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AS LEVEL

Examiners' report

FURTHER MATHEMATICS B (MEI)

H635
For first teaching in 2017

Y413/01 Summer 2019 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper 1 series overview

June 2019 was the second examination series for the H635 AS Further Maths B (MEI) optional Paper Y413, Modelling with Algorithms. It was extremely pleasing to note that most candidates were well prepared for the examination, making good attempts on all questions. Candidates seemed to have enough time to answer the questions and nearly all seemed to make good use of the Printed Answer Booklet (i.e. very few answers to one question appeared in the part for another). Based on the responses seen by examiners this series the following general points should be considered by centres in preparation for candidates for future sittings of this paper.

Diagrams should be completed in pencil so that candidates can rub out and replace their answer rather than trying to correct in pen on what is likely to be a diagram, which does not have enough space for multiple attempts. This was especially true in Question 2(d) (scheduling activities) in which it was difficult at times for examiners to know if crossing out was candidates' genuinely trying to correct an error, or was the candidates attempt to schedule the workers to tasks.

This unit is primarily a methods examination and so it is vital that candidates make their method and application of any corresponding algorithm clear; spotting and writing down the solution (for example in sorting questions) will rarely gain any credit.

Page 8 of the specification contains a section on the meanings of a number of instructions that will be used in examination questions. Candidates should familiarise themselves with these command words. The command word 'determine' appeared in this paper – in this mathematical context, it requires that justification should be given for any results found and not just the writing down or stating of the answer.

Candidates are reminded to use the number of marks available as the main guide to how detailed their answer should be and not the space given in the Printed Answer Booklet. It is always hoped that the Printed Answer Booklet should provide enough space for the candidate's response, which often includes additional space so candidates can correct any errors.

Question 1 (a)

1 24 17 9 25 18 11 23 19 30 15

(a) Show the result of applying the first fit algorithm to pack items with the sizes listed above into bins that have a capacity of 50. **[2]**

Most candidates applied the first fit algorithm correctly in this part with the most common error being the placement of the 15 in a fifth, rather than the third bin.

Question 1 (b)

(b) Use the quick sort algorithm to sort the list of numbers above into descending order. You should use the first value as the pivot for each sublist. **[3]**

While several candidates proceeded to sort the list of numbers into ascending order most correctly applied the quick sort algorithm to sort the list into descending order, using the first value as the pivot for each sublist.

	AfL	Candidates are reminded of a couple of key points when applying this algorithm: 1. If a value, after being used as a pivot, splits the list up into two sublists, then this will generally create two pivots for the next pass and candidates should then use both values as pivots for the next pass, e.g. after the first pass (which used the 24 as a pivot) the list was 25 30 24 17 9 18 11 23 19 15, and so, both the 25 and 17 should now be used as pivots for the second pass – some candidates only used one of these values as a pivot in the second pass, and then used the other in a third (or later) pass and therefore completed a 'slow' sort. 2. Candidates are reminded that when a sublist containing more than one item appears to be in order, a pass, pivoting on the first value in the sublist, still needs to take place, e.g. after the third pass a sublist containing 23 and 19 was created and so a fourth pass (pivoting on the 23) was required even though these two numbers are in the correct order. 3. Candidates are reminded that items should remain in the order from the previous pass as they move into sublists. 4. The algorithm is only completed when either a pass takes place with no change in the order of the items (so no swaps have taken place) or a comment to the effect that all sublists have only one element and therefore it can be concluded that the sort is complete.

Question 1 (c)

(c) Show the result of using the first fit decreasing algorithm to pack items with the sizes listed above into bins that have a capacity of 50. **[2]**

As with part (a) this part was done extremely well with the most notable error being the placement of the 19 in Bin 3 rather than Bin 1. Candidates are reminded that first fit is just that; candidates must decide if the current item will fit in the first bin rather than the most recent bin used.

Question 1 (d)

The number of comparisons is used as a measure of the complexity of both the first fit and first fit decreasing packing algorithms.

