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A LEVEL

Examiners' report

FURTHER MATHEMATICS A

H245
For first teaching in 2017

Y543/01 Summer 2019 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper Y543 series overview

Y543 is one of the optional modules of the A Level Further Mathematics A qualification. The examination is worth 25% of the total A Level. It is a 90-minute paper and there are 75 possible marks to be given. Candidates are expected to attempt all questions and have the use of a scientific or graphical calculator.

There are three overarching themes that are applied across all content:

OT1 Mathematical argument, language and proof

OT2 Mathematical problem solving

OT3 Mathematical modelling

In this module candidates extend their knowledge of particles, kinematics and forces and have to use their knowledge of pure maths to solve problems with more complex systems. Topics include dimensional analysis, work, energy and power, impulse and momentum, centre of mass, circular motion and variable force.

To do well on this paper, candidates need to have a good understanding of each of the topics and be able to communicate their understanding through their solutions. They needed to efficiently use their calculator and be able to apply their knowledge to unfamiliar contexts.

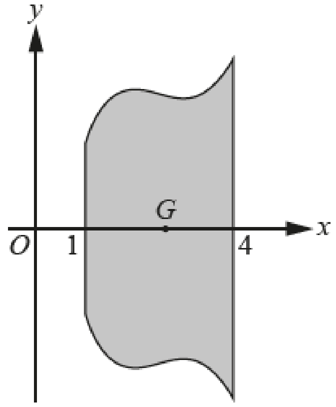
Candidates who did well on this paper generally did the following:	*Candidates who did less well on this paper generally did the following*
• Structured their solutions clearly, ensuring that they were easy to follow • Used their calculator to solve equations and integrals when needed • Communicated which formula or method they were using before substituting in values • Understood momentum in two dimensions • Could use a range of techniques in unstructured questions	• Hadn't learnt key formulae • Didn't read the question carefully and misinterpreted key information (Q7) • Didn't check their solutions for numerical errors • Couldn't show clear steps when asked to 'show that' • Didn't use diagrams to aid their understanding

Candidates appeared to have used the time well and were able to attempt most questions. Some candidates need to be encouraged to express their answers clearly and make sure that their writing is legible.

The OCR A GCE suite of Mathematics and Further Mathematics make use of a defined set of assessment 'command words' to emphasis where clear mathematical justification is required for full credit. Whilst setting work out clearly is good standard practice, questions with the 'Show that' instruction require careful steps that progress from the given information to the stated result.

Question 1 (a)

1 The region bounded by the x-axis, the curve $y = \sqrt{2x^3 - 15x^2 + 36x - 20}$ and the lines $x = 1$ and $x = 4$ is rotated through 2π radians about the x-axis to form a uniform solid of revolution R.
The centre of mass of R is the point G (see diagram).



(a) Explain why the y-coordinate of G is 0. **[1]**

Most candidates answered this correctly. To achieve this mark examiners wanted to see that the candidate understood that the centre of mass lies on the line of symmetry. Common mistakes were reiterating the information that was given in the question (rotated about the x-axis) or by saying it was symmetrical about the y-axis.

?	**Misconception**	Candidates referenced uniform distribution (which suggests a cylindrical shape)

Question 1 (b)

(b) Find the x-coordinate of G. **[4]**

Most candidates knew the formula here and were able to apply it effectively. If candidates didn't know the formula they were unable to access the question. Examiners expected to see calculator use to find the integrals involved. Some students spent far too long working out the integrations, the direction 'Find' means that they do not have to show detailed working.

Question 1 (c)

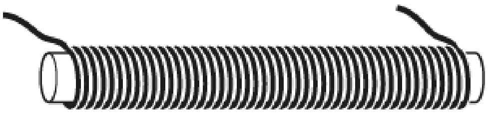
P is a point on the edge of the curved surface of R where $x = 4$. R is freely suspended from P and hangs in equilibrium.

(c) Find the angle between the axis of symmetry of R and the vertical. **[3]**

A very small amount of candidates didn't answer this question and the success rate wasn't as high as on the first two parts. Some candidates lost a mark for finding the angle between the axis and the horizontal and others didn't subtract their value from part (b) from 4. Candidates benefitted from drawing a diagram to aid their understanding.

