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A LEVEL

Examiners' report

FURTHER MATHEMATICS A

H245
For first teaching in 2017

Y540/01 Summer 2019 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects that caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the exam paper can be downloaded from OCR.

Paper Y540 series overview

This first paper of the new specification proved to be very accessible to candidates and some very high marks were achieved. There seemed to be little difficulty in recall of the topics, even though in the previous modular specification the syllabus would have been covered by three units and at different points of the course.

As in other recently reformed mathematics qualifications, the instruction to 'show detailed reasoning' is used with a number of questions, to indicate that the working to produce the answer must be shown. It is recommended that candidates spend more time practising such responses.

Presentation of working

A significant number of candidates seemed to be unable to present their work in a logical and orderly fashion. In extreme cases this could lead to examiners not being able to follow the work. Centres are encouraged to work with students on improving the layout of their responses.

Question 1(a)

1 In this question you must show detailed reasoning.

The quadratic equation $x^2 - 2x + 5 = 0$ has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. **[1]**

This should have been an easy first mark for candidates and indeed it was for the vast majority. It was pleasing to note that most candidates interpreted the command words 'Write down' correctly, meaning that no work should be needed; the values should have been seen in the quadratic equation given. A few candidates chose to find the actual complex values for α and β.

i	**OCR support**	OCR's command words poster is available to download in both A2 and A4 versions from 'Assessment guides' at https://www.ocr.org.uk/qualifications/as-and-a-level/further-mathematics-a-h235-h245-from-2017/assessment/

Question 1(b)

(b) Hence find a quadratic equation with roots $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$. **[3]**

The command word 'Hence' means 'using what you have just obtained' and again most candidates did so. A few disregarded this and did far too much work for the marks available.

Question 2

2 Indicate by shading on an Argand diagram the region

$\{z : |z| \leqslant |z-4|\} \cap \{z : |z-3-2\mathrm{i}| \leqslant 2\}$. **[3]**

Candidates should understand that while only a sketch is required for this question (they are not required to bring into the examination a pair of compasses, for instance), it needs to be clear to examiners that what has been sketched really is a circle and a straight line. A few candidates lost one or more marks because of the lack of clarity here.

A number of candidates treated the inequality $|z| \leqslant |z-4|$ as if it were $x \leqslant 4$ and so shaded the incorrect region.

In this question it was acceptable to shade either the region that was required or the region that was not required.

Question 3

3 In this question you must show detailed reasoning.

You are given that $x = 2 + 5\text{i}$ is a root of the equation $x^3 - 2x^2 + 21x + 58 = 0$.

Solve the equation. **[4]**

Candidates stated correctly that the complex conjugate of 2 + 5i (i.e. 2 – 5i) was also a root.

The process of finding the third root was varied. The most elegant (and requiring the least work) was to state that the product of all three roots is –58 meaning that the third root has to be *x* = –2.

The longer (but more popular) way was to create the quadratic function with the two given factors and to divide that into the cubic function to obtain *x* + 2 and hence the third root.

A number of candidates did not answer the question asked, leaving their answer in terms of factors of the original cubic function.

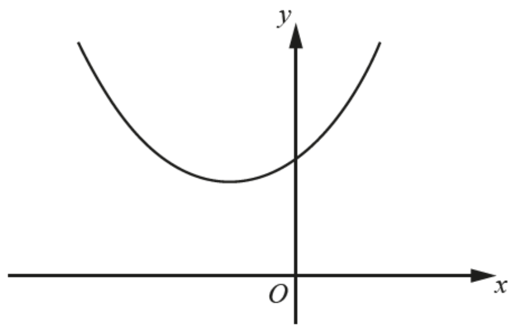
Question 4

4 Using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, show that $\sum_{r=1}^{10} r(3r-2) = 1045$. **[3]**

The vast majority of candidates understood the meaning of the command words 'show that' and gave enough detail in their answer to show that ten terms had not been added together.