(d) Explain why, in the worst case, both packing algorithms have the same order of complexity. **[1]**

Very few candidates managed to score the mark in this part as many simply quoted the result that in the worst case both packing algorithms have the same (quadratic) complexity. Very few explained correctly that in the worst case each number will not fit in any of the previous bins. This therefore implies that each new number has been compared with the spare capacity of all the previous bins before being placed in a separate (new) bin. The fact that the numbers being considered are either in decreasing order or not does not alter the number of comparisons required in the worst case so therefore both algorithms have the same order of complexity.

Question 2 (a)

2 The network in Fig. 2.1 represents a project using activity on arc. The durations of the activities are not yet shown.

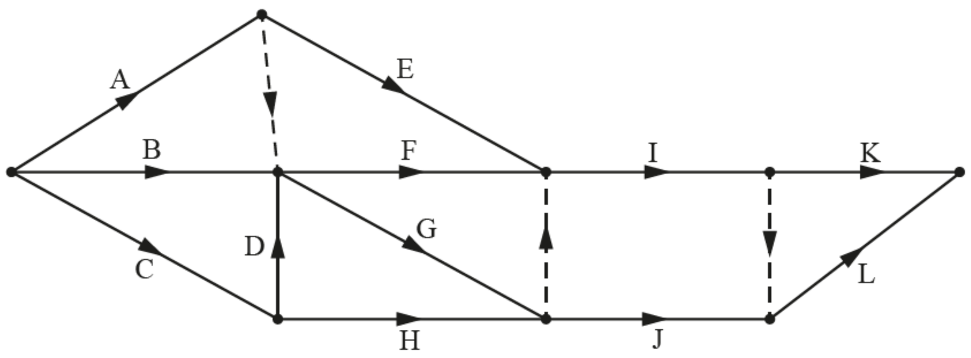


Fig. 2.1

(a) Complete the table in the Printed Answer Booklet to show the immediate predecessor(s) for each activity. **[2]**

This part was answered extremely well with nearly all candidates scoring at least one mark for correctly completing five rows of the table. The most common errors were due to the dummy activities; failing to include A in the immediate predecessors for either F or G or failing to include both of G and H for activity I.

Question 2 (b)

It is given that activity E is a critical activity. The manager of this project therefore claims that activities A, E, I and K form a critical path for the network.

(b) Is the manager's claim correct? Explain your reasoning. **[2]**

Some candidates attempted to give a simple 'yes' or 'no' answer even though the question specifically asked for an explanation. Those candidates who understood that each activity on a critical path must itself be critical usually realised that both A and I are therefore guaranteed to be critical. However, as there are two paths from source to sink that pass through these three activities (AEIK and AEIL) it does not necessarily mean that AEIK is a critical path (as it could be AEIL). It was pleasing to note that many candidates correctly dealt with this type of question on inference.

Question 2 (c) (i)

Fig. 2.2 lists all the activities and their corresponding durations, in hours.

