

## A LEVEL

*Examiners' report*

# **MATHEMATICS A**

**H240**

For first teaching in 2017

## **H240/01 Summer 2019 series**

Version 1

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## Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.



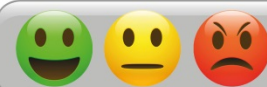
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## Paper 1 series overview

H240/01 is one of the three examination units for the new revised A Level examination for GCE Mathematics A. It is a two hour paper consisting of 100 marks which tests Pure Mathematics topics. Pure Mathematics topics are also tested on the first half of Papers 2 and 3, and any Pure Mathematics topic could be tested on any of the three papers. This is the second series of the reformed linear A Level Maths specification, and the first to be sat by candidates following the standard two year programme.

To do well on this paper, candidates need to be familiar with the entire specification and also have an understanding as to how to apply it to any question. They should know which formulae are given at the start of the paper, quoting them accurately as required, and also be aware of formulae that will not be given to make sure that these have been memorised. They should also be confident with the functions available on their calculator and be able to make effective use of the technology where appropriate.

Some questions contain specific defined 'command words', in particular the instruction '**In this question you must show detailed working**'. In these questions, candidates are required to demonstrate their understanding of the relevant concepts by showing their working, rather than by presenting an answer gained simply by pressing a few buttons. Consequently, in these questions, marks will not be given unless correct working is seen. Remember that this does not preclude candidates from checking their working using the calculator.

	<b>OCR support</b>	A poster detailing the different command words and what they mean is available here: <a href="https://teach.ocr.org.uk/itallddsup">https://teach.ocr.org.uk/itallddsup</a>
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Conversely, some questions have slightly lower mark tariffs than seen in the legacy assessment, reflecting the full range of calculator functions available. These are also signposted by the 'command words' used in the question: 'Find', 'Calculate', 'Write down'.

Candidates should make sure that their solutions have sufficient detail in each step taken and not try to include more than one step in a single line of working. If candidates realise that there is an error in their solution, then they would be well advised to delete that attempt and start afresh. Trying to correct errors in an existing solution can often result in not all necessary amendments being made and the solution still containing errors.

If questions are set in context then the candidate should make sure that their response is also in context, paying careful attention to units and also whether a decimal answer is appropriate or whether an integer answer is required. When commenting on the limitations of a model, explanations should be sufficiently detailed and include specific reasons as appropriate. Candidates should also make sure that their handwriting is legible.

## Question 1

**1 In this question you must show detailed reasoning.**

Solve the inequality  $10x^2 + x - 2 > 0$ . [4]

This question was generally answered well, with many fully correct solutions seen. Candidates generally paid heed to the 'detailed reasoning' instruction, showing how they had solved their quadratic. Some candidates used their calculator to produce the roots and then either quoted them or attempted to work backwards and deduce what the factors might have been. Most candidates were then able to identify the correct region for the inequality, with the majority giving their answer as inequalities whereas some showed good mastery of set notation. A few spoiled an otherwise correct solution by using 'and' to link their inequalities and others attempted to write it as a single inequality.

## Question 2 (a) (i)

**2** The point  $A$  is such that the magnitude of  $\overrightarrow{OA}$  is 8 and the direction of  $\overrightarrow{OA}$  is  $240^\circ$ .

**(a) (i)** Show the point  $A$  on the axes provided in the Printed Answer Booklet. [1]

This proved to be a challenging question; while candidates appeared to be familiar with the term 'magnitude' they were less certain about the 'direction' of a vector and many treated it as a bearing, taking it clockwise from the  $y$ -axis. Candidates should make sure that they show sufficient detail on any sketch; in this question both the length of  $OA$  and a relevant angle were expected to be labelled.

## Question 2 (a) (ii)

**(ii)** Find the position vector of point  $A$ .  
Give your answer in terms of  $\mathbf{i}$  and  $\mathbf{j}$ . [3]

Most candidates were able to make some attempt at finding the components of the position vector, with some using basic trigonometry and others setting up, and solving, simultaneous equations using Pythagoras' theorem and  $\text{invtan}$ . However, the incorrect positioning of the point  $A$  in the previous part of the question inevitably limited the marks gained in this part for many of the candidates.

## Question 2 (b)

The point  $B$  has position vector  $6\mathbf{i}$ .

**(b)** Find the exact area of triangle  $AOB$ . [2]

Most candidates were able to attempt the area of the triangle, using what they believed angle  $AOB$  to be. Notice that this question explicitly asked for the 'exact area'; full marks would not be given for a decimal approximation of the surd, regardless of the number of decimal places quoted.

### Question 2 (c)

The point  $C$  is such that  $OABC$  is a parallelogram.