Question 5(a)

5 The diagram shows part of the curve $y = 5\cosh x + 3\sinh x$.



(a) Solve the equation $5\cosh x + 3\sinh x = 4$ giving your solution in exact form. **[4]**

This question was answered well; only a handful of candidates wrote '$\cosh x = e^x + e^{-x}$' instead of '$\cosh x = \frac{1}{2}(e^x + e^{-x})$' and similarly for sinh*x*. There were other approaches to this question, including the compound angle formulae and the method of squaring and using Pythagoras' theorem; these were usually less successful.

Question 5(b)

(b) In this question you must show detailed reasoning.

Find $\int_{-1}^{1}(5\cosh x + 3\sinh x)\mathrm{d}x$ giving your answer in the form $a\mathrm{e} + \frac{b}{\mathrm{e}}$ where a and b are integers to be determined. **[3]**

The conversion to exponential form could be done before or after integration. A few candidates made sign errors. It was surprising to see so many candidates choosing to state that *a* = 5 and *b* = 5, having obtained the right answer.

Question 6(a)

6 You are given that $y = \tan^{-1}\sqrt{2x}$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. **[2]**

It was best to use the chain rue to perform this differentiation, although a significant number did not do so. The alternative (to make a substitution) was usually correct, although it took more writing (and therefore time) to do so.

Question 6(b)

(b) Show that $\int_{\frac{1}{6}}^{\frac{1}{2}} \frac{\sqrt{x}}{(x+2x^2)}\mathrm{d}x = k\pi$ where k is a number to be determined in exact form. **[4]**

No hint was given about whether or not to use the work done for Q6(a), however it was disappointing to see so many candidates embark on substitution methods, ignoring (or not seeing) the connection between the two parts.

Question 7(a)

7 The function $\operatorname{sech} x$ is defined by $\operatorname{sech} x = \frac{1}{\cosh x}$.

(a) Show that $\operatorname{sech} x = \frac{2\mathrm{e}^x}{\mathrm{e}^{2x}+1}$. **[2]**

This was done well by almost all candidates.

Question 7(b)

(b) Using a suitable substitution, find $\int \operatorname{sech} x \, dx$. **[4]**

Most candidates made a good choice of substitution to obtain the indefinite integral. A number of candidates however lost the final accuracy mark by not including an arbitrary constant of integration.

Question 8

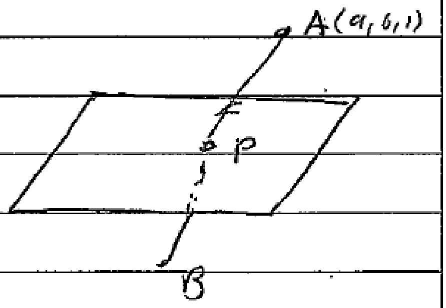
8 The equation of a plane is $4x+2y+z=7$.
The point A has coordinates (9, 6, 1) and the point B is the reflection of A in the plane.

Find the coordinates of the point B. **[6]**

The neatest solution, seen by many candidates, was to find a parametric point on the normal line to the plane through the point A and then, having found the value of the parameter for the point on the plane, doubled it for the point of reflection. A significant number of candidates decided that this question required them to find the shortest distance from the point to the plane and then to double that distance to get to the reflection point; about half of these candidates managed to obtain the correct answer. The question asked for the coordinates of B, so the direction vector of OB was not accepted for the final mark.

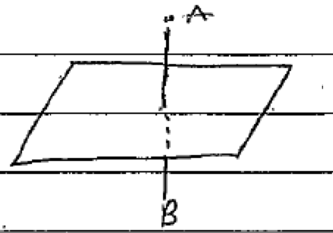
Exemplar 1

coordinates of P:



normal to plane = $\begin{pmatrix}4\\2\\1\end{pmatrix}$

$P = \begin{pmatrix}9\\6\\1\end{pmatrix} + \lambda\begin{pmatrix}4\\2\\1\end{pmatrix}$



$4x + 2y + z = 7$

$4(9+4\lambda) + 2(6+2\lambda) + (1+\lambda) = 7$

$36 + 16\lambda + 12 + 4\lambda + 1 + \lambda = 7$

$49 + 21\lambda = 7$

$21\lambda = -42$

$\lambda = -2$

$B = \begin{pmatrix}9\\6\\1\end{pmatrix} + 2\lambda\begin{pmatrix}4\\2\\1\end{pmatrix}$

$= \begin{pmatrix}9\\6\\1\end{pmatrix} - 4\begin{pmatrix}4\\2\\1\end{pmatrix}$

$= \begin{pmatrix}-7\\-2\\-3\end{pmatrix} \longrightarrow B = (-7, -2, -3)$

midpoint = $(1, 2, -1)$ ✓

This candidate has not found the need to calculate distances, but has used the property that the intersection of the normal line from A with the plane is the midpoint of the line AB. Full marks are earned for this.