Activity	A	B	C	D	E	F	G	H	I	J	K	L
Duration, hours	7	4	6	2	10	6	3	7	5	6	5	3

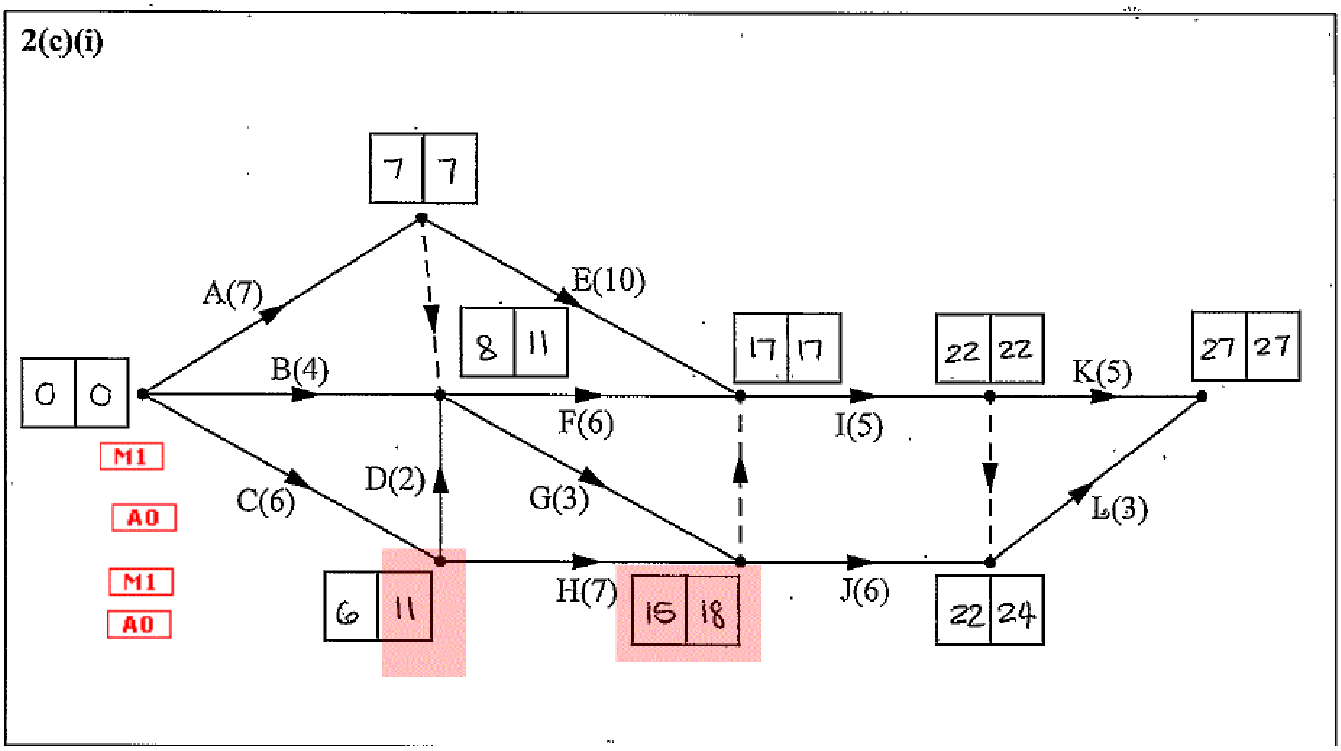
Fig. 2.2

(c) (i) Carry out a forward pass and a backward pass through the activity network, showing the early event time and late event time at each vertex of the network. **[4]**

While nearly all candidates managed to score the method marks for attempting both a forward and backward pass through the network it was uncommon to award all four marks as many candidates did not consider correctly the dummy activities when completing either one or both passes (most notably the backward pass). The most common error was an 18 (rather than 17) at the end of activity H.

?	**Misconception**	The start event node should be labelled with a double zero and neither of these two boxes should be left blank.

Exemplar 1



This response scored the two method marks. For the forward pass the numbers are increasing from source to sink but there is an error at the end of H (15 rather than 13). For the backward pass the numbers are generally decreasing from sink to source (we condone the slip at the end of H as across the dummy here 17 leads into 18) – although two of the late event times are incorrect there is only slip in decreasing values.

Question 2 (c) (ii)

(ii) State the minimum project completion time. **[1]**

Almost all candidates correctly stated the minimum project completion time as 27 hours.

Question 2 (d)

Each activity requires one person.

(d) Use the diagram in the Printed Answer Booklet to show how three people can complete the project in the minimum time. Each column in the diagram represents 1 hour. For each person, write the letter of the activity they are doing in each box, or leave the box blank if the person is resting for that 1 hour. **[2]**

This part was answered extremely well with many candidates scoring at least one mark for producing a schedule that used at most three workers, with at least 10 activities placed, of which 5 were correct. When errors occurred, it was usually due to precedence issues and candidates are reminded that after completing such a schedule it is wise to check that each activity has the correct duration, is taking place in the correct time interval and that precedence with other activities has been maintained.

