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Examiners' report

FURTHER MATHEMATICS A

H245
For first teaching in 2017

Y545/01 Summer 2019 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects that caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper Y545 series overview

This summer was the first assessment of Y545 to run and there was an entry of around 330 candidates. Given that there are at least two new topic areas being examined (number theory and surfaces & partial differentiation work) the quality of the responses received were very promising indeed. Scoring 60 marks or more (out of 75) represented a considerable achievement and this feat was accomplished by well over a quarter of the cohort. At the other end of the scale, only half-a-dozen candidates did not pick up at least 20 marks.

Although much greater detail will follow below in the comments on individual questions, there are a number of broad areas to be mentioned for the benefit of future candidates and a close scrutiny of the performance of this year's cohort will help to inform both teachers and their students.

Questions requiring explanation, justification or supporting reasoning were generally handled far too variably, with many candidates seemingly adopting the approach of constructing excessively lengthy responses in the hope that some of their words would earn the mark(s) allocated. Within the 75-minute time frame for this paper, it should be clear that written comments need to be brief and to the point; paragraphs of essay-style detail are very seldom appropriate. On the other side of this issue for candidates is how to cover the salient point(s) within a written response; many candidates often managed to say something correct, but were unable to state convincing mathematical reasons for their observation(s) and so could not be given credit. This was especially so in Q5 (groups) and Q6 (vector products) as well as to a lesser extent in Q2(b), Q3(b) and Q8.

	AfL	Another major issue arises in the number theory questions, one of the newer aspects of the course. As with the previously-encountered work on matrices, the issue of division is really rather an important one. To all intents and purposes it should be taken to be a non-existent operation, being replaced by the *substitute* operation of multiplication by inverses (i.e. reciprocals), a process reinforced within the group theory topic. Although it is not necessarily always incorrect to write a division statement within a set of finite arithmetic working, it is a challenge for students. Even within ordinary algebraic working, division by any quantity that *might* be zero always needs consideration. Within number theory working it is critical and almost all if not every candidate who begun working where a division was involved came unstuck at that point. Even where it is permissible to think of the process of division, it needs to be justified. Candidates therefore should always stop and make the appropriate remark to support this otherwise questionable practice. For example, $5x \equiv 11 \pmod{23} \Rightarrow 5x \equiv 11$ or 34 or 57 or $80 \ldots \pmod{23}$ by adding 23s; this then leads to the point where we 'divide' by 5, which is only permissible because **<u>hcf(5, 23) = 1</u>** (essentially justifying that 7 has an inverse modulo 23 that can be multiplied throughout the modulus 'equation'). There will generally be a mark for this observation, even though it may only be tested once (as this year, where it was required only in Q8 to support the use of FLT and not in the more routine Q3). Nevertheless, it is important for candidates to be in the good habit of stating the appropriate justification on each occasion where it is relevant. Note that there are usually alternative ways to present working that do not actually require 'division', but the point is still one of major significance within the topic.

	AfL	A second point of considerable importance also relates to the question of how much working is required in various situations, especially regarding calculators. Candidates should not get to the point of sitting the paper itself and needing to consider whether or not they could (or should) use their calculator to replace (for example) an integration using calculus with a numerical integration obtained from a calculator (as in Q7). Again, candidates should be prepared to recognise the key factors involved so that they can move swiftly from one case to the other. Candidates spending several minutes evaluating an integral analytically when all that the examiner is expecting is calculator work is a heart-breaking waste of their time. Key factors to consider are • the wording of the question, including the command word and/or a request for 'detailed reasoning', • the number of marks allocated, • what it is that the question is testing. The first two factors are generally very clear from what appears on the page and candidates would do well to consider more this clear information. The third factor is more of an acquired skill and does take specific practice working further maths papers (in particular) to pick up such background knowledge. This feature *does* raise its head on this paper (Q4) but the likely mark allocation is more deeply embedded in a longer question; hence less obvious. A good example is hence the Y535 (AS Level Further Mathematics A - Additional Pure Mathematics) 2019 paper where Q8b requires partial differentiation to determine values for which a function has a stationary point for 4 marks. Considering the allocation of 4 marks, it seems logical that 1 mark would be for determining $\frac{\partial z}{\partial x}$, 1 mark for determining $\frac{\partial z}{\partial y}$, 1 mark for setting both to zero and solving, plus 1 mark for finally obtaining solutions for *x* and *y*. While solving simultaneous equations by hand can be relatively simple, here the numbers involved didn't make it a simple process and doing this by hand was a challenge (which, when attempted, was often not done successfully) that candidates could have simply used their calculator for. Solving a pair of (albeit awkward) simultaneous equations is actually GCSE Level work that is assumed content knowledge here, so is unlikely to be something examiners are expecting candidates to spend much time with, particularly at the expense of content on this specification such as partial differentiation and setting derivatives to zero.