Question 9(a)

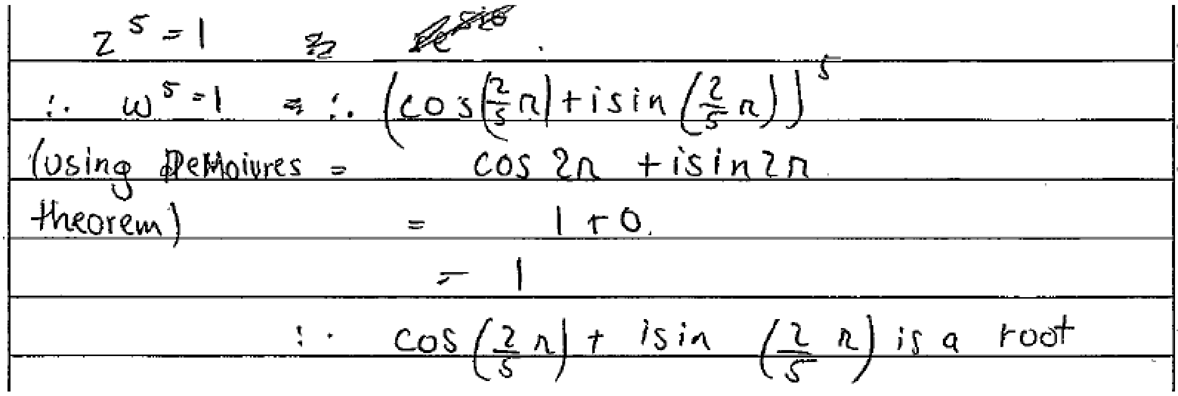
9 In this question you must show detailed reasoning.

You are given the complex number $\omega = \cos\frac{2}{5}\pi + \mathrm{i}\sin\frac{2}{5}\pi$ and the equation $z^5 = 1$.

(a) Show that ω is a root of the equation. **[2]**

This part, as with the other parts of this question, saw many candidates effectively write down the answer and then to say 'so this is the answer'. So, in this part, to write 'ω is a root of the equation so ω is a root' is not credit worthy. It was expected that the de Moivre's theorem would be used.

Exemplar 2


$z^5 = 1$

$\therefore\ \omega^5 = 1 \quad = \therefore \left(\cos\left(\frac{2}{5}\pi\right) + i\sin\left(\frac{2}{5}\pi\right)\right)^5$

(using DeMoivres theorem) $= \cos 2\pi + i\sin 2\pi$

$= 1 + 0$

$= 1$

$\therefore \cos\left(\frac{2}{5}\pi\right) + i\sin\left(\frac{2}{5}\pi\right)$ is a root

This candidate substituted ω for z and showed that ω□ = 1.

Question 9(b)

(b) Write down the other four roots of the equation. **[1]**

Answers were accepted in various forms including arguments in the range $-\pi \leqslant \theta \leqslant \pi$.

Question 9(c)

(c) Show that $\omega + \omega^2 + \omega^3 + \omega^4 = -1$. **[2]**

A response of '$\omega + \omega^2 + \omega^3 + \omega^4 + 1 = 0$, therefore $\omega + \omega^2 + \omega^3 + \omega^4 = -1$' was not acceptable. Candidates who were successful in this part either used the formulae for a geometric progression or argued that the sum of the roots of an equation equals $-\frac{b}{a}$ where b is the coefficient of the z^4 term, which is 0. A common response was to use the calculator, thus bypassing the working required to be seen in a 'show that…' question.