	AfL	Candidates should be reminded that the activity network diagram with the forward and backward passes is extremely useful in completing an accurate schedule of workers to tasks.

Question 2 (e)

The manager claims the project can in fact be completed by two people in the minimum project completion time.

(e) Show that the manager's claim is incorrect. **[1]**

In this part many candidates over complicated this problem by trying to argue that certain activities could not be taking place at the same time and while such arguments, if valid, were acceptable (e.g. B, C, D and F take 18 hours, but these need to be completed in 17 hours to avoid delaying activity I), very few gave enough detail of either the activities and/or corresponding times. The most elegant correct answers were those that realised that the total of all the activities was 64 hours and two workers only had 54 hours available or those that argued that the non-critical activities would take 37 hours to complete which therefore couldn't be completed by a single worker in 27 hours.

Question 3 (a)

3 A network has 9 vertices, A to I. The table in Fig. 3 shows the distance between each pair of vertices for which there is a connecting arc.

	A	B	C	D	E	F	G	H	I
A		5	8	16					
B	5		2			4	11		
C	8	2		8		1			
D	16		8		2				
E				2		3			5
F		4	1		3			2	12
G		11						2	
H						2	2		7
I					5	12		7	

Fig. 3

(a) Apply the tabular form of Prim's algorithm to the network, starting at vertex A, to find a minimum spanning tree for the network. In your solution you should give

- the order in which the arcs are selected, and
- the total length of the arcs in the minimum spanning tree.

[4]

While most candidates answered this part extremely well many did not follow the specific instruction in the question, which was to apply Prim's algorithm in tabular form. So, while some candidates gave the correct order in which the arcs were selected and had a correct total length, they only scored 2 of the 4 marks available as they did not indicate that they had used the given table. The simplest way to indicate the successful application of the algorithm in tabular form is to circle the values in the table and to indicate the order of selection by placing a numerical value (starting at 1) above the node in the order in which it was added to the spanning tree.

Question 3 (b)

(b) Without drawing the network, use Dijkstra's algorithm to find the shortest route from A to I. **[5]**

Many candidates scored full marks in this part. Centres should note that it is vital when applying Dijkstra's algorithm that candidates show all the working values at each node and that the order of labelling is correct. In this case, examiners had to see the correct working values of 8 and 7 at C to indicate that the algorithm was initially being applied correctly. A small number of candidates labelled two different nodes with the same label and a number of candidates correctly applied the algorithm but then did not state the shortest route from A to I.

	AfL	The Printed Answer Booklet provided the standard key for the completion of the boxes at each node together with a specific instruction to candidates that they should not cross out their temporary labels. Several candidates still did so. It is vital that examiners can read all the values at each node (so that they can check that the algorithm has been applied correctly) and it is strongly recommended that teachers' emphasise that no temporary labels are crossed out in classroom practice.

Question 3 (c)

(c) Determine the length of the shortest route from C to G via A. **[1]**

Surprisingly, many candidates did not realise that they could use their answer to part (b) to answer this part by simply adding together the final value at C to the final value at G.

Question 4 (a)