Overall, the skills demonstrated by candidates and their ability to present coherent mathematical arguments was very impressive. A significant minority of scripts however contained writing that was almost illegible, causing considerable problems to the markers who at times were barely able to read what had been written (especially in the text responses). Candidates should be encouraged to produce responses as clearly as they can, with clear writing of digits.

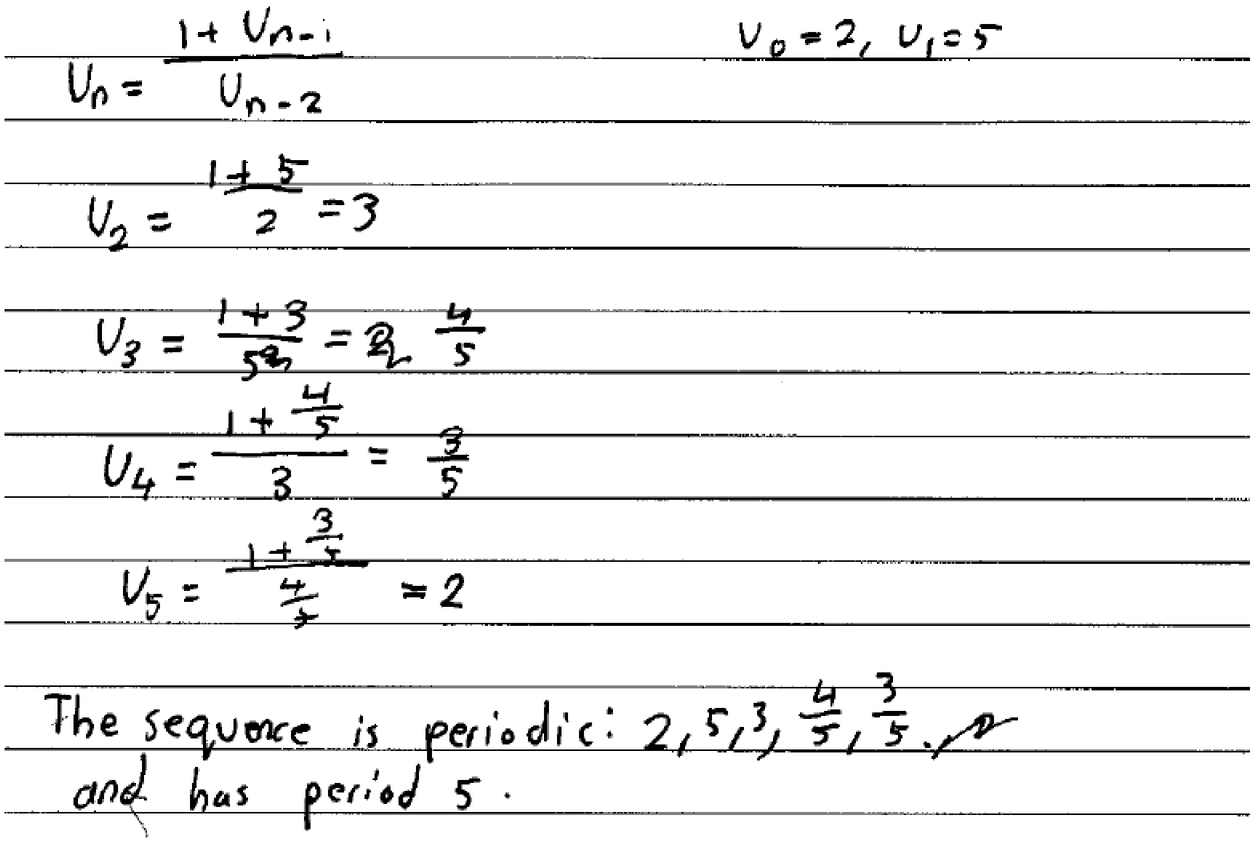
Question 1

1 The sequence $\{u_n\}$ is defined by $u_0 = 2,\ u_1 = 5$ and $u_n = \dfrac{1+u_{n-1}}{u_{n-2}}$ for $n \geqslant 2$.

Prove that the sequence is periodic with period 5. **[4]**

This was intended to be a fairly gentle introduction to the paper. Even without the aid of a modern calculator (which can very speedily generate sequences of this kind), it is very easy to calculate the next few terms. Indeed, just working as far as u_6 earned 3 of the 4 marks. However, around a quarter of all candidates stopped at the very first repeated term (i.e. u_5), thereby losing 2 marks. This represents a misunderstanding, since the given recurrence relationship is *second-order*, each term relying on the **two** previous terms and for this reason, candidates need to find a repeated pair of consecutive terms. Finally, as this was a 'proof' question, a final acknowledgement of this was needed; of the 70% of candidates scoring 3 or 4 marks on this question, the numbers were split fairly evenly between those who did and those who didn't.