Question 2 (a)

2 A solenoid is a device formed by winding a wire tightly around a hollow cylinder so that the wire forms (approximately) circular loops along the cylinder (see diagram).



When the wire carries an electrical current a magnetic field is created inside the solenoid which can cause a particle which is moving inside the solenoid to accelerate.

A student is carrying out experiments on particles moving inside solenoids. His professor suggests that, for a particle of mass m moving with speed v inside a solenoid of length h, the acceleration a of the particle can be modelled by a relationship of the form $a = km^{\alpha}v^{\beta}h^{\gamma}$, where k is a constant. The professor tells the student that $[k] = \mathrm{MLT}^{-1}$.

(a) Use dimensional analysis to find α, β and γ. **[6]**

This was the question with the highest success rate in this examination. The most common error was using units of M as part of the acceleration. Examiners would encourage candidates to show which dimension they are equating to make sure they gain method marks if they make a mistake.

Question 2 (b)

(b) The mass of an electron is 9.11×10^{-31} kg and the mass of a proton is 1.67×10^{-27} kg.

For an electron and a proton moving inside the same solenoid with the same speed, use the model to find the ratio of the acceleration of the electron to the acceleration of the proton. **[3]**

This was generally well answered. In some cases candidates found it challenging to logically express the comparisons and some did not denote which side related to electron or proton. This made it difficult to award method marks. A common error was giving the ratio the wrong way round.

Question 2 (c)

(c) The professor tells the student that a also depends on the number of turns or loops of wire, N, that the solenoid has.

Explain why dimensional analysis **cannot** be used to determine the dependence of a on N. **[1]**

The majority of candidates answered this correctly.

?	**Misconception**	Referencing 'no units' rather than 'dimensionless'

Question 3 (a)

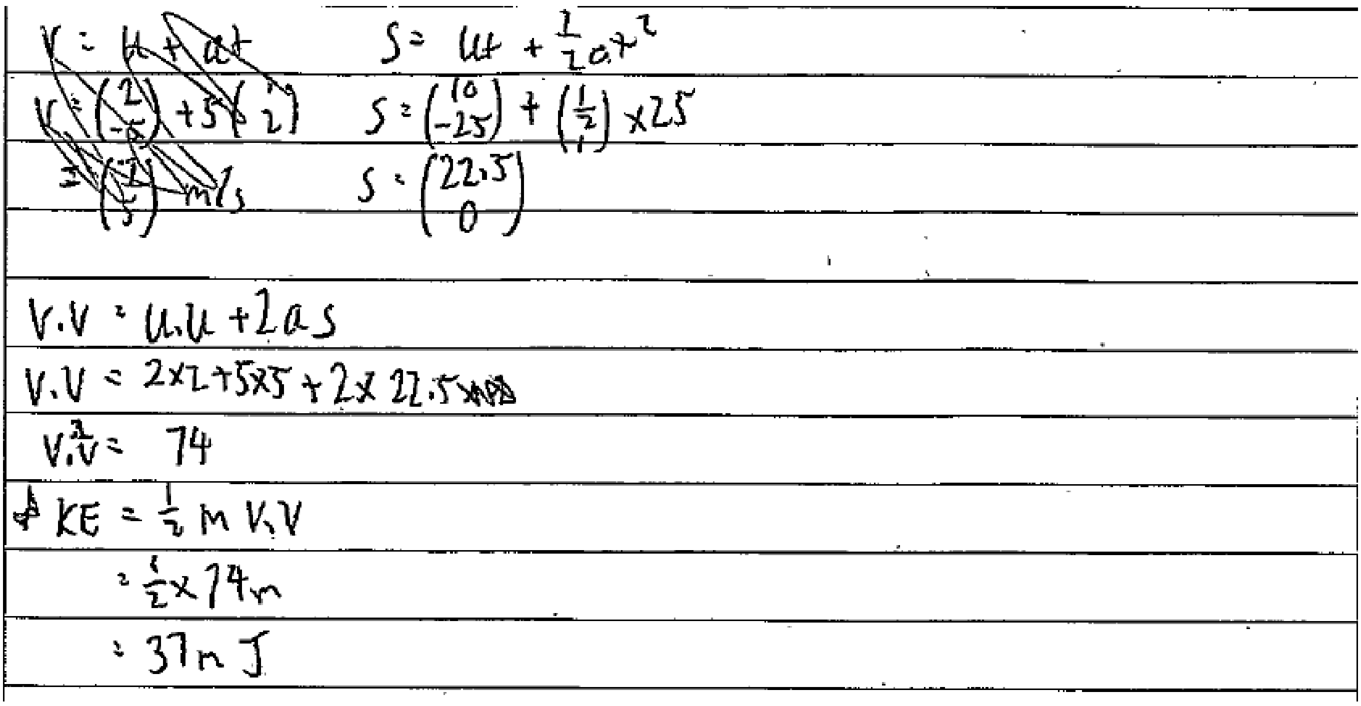
3 A particle Q of mass m kg is acted on by a single force so that it moves with constant acceleration $\mathbf{a} = \begin{pmatrix}1\\2\end{pmatrix}\mathrm{m\,s^{-2}}$. Initially Q is at the point O and is moving with velocity $\mathbf{u} = \begin{pmatrix}2\\-5\end{pmatrix}\mathrm{m\,s^{-1}}$.

