated that candidates were expected either to discuss (briefly) the orders of elements or the cyclic versus non-cyclic structure (which can also be argued from the point of view of the elements of order 6, or lack of them). In the event, around 60% of the candidates failed either to make any suggestion at all or to convey the message with sufficient clarity to earn the mark.

Question 8 (a)

8 The motion of two remote controlled helicopters P and Q is modelled as two points moving along straight lines.

Helicopter P moves on the line $\mathbf{r} = \begin{pmatrix} 2+4p \\ -3+p \\ 1+3p \end{pmatrix}$ and helicopter Q moves on the line $\mathbf{r} = \begin{pmatrix} 5+8q \\ 2+q \\ 5+4q \end{pmatrix}$.

The function z denotes $(PQ)^2$, the square of the distance between P and Q.

(a) Show that $z = 26p^2 + 81q^2 - 90pq - 58p + 90q + 50$. **[3]**

Although based on a simple idea, this question, as a whole, was a good discriminator of ability. This first part demanded of candidates an ability to cope with the squaring of a three-term bracket (thrice) to obtain the given answer. Again, it is important for candidates to show that their working is *visibly* correct and "Additional Further Pure Maths" candidates should have far greater experience in presenting their working clearly. This is important to mention because far too many candidates offered working of an almost microscopic size, hastily scrawled and with overlapping brackets and intermediate expressions, before (*magically*) producing the given answer.

	AfL	The command word 'Show that', along with the given answer should sign post that the marks will be given for clear and correct algebraic manipulation. Care should be taken if work is to be corrected, especially sign errors; it may be advisable for candidates to rewrite their solution rather than attempt to make corrections to an original expression.

Question 8 (b)

(b) Use partial differentiation to find the values of p and q for which z has a stationary point. **[4]**

Starting from a given answer to part (a), and with a specific instruction to use partial differentiation, this proved to be a popular and well handled question with the vast majority of candidates gaining 3 or 4 of the available marks. Most of those who scored 3/4 dropped the mark by failing to solve the simultaneous equations correctly. Once again, it is worth mentioning that all the popular models of calculator that meet the minimum requirements for Further Mathematics have as standard the ability to solve simultaneous linear equations and no reason not to use them in this case.

Exemplar 11

$\frac{\partial z}{\partial p} = 52p - 90q - 58$ $\qquad$ $\frac{\partial z}{\partial q} = 162q - 90p + 90$

$0 = 52p - 90q - 58$ $\qquad$ $0 = 162q - 90p + 90$

$90q - 58 = 52p$

$0 = 162q - 90\left(\frac{45q - 29}{26}\right) + 90$

$\frac{45q - 29}{26} = p$

$= \frac{81}{13}q + \frac{2475}{13}$

Stationary Point

at $\left(-54, \frac{-275}{9}, 2914109\right)$ $\qquad$ $\frac{-2475}{13} = \frac{81}{13}q$

$\frac{-275}{9} = q \quad \therefore p = -54 \quad \therefore Z = 2914109$

~~Stationary point at $(-54, \frac{-275}{9},$~~

In Exemplar 11, the candidate has partially differentiated correctly only to get into trouble solving the ensuing simultaneous equations (since the numbers are not "nice" as such).

Exemplar 12

$f_p = 52p - 90q - 58\cancel{p} = 0$

$f_q = 162q - 90p + 90 = 0 \cancel{16}$

$162q - 90p = -90 \cancel{= 0}$

$-90q + 52p = 58$

$q = \frac{5}{3} \qquad p = 4$

Exemplar 12 has done exactly the same initial calculus work as seen in the previous response, but (presumably) made efficient use of their calculator to solve the pair of equations afterwards. This is not only permissible, but to be encouraged. The working clearly shows the pair of equations rearranged in to the standard format of $\begin{matrix} \square x + \square y = \square \\ \square x + \square y = \square \end{matrix}$. Candidates would be penalised if the final answer given in terms of x and y rather than the p and q from the question.

Question 8 (c)

(c) With the aid of a diagram, explain why this stationary point must be a minimum point, rather than a maximum point or a saddle point. **[2]**

This proved challenging for the majority of candidates, with over half scoring zero here. The anticipated approach (which almost no-one took) optimistically expected that candidates would sketch two (non-intersecting) lines and indicate somehow that any point on one line has a shortest distance from the other line; or that any stationary value **cannot** be a maximum, as this distance between lines can be made indefinitely large; moreover, since this is a symmetric property of the two lines, the stationary value cannot be a saddle-point.

The secondary approach was that candidates would consider sections of the surface in (or parallel to) the *z*-*p* plane and the *z*-*q* plane and note that it must always be a "positive" parabola that appears … in the event, this was the most popular line of reasoning, even though that still accounted for only about 20% of the entry.

Question 8 (d)

(d) Hence find the shortest possible distance between the two helicopters. **[2]**

Some candidates had effectively obtained, unrequested, an answer for *z* (the square of the distance) in part (b) but didn't realise this and offered no response here. Of those providing an answer here, a significant number lost a mark by failing to take the square-root.

Question 8 (e)

The model is now refined by modelling each helicopter as a sphere of radius 0.5 units.

(e) Explain how this will change your answer to part **(d)**. **[2]**

It was pleasing to see candidates attempt this part even if they had not offered responses to some of the previous parts. A few more candidates felt able to interpret the solution and gained at least one of the 2 marks just by observing that the distance would be reduced when the helicopters were no longer being considered as points; the second mark was for adding that this was a reduction of 1 unit relative to (d)'s answer, and this was rewarded even when there was no answer to part (d).

Supporting you

For further details of this qualification please visit the subject webpage.

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