Exemplar 1a


$U_n = \dfrac{1+U_{n-1}}{U_{n-2}}$ $U_0 = 2,\ U_1 = 5$

$U_2 = \dfrac{1+5}{2} = 3$

$U_3 = \dfrac{1+3}{5} = \dfrac{4}{5}$

$U_4 = \dfrac{1+\frac{4}{5}}{3} = \dfrac{3}{5}$

$U_5 = \dfrac{1+\frac{3}{5}}{\frac{4}{5}} = 2$

The sequence is periodic: $2, 5, 3, \frac{4}{5}, \frac{3}{5}, 2$
and has period 5.

Exemplar 1b

$u_n = \frac{1}{u_{n-2}} + \frac{u_{n-1}}{u_{n-2}}.$

$u_0 = 2,\ u_1 = 5,\ u_2 = 3,\ u_3 = \frac{4}{5}\ \ u_4 = \frac{3}{5}.$

$u_5 = 2.\ \ u_6 = 5$

Hence returns to initial values of u_0 and u_1 at u_5 and u_6.

The sequence will then repeat thus periodic w/ period 5.

Exemplar 1a stops at u_5 (the first repeating term) and this scored 2 of the 4 marks. Exemplar 1b has stopped just one term later (scoring the third mark). Then, despite having no explicit statement that the sequence is defined recursively as a function of *two* consecutive terms, there is a clear implication that the required result follows from the fact that $\{u_5, u_6\} = \{u_0, u_1\}$ and this is sufficient to earn the fourth mark for the requested 'proof' (on this occasion, a fairly informal one).

Question 2(a)

2 A surface has equation $z = \mathrm{f}(x, y)$ where $\mathrm{f}(x, y) = x^2 \sin y + 2y \cos x$.

(a) Determine f_x, f_y, f_{xx}, f_{yy}, f_{xy} and f_{yx} . **[5]**

Here candidates that have grasped the basic principles of partial differentiation scored highly and almost all candidates picked up the 5 marks. Almost all of the successful candidates worked out both f_{xy} and f_{yx} separately, with very few just stating that they are equal (due to a result known as the *Mixed Derivative theorem* for 'well-behaved' functions).

Question 2(b)(i)

(b) **(i)** Verify that z has a stationary point at $\left(\frac{1}{2}\pi, \frac{1}{2}\pi, \frac{1}{4}\pi^2\right)$. **[3]**

This part of the question was also well handled by the overwhelming majority of candidates, however almost half lost one of the 3 marks by failing to address all necessary matters. The two that were most prone to oversight being the mark for noting that $\mathrm{f}_x = 0$ and $\mathrm{f}_y = 0$ for a stationary point (which needed also to be demonstrated and not just stated) and, in particular, the mark for verifying the value of z at the point given.

A small number of candidates mistakenly thought they were required to *solve* $\mathrm{f}_x = 0$ and $\mathrm{f}_y = 0$ (or, occasionally, $\mathrm{f}_x = \mathrm{f}_y$).

Question 2(b)(ii)

(ii) Determine the nature of this stationary point. **[3]**

Again, this part of the question was dealt with very capably by almost all candidates, though again one mark was frequently lost due to a lack of care in either the numerical values (exact or decimal) of the four second partial derivatives or the evaluation of the Hessian's determinant.

Exemplar 2a

$$H = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix} = \begin{pmatrix} 2\sin y - 2y\cos x & 2x\cos y - 2\sin x \\ 2x\cos y - 2\sin x & -x^2\sin y \end{pmatrix}$$

$$f_{xx}f_{yy} - 2\,f_{xy}^{\;2} = -2x^2\sin^2 y + 2x^2 y\cos x - 4x^2\cos^2 y - 8x\cos y\sin y + 4\sin^2 x$$

$x = y = \frac{\pi}{2}$

$$-2\times\left(\frac{\pi}{2}\right)^2\sin^2\left(\frac{\pi}{2}\right) + 2\times\left(\frac{\pi}{2}\right)^3\cos\left(\frac{\pi}{2}\right) - 4\left(\frac{\pi}{2}\right)^2\cos^2\left(\frac{\pi}{2}\right) - 8\times\frac{\pi}{2}\cos\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right) + 4\sin^2\left(\frac{\pi}{2}\right) = -2.46$$

$|H| < 0$, therefore saddle point.