Question 9(d)

(d) Hence show that $\left(\omega+\frac{1}{\omega}\right)^2+\left(\omega+\frac{1}{\omega}\right)-1=0$. **[3]**

It was unfortunate to see a number of candidates writing down the left hand expression = 0 through their working. Additionally, candidates would multiply (or divide) by ω^2 without commenting on the value of the function. As in the previous parts, writing down values obtained from a calculator was not acceptable.

Question 9(e)

(e) Hence determine the value of $\cos\frac{2}{5}\pi$ in the form $a+b\sqrt{c}$ where a, b and c are rational numbers to be found. **[4]**

Most candidates were able to convert the quadratic equation in part (d) to one involving $2\cos\frac{2}{5}\pi$ to obtain the correct value, although a few did not recognise that one of the roots of the equation was not valid and should not have been quoted as part of the solution.

Question 10(a)

10 You are given the matrix **A** where $\mathbf{A}=\begin{pmatrix} a & 2 & 0 \\ 0 & a & 2 \\ 4 & 5 & 1 \end{pmatrix}$.

(a) Find, in terms of a, the determinant of **A**, simplifying your answer. **[2]**

These two parts were answered well, but a few thought that the value of the determinant was the reciprocal, i.e. $\frac{1}{a^2-10a+16}$.

Question 10(b)

(b) Hence find the values of a for which **A** is singular. **[2]**

For candidates that had obtained the right quadratic equation in (a), the two values were obtained very easily.

Question 10(c)

You are given the following equations which are to be solved simultaneously.

$$\begin{aligned} ax + 2y \quad\quad &= 6 \\ ay + 2z &= 8 \\ 4x + 5y + z &= 16 \end{aligned}$$

(c) For each of the values of a found in part **(b)** determine whether the equations have

- a unique solution, which should be found, or
- an infinite set of solutions or
- no solution. **[7]**

Many candidates were able to access full marks for this part. However, in a significant number of scripts the logical reasoning leading to the answer was lacking. The command word 'determine' requires reasoning and justification; sometimes it was hard to navigate through candidate's working to discern its accuracy or otherwise.

Question 11(a)

11 A particle is suspended in a resistive medium from one end of a light spring. The other end of the spring is attached to a point which is made to oscillate in a vertical line.

The displacement of the particle may be modelled by the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 10\sin t$$

where x is the displacement of the particle below the equilibrium position at time t.

When $t = 0$ the particle is stationary and its displacement is 2.

(a) Find the particular solution of the differential equation. **[11]**

This was a long question with a number of parts to it. Candidates should be able to write their working clearly and logically (both were sadly lacking in a number of scripts). The complementary function was usually found, although a number of candidates wrote

$x = e^t(A\cos 2t + B\sin 2t)$ or $x = e^{-1}(A\cos 2t + B\sin 2t)$.

A small number of candidates then used the initial and boundary conditions to find the parameters before adding the particular integral.

There was no complication with the particular integral (in that the function in the differential equation did not form part of the complementary function), but even so a few candidates took their particular integral to be $t(p\cos t + q\sin t)$.

Exemplar 3

$x = e^{-t}(A\cos 2t + B\sin 2t) - \cos t + 2\sin t$

$t = 0,\ x = 2,\ \frac{dx}{dt} = 0$

$2 = A - 1$

$A = 3$

$\frac{dx}{dt} = -e^{-t}(A\cos 2t + B\sin 2t) + \sin t + 2\cos t + e^{-t}(2A\sin 2t + 2B\cos 2t)$

$0 = -A$ ~~×~~ $2 + 2B$

$0 = -3 + 2 + 2B$

$B = 1/2$

$x = e^{-t}(3\cos 2t + \tfrac{1}{2}\sin 2t) - \cos t + 2\sin t$

This candidate found the general solution and then differentiated appropriately to use the boundary conditions to find the particular solution. At this stage of the question it was disappointing to see so many candidates not differentiate using the product rule, which at this level should be a basic skill.

Question 11(b)

(b) Write down an approximate equation for the displacement when *t* is large. **[2]**

This was generally answered well. Follow through marks could be given here following an incorrect particular solution from part (a).

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