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Examiners' report

FURTHER MATHEMATICS A

H245
For first teaching in 2017

Y542/01 Summer 2019 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects that caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question/part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the exam paper can be downloaded from OCR.

Paper Y542 series overview

This was the first sitting for the new A Level Further Mathematics syllabus. Y542 is the Statistics paper. To do well on this paper, candidates needed not only to do calculations correctly, but to set out hypothesis tests correctly (stating hypotheses and conclusions in the correct form) and to identify which of the standard conditions and assumptions for the validity of probability models or regression were relevant in the given situations.

The standard was generally high with most candidates having prepared well. Most candidates carried out very many of the calculations correctly; in particular the test statistics in the hypothesis tests involving the normal and chi-square distributions (Questions 4(a) and 6(a)) were often correct.

However, some centres could have made more use of the specimen and practice material. This was particularly clear in stating hypotheses. The mark schemes for the specimen and practice papers had made it clear that hypothesis tests for a population mean or for correlation using either Pearson's or Spearman's correlation coefficients need hypotheses to be given in terms of parameter values – μ, ρ or ρ_s – and to give a clear definition of each symbol, preferably using the word 'population' (for example, 'where μ is the population mean of the greatest weight that a shelf can support'). The requirement to give such a definition marks a change from the legacy modular syllabus.

In questions requiring verbal statements of the hypotheses, many candidates stated them the wrong way around, or made the wrong inference from comparison of the test statistic and the critical value (giving 'reject H_0' when it should have been 'do not reject H_0', or vice versa). While it is conceivable that similar methods are easily confused; more practice is clearly needed when there are more methods to choose from.

?	**Misconception**	If the result of a hypothesis test is not to reject the null hypotheses, the correct statement is 'there is insufficient evidence that [for example] the mean weight that can be supported has been reduced'. It is not correct to state 'there is evidence that the mean weight has not been reduced'.

In carrying out hypothesis tests it was anticipated that most candidates would use their calculators to obtain *p*-values directly, as this is both the quickest method and the method most in tune with modern statistical practice. However, responses in this paper showed that many candidates calculated critical values. They will of course obtain credit if done correctly, but with modern technology the method is not advised.

Many candidates struggled to identify the correct reason, condition or assumption when faced with a possible choice. The ability to select such answers correctly is a significant part of this assessment. Candidates responding with several different reasons will usually receive no credit. In fact, it seemed that many candidates were often reduced to guesswork and were unable to analyse scenarios well enough to identify which issue was relevant.

Question 1(a)

1 A set of bivariate data (X, Y) is summarised as follows.

$$n = 25, \Sigma x = 9.975, \Sigma y = 11.175, \Sigma x^2 = 5.725, \Sigma y^2 = 46.200, \Sigma xy = 11.575$$

(a) Calculate the value of Pearson's product-moment correlation coefficient. **[1]**

For most candidates this was a very straightforward start to the paper.

Question 1(b)

(b) Calculate the equation of the regression line of y on x. **[2]**

Correct answers were very common here, although a few candidates used their product-moment correlation coefficient as *a*. The variable letters *x* and *y* were almost always correctly given.

Question 1(c)

It is desired to know whether the regression line of y on x will provide a reliable estimate of y when $x = 0.75$.

(c) State one reason for believing that the estimate will be reliable. **[1]**

Many candidates produced unnecessarily complicated answers. There are only two standard criteria to consider for the reliability of an estimate, correlation and whether the given data value (here 0.75) is in the data range. As candidates should have got in Question 1(a) a value of *r* that is high, there was no reason for them to respond with an answer any more complicated than '*r* is close to 1 so there is strong correlation'. Those who attempted to use the other condition needed to justify that 0.75 was likely to be within the range of the data (for instance by finding both the sample mean (0.399) and the standard deviation), but this cannot be known with certainty from the information given.

Exemplar 1

1(c) The estimate will be reliable when $x = 0.75$ because n is a large number of 25 so a large sample is used to find the regression line, so anomalies will not affect it too much.

This is a typical irrelevant answer.