Exemplar 2b

at $\left(\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi^2}{4}\right)$, $H = \begin{pmatrix} 2 & -2 \\ -2 & -\frac{\pi^2}{4} \end{pmatrix}$

$|H| = -\frac{\pi^2}{2} - 4 < 0$

$\therefore$ it is a saddle-point

Exemplar 2a has made life very hard by attempting to work the determinant of the Hessian *algebraically*. For Exemplar 2b, a simple statement of the relevant values (worked out by calculator, presumably) suffices and then the following working and conclusion are made to look very straightforward

Question 3(a)

3 **(a)** Solve $7x \equiv 6 \pmod{19}$. **[2]**

This was another early question where a large majority of the candidates scored successfully. Of those that didn't pick up both marks, many of them lost one mark by arriving at the answer "$x = 9$", which appears to give the impression that there IS only the one answer, rather than infinitely many (of which 9 is a representative). "$x = 9 \pmod{19}$" is acceptable, although the use of the correct notation in the proper way is preferred.

Although not allocated a mark here in Q3, candidates using the 'division' approach (see comments in the introductory overview above) could (in similar future questions) be required to justify this move by noting the co-primality of some relevant factor with 19. Most of those who *did* mention that (for instance) hcf(7, 19) = 1 did so in order to note that a (unique) solution did indeed exist, rather than to justify the 'division' step.

Question 3(b)

(b) Show that the following simultaneous linear congruences have no solution.

$x \equiv 3 \pmod 4$, $x \equiv 4 \pmod 6$. **[2]**

The intention here was that candidates would note that the first statement necessarily requires x to be odd, while it must be even in the second, i.e. there is a clear contradiction. In summer 2019 responses, this was matched for frequency with responses quoting a result that is not actually on the syllabus, namely that for $x \equiv a \pmod{m_1}$ and $x \equiv b \pmod{m_2}$, solutions exist only when $\text{hcf}(m_1, m_2) \mid (b - a)$. While this is a perfectly acceptable approach to take, there were still quite a number who tried to quote it, but couldn't quite recall the details accurately and ended up without marks. It also raises the issue that, while *external* results may be valid and useful, the questions are actually constructed on the basis of what is on the syllabus and that all questions can be answered without recourse to additional material.

Question 4(a)

4 (a) Solve the second-order recurrence relation $T_{n+2} + 2T_n = -87$ given that $T_0 = -27$ and $T_1 = 27$. [8]

Although a question such as this (involving the solution of a second-order recurrence system (linear, with constant coefficients)) is relatively routine in principle, in practice it was relatively demanding due to the appearance of either complex numbers or trigonometric terms. Very few candidates elected to pursue the second of these two solution-paths and this was almost certainly for the best; candidates opting for trigonometric terms invariably did not identify the correct fundamental angle (simply $\frac{1}{2}\pi$) and performed poorly overall. Otherwise, the high level of success on this question was a fine testimony to the capabilities of the candidates' background knowledge in this respect, with around half of the candidates managing at least 7 of the 8 marks for this part.

It is an interesting point to note that part of this question required the solution of a pair of simultaneous equations and that there was no reason *not* to resort to the calculator, most of which cope readily with complex numbers.

A very small number of candidates (very shrewdly) recognised that the nature of the given recurrence relationship means that the odd- and even-numbered terms are independent of each other. Giving the answers for each of these two sub-sequences was usually completely successfully and for the record is below.

$$T_n = \begin{cases} 56\times(-2)^{\frac{1}{2}(n+1)} & n \text{ odd} \\ 2\times(-2)^{\frac{1}{2}n} & n \text{ even} \end{cases}$$

Question 4(b)

(b) Determine the value of T_{20}. [2]

Even those candidates who hadn't been able to sort out part (a) entirely satisfactorily could still work part (b) of the question, since T_{20} can be calculated directly from the sequence's definition, possibly even by means of a suitable calculator.

Question 5(a)

5 The group G consists of a set S together with $\times_{80}$, the operation of multiplication modulo 80. It is given that S is the smallest set which contains the element 11.

(a) By constructing the Cayley table for G, determine all the elements of S. [5]

The key here is to keep finding powers of 11 until one arrives at the identity (and a repeating cycle of elements following it). Looking to the following parts of the question, it should have been clear that G was going to be of order 4. The vast majority of candidates seemed perfectly at ease here and could be given all 5 marks on offer. Of the remaining candidates, the most popular response seemed to incorporate the integers from 1 to 11 before stopping, earning only the one mark for identifying the obvious identity '1'.