After Q has been moving for 5 seconds it reaches the point A.

(a) Use the equation $\mathbf{v.v} = \mathbf{u.u} + 2\mathbf{a.x}$ to show that at A the kinetic energy of Q is $37m$ J. **[5]**

Most candidates achieved 3 or more marks on this question. Some students didn't 'use' the equation given to find the kinetic energy which resulted in them only being given 2 marks – candidates need to be aware that they must use the equation given in this type of question. As detailed in the specification, in a 'show that' question, the explanation has to be sufficiently detailed to get full marks. Examiners wanted to see values substituted into the equation and the dot product applied correctly (the dot product gives a scalar value and not a vector).

Exemplar 1


$s = ut + \frac{1}{2}at^2$

$s = \begin{pmatrix}10\\-25\end{pmatrix} + \begin{pmatrix}\frac{1}{2}\\1\end{pmatrix} \times 25$

$s = \begin{pmatrix}22.5\\0\end{pmatrix}$

$v.v = u.u + 2as$

$v.v = 2\times2 + 5\times5 + 2\times 22.5$

$v.v = 74$

$KE = \frac{1}{2}m\, v.v$

$= \frac{1}{2}\times 74m$

$= 37m$ J

In Exemplar 1, the candidate has lost a method mark for not showing their vectors substituted into the equation and therefore their accuracy mark. Candidate given a total of 3 out of 5.

Question 3 (b) (i)

(b) **(i)** Show that the power initially generated by the force is $-8m$ W. **[2]**

Again, most candidates were able to answer this question but some lost a mark for not giving a detailed enough method. For full marks it was important to see the values substituted in, the dot product calculated and then the final answer.

Question 3 (b) (ii)

(ii) The power in part **(b)(i)** is negative. Explain what this means about the initial motion of Q. **[1]**

Most candidates answered this incorrectly by stating the misconception below. Some candidates were able to understand that the force is opposing the motion, however neglected to translate this to a loss of speed or kinetic energy.

?	**Misconception**	Candidates thought that negative power means that the particle is 'travelling backwards'.

Question 3 (c) (i)

(c) **(i)** Find the time at which the power generated by the force is instantaneously zero. **[3]**

The majority of candidates answered this well. The most common error was to set the velocity to 0. Some candidates forced the velocity vector to be perpendicular to the acceleration vector which was a lot longer method. Candidates were more prone to making errors using this method.

As the direction was 'Find' candidates didn't have to be as thorough with their methods as in the first two parts of this question, but accurate working allows for partial credit if mistakes made, and helps avoid the risk of making mistakes.

Question 3 (c) (ii)

(ii) Find the minimum kinetic energy of Q in terms of m. **[2]**

Most of the candidates who answered Q3 (c)(i) correctly were able to access this question. Some candidates didn't realise that they could use the time at which the power is zero to find the minimum kinetic energy, and instead spent time restarting the solution.