Exemplar 2

1(c) The average value of $\bar{x} = 0.399$ and $x = 0.75$ is close to this. ~~so it's in the range of the data.~~

This assertion is not valid without further calculation, for example of the standard deviation. The deleted words make no difference.

Question 1(d)

(d) State what further information is needed in order to determine whether the estimate is reliable. **[1]**

Following from the comments on part (c), it should be clear that all that was needed here was 'the range of values of *x*'. It is not necessary to know all the data values and those who said that they need to know the correct corresponding value of *y* were missing the point. Some said that the data had to be bivariate normal, or elliptically distributed, but this is not necessary unless a formal hypothesis test is being carried out. About half of the candidates picked up the mark here.

Question 2(a)

2 The average numbers of cars, lorries and buses passing a point on a busy road in a period of 30 minutes are 400, 80 and 17 respectively.

(a) Assuming that the numbers of each type of vehicle passing the point in a period of 30 minutes have independent Poisson distributions, calculate the probability that the total number of vehicles passing the point in a randomly chosen period of 30 minutes is at least 520. **[3]**

This was almost always answered correctly. The only common error was to find the probability of fewer than or equal to 520 vehicles passing, instead of 519.

Question 2(b)

(b) Buses are known to run in approximate accordance with a fixed timetable.

Explain why this casts doubt on the use of a Poisson distribution to model the number of buses passing the point in a fixed time interval. **[1]**

The key issue here is that events (arrival of buses) occur to a fixed pattern, so they do not occur randomly. The constant average rate condition (which candidates often misquoted) is irrelevant and the relevance of independence had to be explained very carefully to gain credit. It remains a matter of concern that many candidates seem to think that events occurring at exactly regular intervals is exactly what is needed for a Poisson distribution to apply.

Exemplar 3

2(b) Poisson distribution assumes a constant rate of events occurring in a time period. Howeve a timetable may have different numbers of buses per hour to cater to peek and off peak time.

Constant average rate is irrelevant if events are not occurring randomly. If buses run exactly to timetable, the number of events in any given time period is *fixed*.

Exemplar 4

2(b) Poisson requires that each event occurs randomly. With a time table there would be a pattern.

This shows a concise correct answer.

Question 3(a)

3 Six red counters and four blue counters are arranged in a straight line in a random order.

Find the probability that

(a) no blue counter has fewer than two red counters between it and the nearest other blue counter, **[3]**

Over half the candidates answered this question well. The easiest way is to say that 10 objects, of which 4 make up one identical set and the other 6 make up another identical set, can be arranged in ${}^{10}C_4$ ways and exactly one of them is BRRBRRBRRB.

Question 3(b)

(b) no two blue counters are next to one another. **[3]**

This is a hard technique, but a standard example of it. About half of the candidates knew what to do and they often answered it concisely and accurately. Others tried to improvise and rarely made useful progress. The best method is to consider the 7 'gaps' between and outside the six red counters and calculate how many ways four can be chosen as the location of the blue counters, giving ${}^7P_4$ x 6! / 10!. Other methods rarely looked like leading to the correct answer.

Question 4(a)

4 The greatest weight W N that can be supported by a shelving bracket of traditional design is a normally distributed random variable with mean 500 and standard deviation 80.

A sample of 40 shelving brackets of a new design are tested and it is found that the mean of the greatest weights that the brackets in the sample can support is 473.0 N.

(a) Test at the 1% significance level whether the mean of the greatest weight that a bracket of the new design can support is less than the mean of the greatest weight that a bracket of the traditional design can support. **[7]**

This question was generally well answered. Candidates appeared well trained in standard hypothesis tests, with few omitting the factor of 40 in the variance. However, in stating hypotheses many had not realised (from seeing the specimen material) that they were expected to explain what the symbol μ represented and some used words.

Exemplar 5

4(a) $W \sim N(500, 80^2)$

$H_0 : \mu = 500$ where μ is the population mean of the greatest weight that a bracket of new design can support

$H_1 : \mu < 500$

An example of good practice in stating hypotheses.

Question 4(b)

(b) State an assumption needed in carrying out the test in part **(a)**. **[1]**

About half of the candidates gave one of the two acceptable answers, which were 'the standard deviation is unchanged' or 'the sample is random'. Most of the others said that you had to assume a normal distribution (which is not the case as the central limit theorem can be applied) or gave irrelevant answers such as 'the new brackets must be made in the same way'.