Question 5(b)

The Cayley table for a second group, H, also with the operation $\times_{80}$, is shown below.

$\times_{80}$	1	9	31	39
1	1	9	31	39
9	9	1	39	31
31	31	39	1	9
39	39	31	9	1

(b) Use the two Cayley tables to explain why G and H are not isomorphic. **[2]**

There were various approaches used by candidates here. The most popular one involved noting the differences between the orders of elements (while *H* consists of all self-inverse elements, of order 2, *G* has at least one element of order 4). Others pointed out that *G* was an example of the cyclic group of order 4, C_4, while *H* was an example of K_4, the Klein group of order 4.

To earn full marks here it was important for candidates to make clear and relevant comments. For instance, '*G* has a generator, *H* doesn't' is too vague and needs amplifying with a bit more detail as to what this means. Comments like 'The orders of the elements in *H* are…, while the orders of the elements in *G* are…' need to be supported with correct orders to count.

AfL

There is a significant discrepancy concerning the treatment (and description) of the properties of a group's identity element. In general, it is taken to have order 1, even though it satisfies the criteria which define a self-inverse element (of order 2). In this question, the orders of the elements of *G* are 1, 2, 4, 4 and the orders of the elements of *H* are 1, 2, 2, 2.

The reader may be reassured on this front, however; as a result of the different ways of referring to the identity and its properties, claims made by candidates are only considered for the *non-identity* elements of groups.

Exemplar 3

Every element in H has order 2 whereas 2 of the elements in G have order 4 (11,51)

The main purpose of this exemplar is to indicate that it is really not necessary to write lengthy responses. This candidate identifies a suitable method for explaining a fundamental difference in group structure and then does so in an admirably succinct way. Bearing in mind the immediately preceding remarks about the identity, the claim that *all* of *H*'s elements are self-inverse is taken to refer to the non-identity elements. In comparing this property with *G*'s elements, even this candidate has gone slightly further than necessary, since one actually only needs to note that there is ***one*** element in *G* that is not self-inverse to be credited.

Question 5(c)(i)

(c) **(i)** List

- all the proper subgroups of G,
- all the proper subgroups of H. **[3]**

The vast majority of candidates were able to identify the proper subgroups (all of order 2) of both groups.

	AfL	There is another significant discrepancy relating to the topic of groups that arises here and it is not quite so easy to tidy up definitively, as there are differences of opinion even among standard group theory texts. Teachers and students can be reassured however that the examiners are aware of this and (as with the identity issue mentioned above) fully intend to ignore the inclusion or exclusion of the sets $\{e\}$ and G in these sorts of questions. There are two main standpoints. The first is that a *proper* subgroup does **not** include the identity subgroup and the whole group itself; these being referred to as the *trivial* subgroups, since every group obviously has these two subgroups. The second view is that a proper subset H of a set G, written $H \subset G$ (as opposed to $H \subseteq G$), is not equal to the set G and hence G is, by this definition, **not** a *proper* subgroup of itself. However, this definition does not preclude the identity subgroup as being described as '*proper*' and this can cause confusion for candidates.

Question 5(c)(ii)

(ii) Use your answers to **(c) (i)** to give another reason why G and H are not isomorphic. **[1]**

Candidates who had any kind of sensible answer to part (c)(i) realised that, in order to be isomorphic, the two groups would necessarily have to contain equal numbers of subgroups (of the same kind/order).

Exemplar 4

all elements in H have the order 2 and can generate a subgroup itself whereas only one element in G can.

Exemplar 4 demonstrates that it is possible to make completely correct statements that still do not achieve the mark. In this case, the candidate's reasons might have earned marks in part (b), but here the question requires the solver to refer back to the proper subgroups found in (c)(i).

Question 6(a)

6 **(a)** For the vectors $\mathbf{p} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 2 \\ -4 \\ 5 \end{pmatrix}$, calculate

- $\mathbf{p} \cdot \mathbf{q} \times \mathbf{r}$,
- $\mathbf{p} \times (\mathbf{q} \times \mathbf{r})$,
- $(\mathbf{p} \times \mathbf{q}) \times \mathbf{r}$. **[6]**

Almost all candidates found this reasonably straightforward. That said there were quite a number of slip-ups with negative signs, but mostly this was handled very capably indeed. A small minority of candidates found a vector answer for the scalar triple product.

Question 6(b)

(b) State whether the vector product is associative for three-dimensional column vectors with real components. Justify your answer. **[1]**

Technically, part (a) provided one example for which **p** × (**q** × **r**) ≠ (**p** × **q**) × **r**, but this is sufficient to conclude that the vector product is not an associative operation. Most candidates duly noted this.