Question 4 (a)

4 A right circular cone C of height 4 m and base radius 3 m has its base fixed to a horizontal plane. One end of a light elastic string of natural length 2 m and modulus of elasticity 32 N is fixed to the vertex of C. The other end of the string is attached to a particle P of mass 2.5 kg.

P moves in a horizontal circle with constant speed and in contact with the smooth curved surface of C. The extension of the string is 1.5 m.

(a) Find the tension in the string. **[2]**

The vast majority of candidates answered this correctly. There was an occasional calculation error with other candidates trying to resolve forces. Some halved the expected value (getting confused with the elastic potential energy formula) resulting in an incorrect answer.

Question 4 (b)

(b) Find the speed of P. **[7]**

The majority of candidates were able to get some marks on this question, although few received full marks. Pretty much all the candidates were able to find the trig ratios and use $a = \frac{v^2}{r}$ for 2 marks. Fewer candidates used the correct value for the radius. Some candidates were able to set up equations to solve. Common mistakes were a lack of understanding of resolving components and a misunderstanding of the direction of acceleration. Some candidates discounted R or did not resolve it. Candidates who didn't draw a good force diagram significantly disadvantaged themselves here.

Question 5 (a)

5 A particle P of mass 4.5 kg is free to move along the x-axis. In a model of the motion it is assumed that P is acted on by two forces:

- a constant force of magnitude f N in the positive x direction;
- a resistance to motion, R N, whose magnitude is proportional to the speed of P.

At time t seconds the velocity of P is $v\,\mathrm{m\,s^{-1}}$. When $t = 0$, P is at the origin O and is moving in the positive direction with speed $u\,\mathrm{m\,s^{-1}}$, and when $v = 5$, $R = 2$.

(a) Show that, according to the model, $\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{10f - 4v}{45}$. **[2]**

Almost all candidates could demonstrate how to get to the equation with few dropping 1 or 2 marks for finding the wrong *k*, or by omitting steps in this 'show that' question.

Question 5 (b) (i)

(b) **(i)** By solving the differential equation in part **(a)**, show that $v = \frac{1}{2}\left(5f - (5f - 2u)\mathrm{e}^{-\frac{4}{45}t}\right)$. **[5]**

Candidates demonstrated a good understanding of pure mathematics with most candidates being given full marks. The majority of solved this by separating the variables (with only a few unable to do this) with the alternative method being to use an integrating factor. Some candidates lost the accuracy mark for making 'slips' in their workings and recovering from this. As it is a 'show that' question, all workings are taken into consideration to get full marks. Common errors were mistakes with rearranging or exponentiating and care needs to be taken to treat constants appropriately.

Question 5 (b) (ii)

(ii) Describe briefly how, according to the model, the speed of P varies over time in each of the following cases.

- $u < 2.5f$
- $u = 2.5f$
- $u > 2.5f$ **[3]**

Most candidates were able to identify that when u = 2.5f then the speed is constant. Some candidates identified that there was a limit for cases 1 and 3 but omitted if speed was increasing or decreasing. Higher ability candidates supported their reasoning with a sketch.

Question 5 (c)

(c) In the case where $u = 2f$, find in terms of f the exact displacement of P from O when $t = 9$. **[4]**

Most candidates were able to substitute in to find a correct equation to be given 1 mark. However there were few candidates who gained full marks. Common mistakes were forgetting the constant of integration, poor use of brackets, with some candidates substituting t = 9 before integrating. Some candidates lost a mark for not being able to simplify their final answer appropriately.

Question 6 (a)

6 Two particles A and B, of masses m kg and 1 kg respectively, are connected by a light inextensible string of length d m and placed at rest on a smooth horizontal plane a distance of $\frac{1}{2}d$ m apart. B is then projected horizontally with speed $v\,\mathrm{m\,s^{-1}}$ in a direction perpendicular to AB.

(a) Show that, at the instant that the string becomes taut, the magnitude of the instantaneous impulse in the string, I N s, is given by $I = \dfrac{\sqrt{3}mv}{2(1+m)}$. **[4]**

Candidates found it difficult to justify their thinking on this question. To be given the first mark candidates needed to find a trig ratio. As it is a 'show that' question the examiner needed to be certain that the candidate had found this from using the lengths of the string. Some candidates did not show detailed enough workings with a diagram to be given this mark. From this point candidates needed to use the conservation of momentum (this is easiest in the direction of the string) to find an equation for the final velocity in terms of m and v. Most candidates were unable to show logical reasoning and seemed to use the answer given in the question to form their answer.