Question 4(c)

(c) Explain whether it is necessary to use the central limit theorem in carrying out the test. **[1]**

This type of question was often asked on the legacy 4733 (S2) papers and (as in the past) many candidates expressed confusion between necessary and sufficient conditions. A large enough sample is never a necessary condition (it is often a sufficient condition), but many candidates said 'because the sample size is large'. There were two possible answers, 'Yes as we do not know that the distribution of weights for the new design is normal' or 'No as the population distribution is known to be normal'. The sample size has nothing to do with it.

Some candidates wrongly thought that the central limit theorem is a theorem about the variance of the sample mean. Sigma-squared-over-n has nothing to do with it either.

Question 5(a)

5 Five runners, A, B, C, D and E, take part in two different races.

Spearman's rank correlation coefficient for the orders in which the runners finish is calculated and a test for positive agreement is carried out at the 5% significance level.

(a) State suitable hypotheses for the test. **[1]**

Again, many candidates had not appreciated that hypotheses should be stated in terms of a population parameter (ρ (or preferably ρ_s) as opposed to r), as was flagged in the specimen material. On this particular occasion credit was given for verbal explanations.

Question 5(b)

(b) Find the largest possible value of Σd^2 for which the result of the test is to reject the null hypothesis. **[3]**

Most candidates correctly obtained 0.9 from the tables (although a few used the pmcc value), and most also used the correct formula for Spearman's rank correlation coefficient. The use of inequalities was very poor, however. Many used = throughout and then often went on to assume that Σd^2 was greater than their value, instead of equal to (or less than) it.

Question 5(c)

(c) In the first race, the order in which the five runners finished was: A, B, C, D, E. In the second race, three of the runners finished in the same positions as in the first race. The result of the test is to reject the null hypothesis.

Find a possible order for the runners to finish in the second race. **[3]**

As above, some candidates found rank orders that gave totals bigger than their answer to part (b), rather than less than it. However, most who used $\sum d^2$ = 2 gave one of the possible correct orders.

Question 6(a)

6 Yusha investigates the proportion of left-handed people living in two cities, A and B. He obtains data from random samples from the two cities. His results are shown in the table, in which L denotes "left-handed".

	L	L'
A	14	9
B	26	51

(a) Test at the 10% significance level whether there is association between being left-handed and living in a particular city. **[7]**

Most stated the hypotheses correctly, but a significant minority got them the wrong way around.

The expected values were often found correctly and many attempted to use Yates's correction, but there were also many wrong formulae, for example $\sum \frac{(O-E-0.5)^2}{E}$. Most used the correct critical value from tables. Conclusions were often well stated.

Question 6(b)

A person is chosen at random from one of the cities A and B.
Let A denote "the person lives in city A".

(b) State the relationship between $\mathrm{P}(L)$ and $P(L|A)$ according to the model implied by the null hypothesis of your test. **[1]**

Those who understood the concept here found this very simple, giving $\mathrm{P}(L) = \mathrm{P}(L \mid A)$. Others (about half the candidates) floundered.

Question 6(c)

(c) Use the data in the table to suggest a value for $P(L|A)$ given by an improved model. **[2]**

Most of the candidates were able to give the straightforward answer of 14/23.

Question 7(a)

7 The random variable D has the distribution $\mathrm{Geo}(p)$. It is given that $\mathrm{Var}(D) = \frac{40}{9}$.

Determine

(a) $\mathrm{Var}(3D+5)$, **[1]**

This was almost always right. Very few multiplied by 3 instead of 9, or added 5.

Question 7(b)

(b) $\mathrm{E}(3D+5)$, **[6]**

The question was very well answered. Almost all candidates found a quadratic equation, found a positive solution, took its reciprocal, multiplied by 3 and added 5. Candidates needed to show and then discard the spurious solution $p = -0.6$. A significant number showed only the positive solution, but explicit rejection of invalid solutions to equations is specifically required by the syllabus. A few used 40 instead of 40/9 and seemed to assume that $3D + 5$ has a geometric distribution, which is not the case.