Question 6(c)

It is given that **a**, **b** and **c** are three-dimensional column vectors with real components.

(c) Explain geometrically why the vector $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$ must be expressible in the form $\lambda\mathbf{b} + \mu\mathbf{c}$, where λ and μ are scalar constants. **[2]**

Although there is a little more to it than this, essentially all that was being looked for here was some explanation that **b**×**c** is a vector perpendicular to both **b** and **c**, and that **a**×(**b**×**c**) is perpendicular to this normal. Whether this is 'in' some plane or merely 'parallel to' it requires a little more care in one's description and many candidates bravely made a fuller description in order to justify the final given form.

Question 6(d)

It is given that the following relationship holds for **a**, **b** and **c**.

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \bullet \mathbf{c})\,\mathbf{b} - (\mathbf{a} \bullet \mathbf{b})\,\mathbf{c} \qquad (*)$$

(d) Find an expression for $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$ in the form of (*). **[3]**

Unfortunately, rather a lot of candidates seemed to just guess here, with the most popular choice being exactly the one quoted in **(*)**. All that was wanted was an application of the anti-commutative property of the vector product followed by a re-ordering of the vectors contained within **(*)** to match.

Question 7(a)

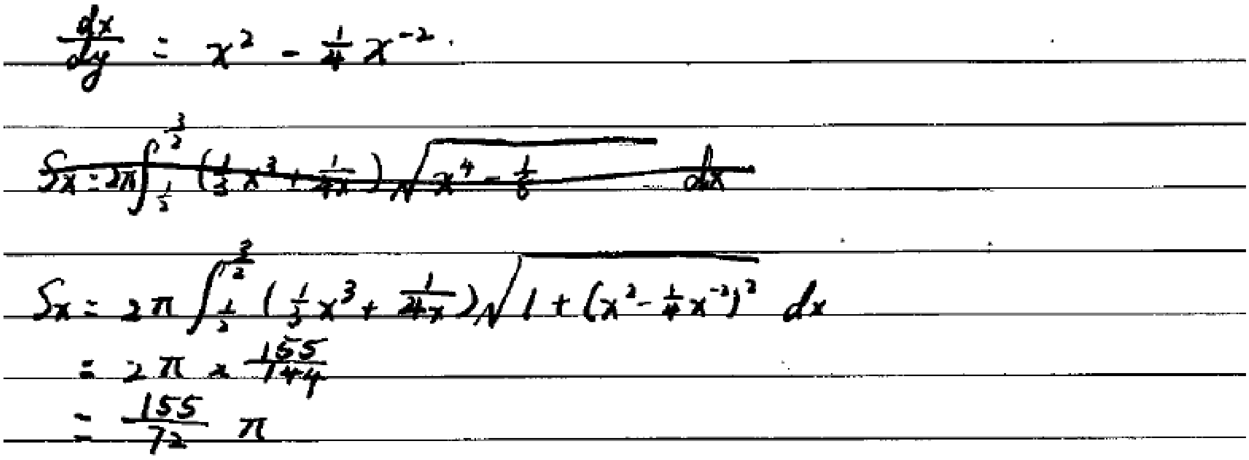
7 The points $P\left(\frac{1}{2}, \frac{13}{24}\right)$ and $Q\left(\frac{3}{2}, \frac{31}{24}\right)$ lie on the curve $y = \frac{1}{3}x^3 + \frac{1}{4x}$.

The area of the surface generated when arc PQ is rotated completely about the x-axis is denoted by A.

(a) Find the exact value of A. Give your answer as a rational multiple of π. **[4]**

A large majority of candidates scored 3 or 4 here. Candidates begun by quoting the correct surface area integral formula and working out the integrand, but then candidates seemed to be split equally among those who completed the integration process by hand and those who used a calculator to identify the correct rational multiple of π. Both were acceptable, though the second approach was probably quicker and less prone to small numerical errors.

Exemplar 5



Exemplar 5 nicely illustrates the correct application of the 'principle of least effort' and serves as an excellent exemplar for any of the three parts of Q7.

The correct derivative has been written down (despite calling it dx/dy for no obvious reason; such irrelevant 'typos' are generally ignored within working until such a point where they affect the correctness or validity of the ensuing mathematical working). The surface area formula has then been quoted in context (i.e. with the relevant content for this question), followed by a clear statement of the integral to be evaluated.