4

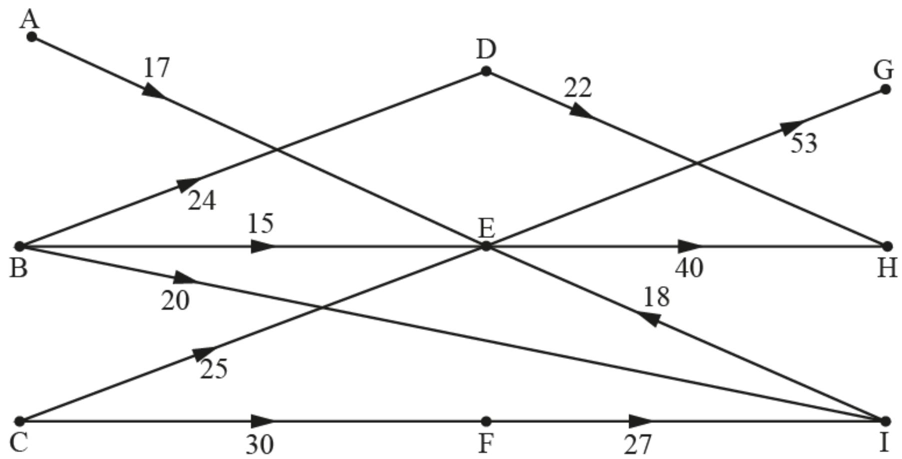


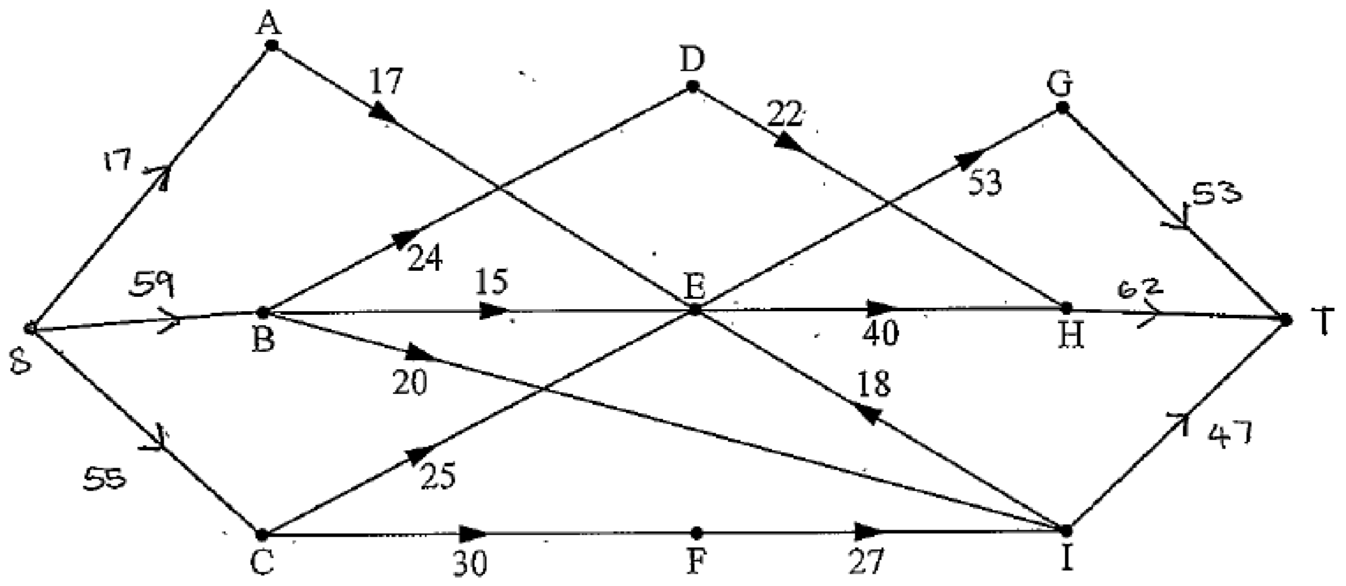
Fig. 4.1

Fig. 4.1 shows a network representing a system of pipes. Fluid flows continuously from three sources, A, B and C, to two sinks, G and H. The weights on the arcs show the capacities of the pipes in gallons per hour.

(a) Add a supersource, S, and a supersink, T, to the network in the Printed Answer Booklet. Give appropriate weightings and directions to the connected arcs. **[2]**

It was common for candidates to correctly add a supersource, S, with three arcs leading from S to A, B and C (with correct corresponding weights). It was also common to see three arcs leading from G, H and I to the supersink, T, even though node I was not a sink and the question specifically said that the network had only two sink nodes (G and H).