Question 7(c)

(c) $\mathrm{P}(D > \mathrm{E}(D))$. **[3]**

This was less well answered. Only a minority realised that D could take only integer values, so $\mathrm{P}(D > \frac{8}{3})$ had to be converted into $\mathrm{P}(D \geq 3)$. Many used $(1-p)^{\frac{8}{3}}$. Some found $\mathrm{P}(D > \mathrm{E}(3D + 5))$.

Question 8

8 A university course was taught by two different professors. Students could choose whether to attend the lectures given by Professor Q or the lectures given by Professor R. At the end of the course all the students took the same examination.

The examination marks of a random sample of 30 students taught by Professor Q and a random sample of 24 students taught by Professor R were ranked. The sum of the ranks of the students taught by Professor Q was 726.

Test at the 5% significance level whether there is a difference in the ranks of the students taught by the two professors. **[10]**

Good statements of the hypotheses involved the medians of the rankings, or more generally their distributions. However, many stated their hypotheses as 'the ranks of the candidates are the same', which is incorrect; it suggests that all the candidates had exactly the same rankings, which is obviously either true or false.

?	**Misconception**	Hypotheses always refer to an underlying distribution, never to the actual data values. Hypothesis tests use data values to make inferences about the underlying distributions.

About half the candidates realised that they needed not the sum of the rankings of candidates taught by Professor Q, but the sum of the rankings of the <u>smaller</u> group of the data, in other words those taught by Professor R. It was therefore necessary to find the sum of all 54 rankings (which is 1485) and subtract 726 from this. However, when using a normal approximation it is <u>not</u> necessary to use the smaller of R_m and $m(m + n + 1) - R_m$, as the symmetry of the normal distribution will give the same answer for either. Some, having correctly found 759, unfortunately went on to use 726 as it is less than 759.

The choice of R_m and $m(m + n + 1) - R_m$ is relevant only to the rank sum of the smaller sample and even then it is not needed when a normal approximation is used. The rule ('use the smaller of the two') is there to save space in the tables and to cope with cases when the data is ranked top-to-bottom as opposed to bottom-to-top.

Question 9(a)

9 The continuous random variable T has cumulative distribution function

$$F(t) = \begin{cases} 0 & t < 0, \\ 1 - e^{-0.25t} & t \geqslant 0. \end{cases}$$

(a) Find the cumulative distribution function of $2T$. **[3]**

Although the calculation of CDFs of related random variables is a standard syllabus item that has appeared in the specimen and practice papers (and on the old S3), many candidates did not choose the right method for this question (perhaps because it appears to be very simple). Many replaced x by $2T$, or doubled or halved the given formula. The correct method is to follow the standard procedure of saying that $P(X < x) = P(2T < x) = P(T < ½x) = F(½x)$. Here the method of using F of the inverse function also works, but that method is not recommended in general as it needs awkward adaptation for decreasing functions.

Question 9(b)

(b) Show that, for constant k, $E(e^{kt}) = \dfrac{1}{1-4k}$.

You should state with a reason the range of values of k for which this result is valid. **[7]**

We are sorry for a misprint on this question (t should have been T) which made the question unanswerable. After analysing candidate performance during marking, we decided the fairest approach was to award all candidates full marks for Question 9b.

Question 9(c)

(c) T is the time before a certain event occurs.

Show that the probability that no event occurs between time $T = 0$ and time $T = \theta$ is the same as the probability that the value of a random variable with the distribution $\text{Po}(\lambda)$ is 0, for a certain value of λ. You should state this value of λ in terms of θ. **[4]**

This was a challenging last question, but one that should not have been entirely unfamiliar; the Specification in Section 5.03a includes 'understanding informally the link between the exponential and Poisson distributions'. Few correct answers were seen. Unexpectedly, the main reason for candidates' difficulty was that they found the probability that T was between 0 and θ ; this is the probability that <u>at least one</u> event occurs between times 0 and ϑ. What was needed for the probability that <u>no</u> event occurs in this interval, which is simply $e^{-0.25\vartheta}$. Some were unable to write down a simplified expression for P(0) from Po(λ), but those who did both correctly had no difficulty in completing the question.

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