To cap off this excellently concise solution, this candidate has then resorted to their calculator to feed them the correct output value for the integral. This is a case in point regarding the general remark made earlier about when it is appropriate to use a calculator and this candidate has judged it to perfection. Although one might expect good candidates at this level to be able to deal quite easily with an integral of this kind (once one has obtained a square-root of a perfect square), an understanding of the assessment should reveal that this is work that does not need to be done that way, as what is being Y545 assesses does not include basic A Level integration methods. In the absence of a "detailed reasoning…" demand or similar command word, candidates are quite at liberty to presume that their solution need not be any more algebraic than this one displayed here. In addition, many candidates who did attempt the integration lost time in so doing and often lost the single mark for which they were striving through some often minor numerical slip along the way.

Question 7(b)

Student X finds an approximation to A by modelling the arc PQ as the straight line segment PQ, then rotating this line segment completely about the x-axis to form a surface.

(b) Find the approximation to A obtained by student X. Give your answer as a rational multiple of π. **[4]**

Almost all candidates approached this using integration very successfully, through finding the equation of the line joining P and Q and then using the same process as for (a). A few lost marks by being careless with this line equation. Again, calculators were frequently used to find the multiple of π required for the answer.

Question 7(c)

Student Y finds a second approximation to A by modelling the original curve as the line $y = M$, where M is the mean value of the function $f(x) = \frac{1}{3}x^3 + \frac{1}{4x}$, then rotating this line completely about the x-axis to form a surface.

(c) Find the approximation to A obtained by student Y. Give your answer correct to four decimal places. **[4]**

Many more candidates actually turned to an entirely calculus approach for the first part before going decimal for the final answer. A lot of mistakes were made with the integral of $\frac{1}{4x}$, but otherwise this was also handled very capably with around half of all candidates gaining all 4 marks.

Question 8(a)

8 In this question you must show detailed reasoning.

(a) Prove that $2(p-2)^{p-2} \equiv -1 \pmod p$, where p is an odd prime. **[4]**

There is little doubt that candidates found this to be the hardest question on the paper and it elicited a lot of poorly-constructed answers and conceptual uncertainties as a result. Most of those who did make a serious attempt at least recognised that Fermat's little theorem (FLT) was needed at some stage. Unnecessarily, though not incorrectly, the use of the binomial theorem was frequently used, but a lot of attempts suffered from the impulse to start dividing, or working with an unspecified inverse/reciprocal of (p – 2) that then proved an insurmountable problem. As mentioned earlier on in the introductory overview section, those candidates who started trying to divide were rarely triumphant.

Successful candidates were those who pointed out that, mod p, $(p - 2) \equiv -2$, since multiples of the modulus can always be removed (or introduced) without harm. Some of these candidates justified this step by deploying the binomial theorem, but this is an unnecessary distraction as it should be obvious that this is so. Having reduced $(p - 2)^{p-2}$ to $(-2)^{p-2}$, it is then simple to note that the exponent must be odd (as p is an odd prime) and so this is now equal to $-(2^{p-2})$. At which point, the extra factor of 2 in the question leads to the use of FLT and the result comes out after only three or four short lines of working. However, the use of FLT in the form $a^{p-1} \equiv 1 \pmod p$ does require the checking of the appropriate condition (on this occasion that hcf(2, p) = 1) and many otherwise successful solutions missed out on the mark for this.

Question 8(b)

