

ADVANCED GCE

MATHEMATICS (MEI)

Methods for Advanced Mathematics (C3)

4753/01

QUESTION PAPER

Candidates answer on the printed answer book.

OCR supplied materials:

- Printed answer book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Monday 20 June 2011

Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets [] at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The printed answer book consists of **16** pages. The question paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

Section A (36 marks)

- 1 Solve the equation $|2x - 1| = |x|$. [4]
- 2 Given that $f(x) = 2 \ln x$ and $g(x) = e^x$, find the composite function $gf(x)$, expressing your answer as simply as possible. [3]
- 3 (i) Differentiate $\frac{\ln x}{x^2}$, simplifying your answer. [4]
- (ii) Using integration by parts, show that $\int \frac{\ln x}{x^2} dx = -\frac{1}{x}(1 + \ln x) + c$. [4]
- 4 The height h metres of a tree after t years is modelled by the equation
- $$h = a - be^{-kt},$$
- where a , b and k are positive constants.
- (i) Given that the long-term height of the tree is 10.5 metres, and the initial height is 0.5 metres, find the values of a and b . [3]
- (ii) Given also that the tree grows to a height of 6 metres in 8 years, find the value of k , giving your answer correct to 2 decimal places. [3]
- 5 Given that $y = x^2\sqrt{1+4x}$, show that $\frac{dy}{dx} = \frac{2x(5x+1)}{\sqrt{1+4x}}$. [5]
- 6 A curve is defined by the equation $\sin 2x + \cos y = \sqrt{3}$.
- (i) Verify that the point $P\left(\frac{1}{6}\pi, \frac{1}{6}\pi\right)$ lies on the curve. [1]
- (ii) Find $\frac{dy}{dx}$ in terms of x and y .
- Hence find the gradient of the curve at the point P . [5]
- 7 (i) Multiply out $(3^n + 1)(3^n - 1)$. [1]
- (ii) Hence prove that if n is a positive integer then $3^{2n} - 1$ is divisible by 8. [3]

Section B (36 marks)

8

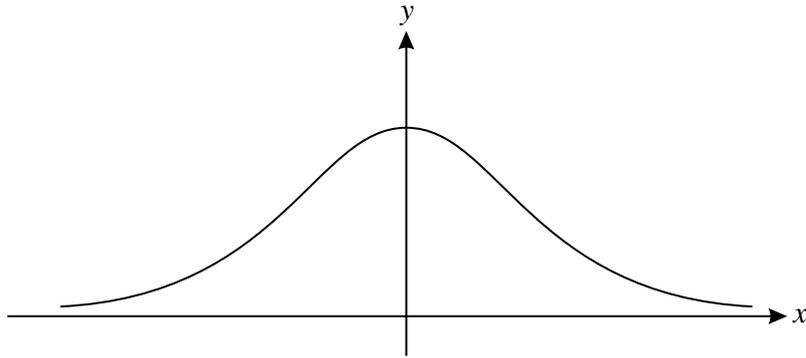


Fig. 8

Fig. 8 shows the curve $y = f(x)$, where $f(x) = \frac{1}{e^x + e^{-x} + 2}$.

- (i) Show algebraically that $f(x)$ is an even function, and state how this property relates to the curve $y = f(x)$. [3]
- (ii) Find $f'(x)$. [3]
- (iii) Show that $f(x) = \frac{e^x}{(e^x + 1)^2}$. [2]
- (iv) Hence, using the substitution $u = e^x + 1$, or otherwise, find the exact area enclosed by the curve $y = f(x)$, the x -axis, and the lines $x = 0$ and $x = 1$. [5]
- (v) Show that there is only one point of intersection of the curves $y = f(x)$ and $y = \frac{1}{4}e^x$, and find its coordinates. [5]

[Question 9 is printed overleaf.]

- 9 Fig. 9 shows the curve $y = f(x)$. The endpoints of the curve are $P(-\pi, 1)$ and $Q(\pi, 3)$, and $f(x) = a + \sin bx$, where a and b are constants.

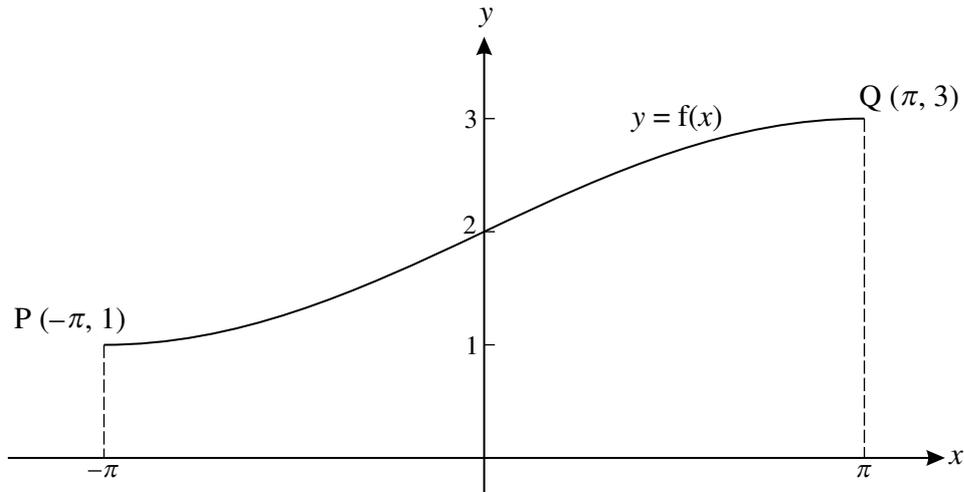


Fig. 9

- (i) Using Fig. 9, show that $a = 2$ and $b = \frac{1}{2}$. [3]

- (ii) Find the gradient of the curve $y = f(x)$ at the point $(0, 2)$.

Show that there is no point on the curve at which the gradient is greater than this. [5]

- (iii) Find $f^{-1}(x)$, and state its domain and range.

Write down the gradient of $y = f^{-1}(x)$ at the point $(2, 0)$. [6]

- (iv) Find the area enclosed by the curve $y = f(x)$, the x -axis, the y -axis and the line $x = \pi$. [4]

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