Exemplar 2



This response scored one mark. The supersource, S, was added correctly but the supersink, T, included an incorrect arc from I to T.

Question 4 (b)

(b) Calculate the capacity of the cut X = {S, A, B, C, E}, Y = {D, F, G, H, I, T}. **[1]**

This part was answered extremely well by most candidates with nearly all giving the correct answer of 167.

Question 4 (c) (i)

An LP formulation is set up to find the maximum flow through the system.

Part of the LP formulation is shown below.

Maximise $AE + BD + BE + BI + CE + FI$

Subject to
$BD - DH = 0$
$CF - FI = 0$
$BD \leqslant 24$
$BE \leqslant 15$
$BI \leqslant 20$
$CE \leqslant 25$
$CF \leqslant 30$

(c) Explain the purpose of each of the following lines from the LP formulation.

(i) Maximise $AE + BD + BE + BI + CE + FI$. **[2]**

As this part had two marks candidates were required to make two points regarding this line from the LP formulation. The first point was that the arcs listed formed a cut for the network (and not as some candidates suggested were the arcs incident to the three source nodes), or as some candidates deduced that all the flow for the network passed through these six arcs. The second point was that because all the flow for the network passes through these six arcs this line of the formulation maximises the total flow in the network.

Question 4 (c) (ii)

(ii) $BD - DH = 0$. **[1]**

Nearly all candidates correctly inferred that this line of the LP formulation guaranteed that the flow into node D was equal to the flow out of node D.

Question 4 (d)

(d) Complete the LP formulation. **[2]**

The complete LP was run in an LP solver and the output is shown in Fig. 4.2.

VARIABLE	VALUE
AE	17.00000
BD	22.00000
BE	15.00000
BI	18.00000
CE	25.00000
CF	0.00000
DH	22.00000
IE	18.00000
EH	22.00000
EG	53.00000
FI	0.00000

Fig. 4.2

Candidates found this part rather demanding with very few scoring both marks. While some attempted to formulate the constraints for the flow through the remaining arcs based on their corresponding capacity it was quite common for some arcs to be left out. While some did try to write down the conservation constraints for nodes E and I it was common for arcs to be missing or for incorrect signs to be used.

?	**Misconception**	In any LP formulation concerning directed arcs it is vital that the order in which the letters are used is correct. Many candidates had no hesitation in using EI even though in this context it had to be stated as IE (as the flow could only be from I to E).

Question 4 (e) (i)

(e) **(i)** State the value of the flow given by this output. **[1]**

Nearly all candidates correctly stated this flow as 97 gallons per hour.

Question 4 (e) (ii)

(ii) Use a suitable cut to prove that this is the maximum flow. **[2]**

It was extremely rare for candidates to either state a correct cut with capacity 97 or for them to give enough detail in proving that this flow was indeed maximal.

	AfL	Proving a flow is maximal: Step 1: state a cut, based on the capacity of saturated arcs directed from source to sink and arcs with zero flow directed from sink to source, as either a set of nodes or as a list of arcs which the cut passes through, and conclude that therefore the minimum cut is less than or equal to this value. Step 2: state the value of the current flow through the network and conclude that therefore the maximum flow is greater than or equal to this value. Step 3: if these two values are the same then by the maximum flow-minimum cut theorem a maximum flow for the network has been found.

?	**Misconception**	When calculating the capacity of a cut, candidates must use the capacity of the arcs and not the current flow which is passing through those arcs (otherwise all cuts for the network would have the same capacity).

Exemplar 3

{S, A, B, C, D, F, I} {E, H, G, T} = 97

This common response scored one mark for a correct cut but gave none of the required detail for proving that the flow was maximal.

Question 4 (f) (i)

The capacity of pipe EH is increased by an additional 20 gallons per hour.