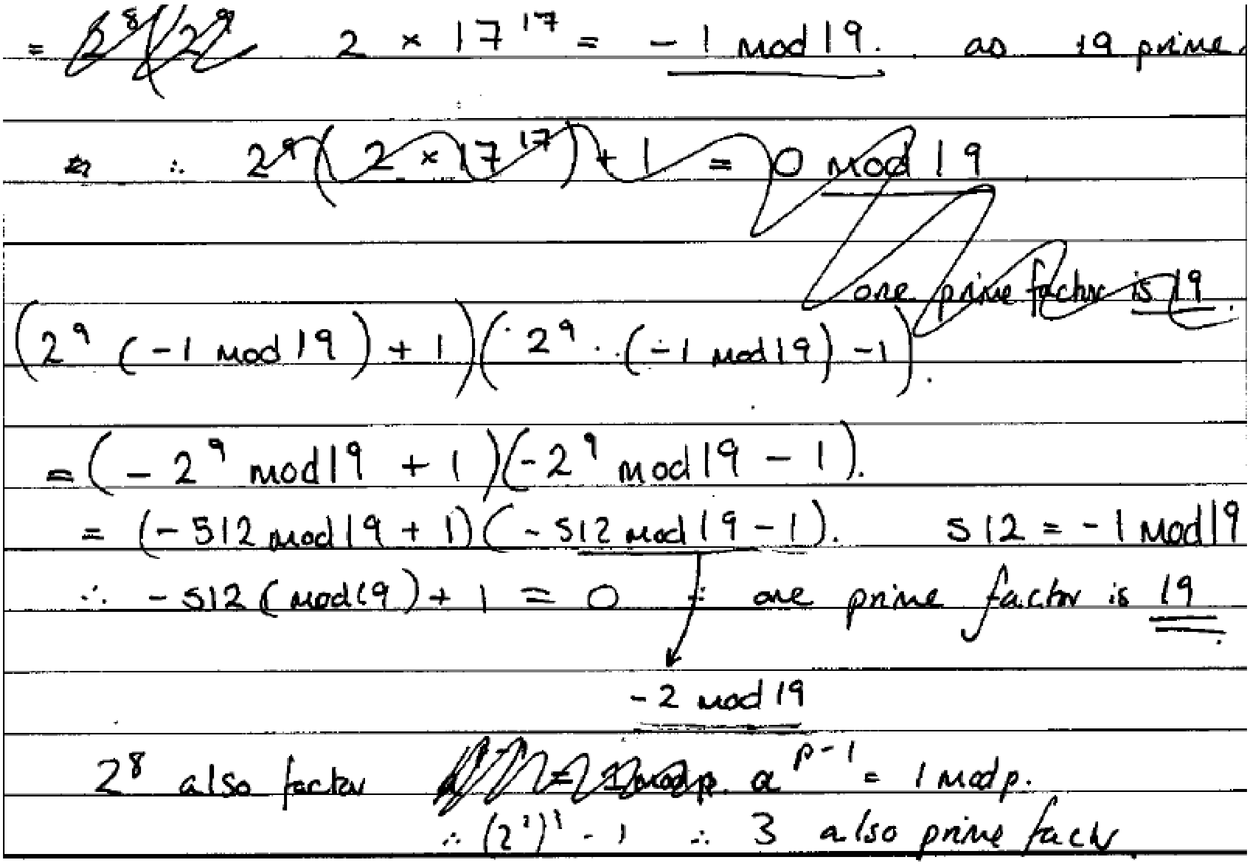
(b) Find two odd prime factors of the number $N = 2 \times 34^{34} - 2^{15}$. **[7]**

Many candidates that had struggled with (a) seemed to have little appetite for this and made no attempt here, but it is good exam technique to recognise that one can use given results (as here in (a)) without having proved them. Among those who did produce an uneasy response, far too many looked to test for divisibility by 17 (or 34, as if 36 were the prime being referred to).

The most sensible starting point (unfortunately taken by few candidates) when asked to look for odd factors is to remove as many even factors (i.e. powers of 2) as possible before proceeding. Extracting, and then ignoring, the 2^{15} then leaves you with $2^{20}\times17^{34} - 1$. Those who made this obvious step were then the beneficiaries of an easy mark for using the 'difference of two squares factorisation, which led to $(2^{10}\times17^{17} - 1)(2^{10}\times17^{17} + 1)$ and $p = 19$ follows. Candidates taking these steps were then able to continue to show that 19 is one of the required odd factors of N.

Only a few candidates spotted the obvious factor of 3, even among those who made progress towards the 19; arguments here tended to vary from far too concise to excessive detail. In simple terms, again it is far better with the maximum number of 2s gone, then to note that $34 \equiv 1 \pmod 3$ and 2^2 1 (mod 3) also.

Exemplar 6



$2 \times 17^{17} = -1 \mod 19$. as 19 prime

$(2^9(-1 \mod 19)+1)(2^9(-1 \mod 19)-1)$

$= (-2^9 \mod 19 + 1)(-2^9 \mod 19 - 1)$

$= (-512 \mod 19 + 1)(-512 \mod 19 - 1)$. $512 = -1 \mod 19$

$\therefore -512 (\mod 19) + 1 = 0$ ∴ one prime factor is 19

$-2 \mod 19$

2^8 also factor $a^{p-1} = 1 \mod p$.

$\therefore (2^2)^1 - 1$ ∴ 3 also prime factor

Exemplar 6 is an excerpt from a solution to Q8(b) that nicely illustrates some very interesting points. Firstly, by factoring out the powers of 2 in two stages a mark has been lost by overlooking that 2^7 has been extracted from each of the two brackets. Nonetheless, the usefulness of a 'difference of two squares factorisation' has still been spotted and this has been left unaffected by the incorrect power of 2 (since it can now safely be ignored). Having taken two steps into this approach, it is now clear that what remains relates back to part (a)'s result, with $p - 2 = 17$ emerging. This candidate went on to score 4 of the 5 marks available for the factor of 19, losing only the mark noted.

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