For the final mark examiners had to be certain that the candidate had considered change of momentum on either particle. Just multiplying their expression for velocity by 'm' wasn't enough to demonstrate this.

A significant amount of candidates were unable to access this question because they were trying to model the problem in 1 dimension.

	AfL	When answering a question (particularly 'show that') encourage candidates to write what they are using to set up the equation and in what direction to communicate clearly what the intention is. For instance: 'CoM parallel to the string' or 'Impulse on A'

Question 6 (b)

(b) Find, in terms of m and v, the kinetic energy of B at the instant after the string becomes taut. Give your answer as a single algebraic fraction. **[3]**

This was the most difficult question on the paper for candidates to access. To be given the first 2 marks candidates needed to identify the 2 components of B's final speed and square each of them. Out of the minority of candidates who managed this, the main error was to forget that m = 1 in the kinetic energy formula leading to an extra m in their final answer.

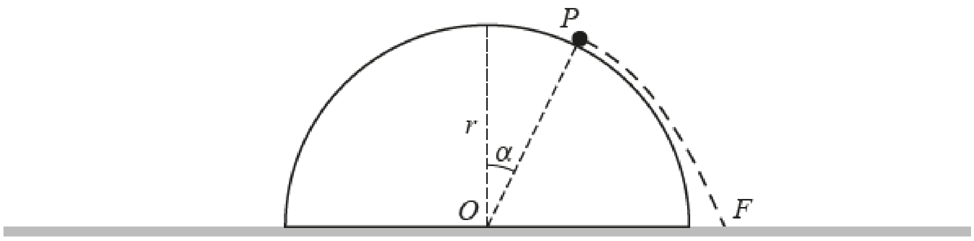
Question 6 (c)

(c) In the case where m is very large, describe, with justification, the approximate motion of B after the string becomes taut. **[2]**

Some candidates omitted this question after struggling with the first 2 parts; however this question was accessible if candidates had modelled the situation. Essentially if *m* is very large, then A will be stationary as B will be unable to move it. As A is a fixed point then B will travel in circular motion around it. Some candidates were able to infer circular motion but didn't refer to A being stationary. The most common incorrect answers were that B would stop or performing simple harmonic motion.

Question 7 (a)

7



The flat surface of a smooth solid hemisphere of radius r is fixed to a horizontal plane on a planet where the acceleration due to gravity is denoted by γ. O is the centre of the flat surface of the hemisphere.

A particle P is held at a point on the surface of the hemisphere such that the angle between OP and the upward vertical through O is α, where $\cos\alpha = \frac{3}{4}$.

P is then released from rest. F is the point on the plane where P first hits the plane (see diagram).

(a) Find an exact expression for the distance OF. **[11]**

Candidates found this question difficult but most candidates were given some marks. Higher ability candidates drew good diagrams and considered conservation of energy between the start point and the point where the particle leaves the surface. Newton's Second Law was then used to find $\cos\theta$ and an expression for v. These candidates were given 6 marks for doing this successfully. Not many candidates were given any further marks for considering vertical motion to find an expression for t and then considering horizontal motion (including adding on $r\sin\theta$). In this second part of the question candidates who took time to organise their projectile equations generally made better progress. This was a challenging question where the best candidates were able to produce a detailed and well-structured solution.

Some candidates set the start point at the top of the hemisphere and that the particle left the hemisphere at α which made it very difficult for them to be given any marks. Incorrect signs were used in the conservation of energy equation (mainly from candidates having difficulties understanding their zero level of potential energy) and other candidates didn't introduce another angle (i.e. Θ) where the particle left the surface.

Question 7 (b)

The acceleration due to gravity on and near the surface of the planet Earth is roughly 6γ.

(b) Explain whether OF would increase, decrease or remain unchanged if the action were repeated on the planet Earth. **[1]**

If candidates had worked out part (a) correctly they were able to argue that OF didn't depend on gravity so it would remain unchanged.

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