(f) (i) Determine the effect of this change on the maximum flow through the system. **[1]**

The command word 'determine' meant that a full explanation was required. Most candidates simply said that there would be no change but to gain credit they had to explain that this was because all the pipes leading into node E are already saturated, so even if EH was increased no more flow can be sent through E.

Question 4 (f) (ii)

(ii) State the effect of this change on the LP formulation. **[1]**

Nearly all candidates correctly stated that the only change to the LP formulation would be that $\text{EH} \le 60$.

Question 5 (a)

5 An LP problem in x, y and z is formulated as follows.

Maximise $P = 3x + 2y + 5z$

subject to
$$x + y + z \geqslant 26$$
$$2x + y + z \leqslant 34$$
$$2x - z = 10$$
$$x, y, z \geqslant 0$$

(a)
- Complete the initial tableau in the Printed Answer Booklet so that the two-stage simplex method may be used to solve this problem.
- Show how the constraints for the problem have been made into equations using slack variables, surplus variables and artificial variables.
- Show how the rows for the two objective functions are formed. **[9]**

It is vital in questions like this that the candidate reads the entire question carefully before beginning as several candidates only completed the initial tableau and did not show the constraints (as equations) or show how the two objective functions were formed. It was clear that some candidates did not know how to deal with the constraint $2x - z = 10$ as not all replaced this with the two constraints $2x - z \le 10$ and $2x - z \ge 10$. It was clear that candidates were much more confident in adding slack variables to the two 'less than or equal to' constraints and re-writing the *P* objective function than adding the surplus and artificial variables to the two 'greater than or equal to' constraint and forming the second (*Q*) objective function.

	AfL	When candidates are asked to derive equations or constraints that need to include variables that have not been defined in the question it is the responsibility of the candidate to define the variables they use. In this case, for example, many candidates did not make it explicitly clear from the letters they used which was the artificial and which was the surplus variable. Candidates should also be reminded that when filling in an initial tableau that all cells must be completed (especially zeros).

Exemplar 4

$x + y + z - s_1 + a_1 = 26 \Rightarrow Q + x + y + z - s_1 = 26$

Minimise $Q + x + y + z - s_1 = 26$

Maximise $P - 3x - 2y - 5z = 0$

subject to: $x + y + z - s_1 + a_1 = 26$

$2x + y + z + s_2 = 34$

$2x - z + s_3 = 10$

$x, y, z \geq 0$

Q	P	x	y	z	s_1	s_2	s_3	s_4	a_1	a_2	RHS
1	0	1	1	1	−1	0	0	0	0	0	26
0	1	−3	−2	−5	0	0	0	0	0	0	0
0	0	1	1	1	−1	0	0	0	1	0	26
0	0	2	1	1	0	1	0	0	0	0	34
0	0	2	0	−1	0	0	1	0	0	0	10

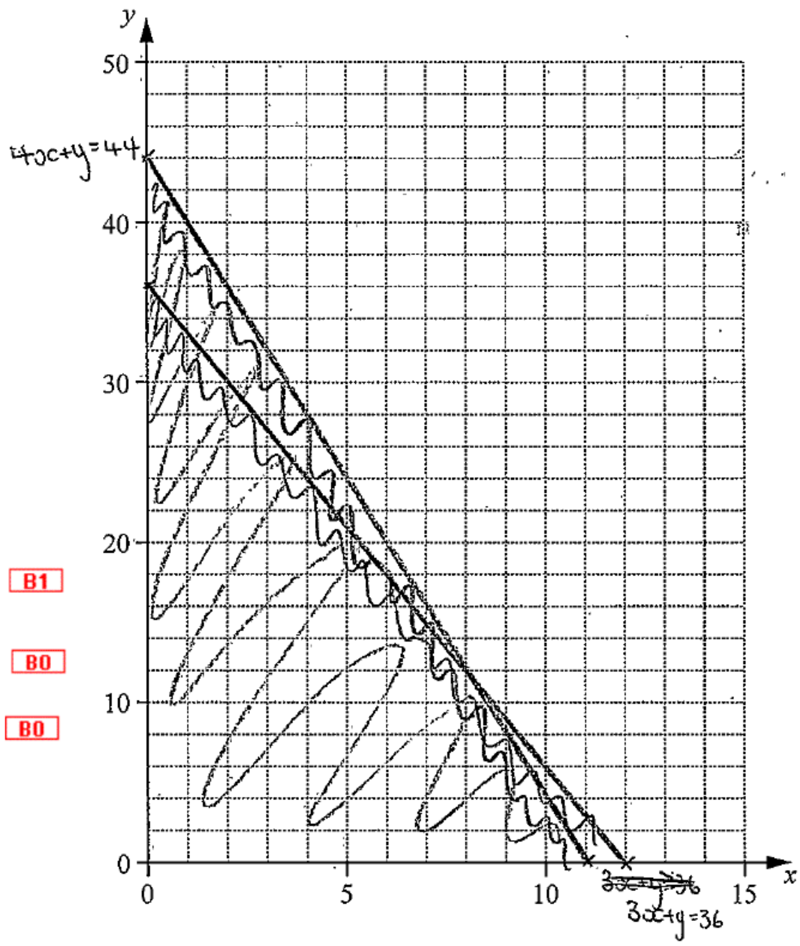
This response scored five of the nine marks available in this part. As expected for the majority of candidates issues arose when dealing with the constraint $2x - z = 10$. This candidate only considered the case $2x - z \leq 10$ (which can be seen from the fact that they wrote down the equation $2x - z + s_3 = 10$) and they did not realise that $2x - z \geq 10$ had to be considered too. By not introducing two artificial variables, this candidate could not score the first method mark or either of the two accuracy marks in this part.

Question 5 (b)

(b) By re-writing the LP problem in terms of x and y only, use a 2-D graphical method to determine the maximum value of P, and the corresponding values of x, y and z. **[8]**

Very few candidates re-wrote the LP formulation as requested and most simply ignored the *z* variable and therefore scored no marks in this part. It was intended that candidates would use the constraint $2x - z = 10$ to eliminate *z* in the constraints and the objective function and while some did very few also realised that the trivial constraint of $z \geq 0$ in the original formulation would imply that $x \geq 5$. Of those that did formulate the problem correctly most went on to obtain the correct maximum value of *P*, together with the corresponding values of *x*, *y* and *z*.

Exemplar 5



$2x - z = 10$

M1 $z = 2x - 10$ $P = 3x + 2y + 5(2x - 10)$

$x + y + (2x - 10) = 26$ $P = 13x + 2y - 50$ A1

$3x + y \geq 36$ (0, 36) (12, 0)

$2x + y + (2x - 10) = 34$

A0 $4x + y = 44$ (0, 44) (11, 0) **(answer space continued on next page)**

5(b)	(continued)
	$4x + y = 44$ ①
	$3x + y = 36$ ②
	① - ② = $x = 8$ ∴ $3(8) + y = 36$
	$y = 12$
	$(8, 12)$
B0	at $(8, 12)$ $P = 13(8) + 2(12) - 50 = 78$
	at $(12, 0)$ $P = 13(12) + 2(0) - 50 = 106$ ✗
B0	at $(0, 44)$ $P = 13(0) + 2(44) - 50 = 38$

This response scored three marks. This candidate had the correct idea of substituting $z = 2x - 10$ into the objective function and at least one constraint and so therefore scored the first method and accuracy mark for correctly deriving the objective as $P = 13x + 2y - 50$. The candidate correctly derived the constraint $3x + y \geq 36$ but did not obtain the other two constraints $(4x + y \leq 44,\ x \geq 5)$ and so did not obtain the second accuracy mark. Their graph only had two lines correct and so scored the first B mark. For the second B mark all three lines had to be correct and the third B mark was for a correct feasible region. Finally, the candidate did not obtain the last two marks in this part as they did not obtain the correct value for *P* and the corresponding values for *x*, *y* and *z*.

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