- (c) Find the position vector of  $C$ .  
Give your answer in terms of  $\mathbf{i}$  and  $\mathbf{j}$ . [2]

Many candidates seemed unfamiliar with the labelling convention for a polygon, and placed the point  $C$  such that the parallelogram being considered was actually  $OACB$ . The more successful candidates made good use of a diagram, with clear calculations to justify the relevant vectors that were being found.

### Question 3 (a)

- 3 The function  $f$  is defined by  $f(x) = (x-3)^2 - 17$  for  $x \geq k$ , where  $k$  is a constant.

- (a) Given that  $f^{-1}(x)$  exists, state the least possible value of  $k$ . [1]

The more able candidates appreciated that the domain had to be restricted as  $f(x)$  had to be a one to one function, and were able to write down the value of  $k$  with ease. However, many candidates seemed unsure as to how to proceed, with the most common error being to solve  $f(x) = 0$ .

### Question 3 (b)

- (b) Evaluate  $ff(5)$ . [2]

The most efficient method was to first find  $f(5)$ ; those candidates who first attempted at algebraic expression for  $ff(x)$  were rarely successful. Having found that  $f(5) = -13$ , candidates were expected to observe that this was outside the domain and hence  $ff(5)$  was undefined, and more astute candidates did indeed give this solution. However the subtlety of the question, combined with a number of incorrect solutions to part (a), meant that this was missed by many who simply gave their final answer as 239. This approach was condoned as a special case when marking.

### Question 3 (c)

- (c) Solve the equation  $f(x) = x$ . [3]

Most candidates were able to gain two marks for setting up and solving the relevant equation, but only a minority of the candidates gained the final mark for discarding  $x = -1$ , noting that it was outside the domain.

### Question 3 (d)

- (d) Explain why your solution to part (c) is also the solution to the equation  $f(x) = f^{-1}(x)$ . [1]

Some candidates were able to give clear and concise explanations about the two functions being reflections in  $y = x$  and thus intersecting on this line; but a number of explanations lacked precision.

### Question 4 (a)

- 4 Sam starts a job with an annual salary of £16 000. It is promised that the salary will go up by the same amount every year. In the second year Sam is paid £17 200.

(a) Find Sam's salary in the tenth year. [2]

The majority of candidates could identify that this was an arithmetic progression, find the common difference and attempt the tenth term. As this was a question posed in a real-life context it was expected that solution should reflect this; a number of candidates did not include units in their answer thus not gaining full credit.

### Question 4 (b)

(b) Find the number of complete years needed for Sam's **total** salary to first exceed £500 000. [4]

Most candidates could attempt the sum of the arithmetic progression, equate it to 500,000 and attempt to solve. There was an expectation that candidates would use their calculators to solve the resulting quadratic. While some did so, they were in the minority and errors in handling the 'unfriendly' numbers were seen. Candidates then had to appreciate that an integer solution was required, and most gained full marks by doing so. Rather than attempting the sum, the most common error was to instead use the  $n$ th term formula and the answer that Sam would have to work for 405 years did not seem to result in candidates reconsidering their solution.

### Question 4 (c)

(c) Comment on how realistic this model may be in the long term. [1]

When commenting on a model, candidates are first expected to state whether or not the model is realistic and then use evidence to support their answer. A variety of solutions were seen, including those which referred to inflation, getting a promotion (or taking on a different role within the company) or there being a pay cap on annual salary within the company. However, too many solutions were too vague and simply referred to Sam being paid too much money without giving any kind of reason or example as to why this was unrealistic. Explanations that referred to death, retirement or moving to a different company were also not accepted.

## Exemplar 1

Unrealistic - due to possible promotions / changes in economy, salary is unlikely to go up exactly same amount each year. It is also likely to be capped at some point, as the company don't have unlimited funds.

The candidate identifies that this model may be unrealistic in the long term and gives a number of specific examples as to why this may be the case.

## Exemplar 2

Unrealistic to think salary will increase every year at a constant rate.

This candidate also identifies that the model may be unrealistic as the salary is unlikely to increase at a constant rate every year but does not support this with an example or specific reason, so did not gain the mark.



### Question 5 (a)

5 A curve has equation  $x^3 - 3x^2y + y^2 + 1 = 0$ .

(a) Show that  $\frac{dy}{dx} = \frac{6xy - 3x^2}{2y - 3x^2}$ . [4]

This question was very well answered, with many fully correct solutions seen. Candidates seemed familiar with implicit differentiation, and were then able to provide convincing evidence of how the given answer was obtained. The most common error was for the sign to be incorrect on the second term coming from the differentiation of  $-3x^2y$ . It was clear that some candidates realised from their final answer that a sign error had occurred, and attempted to correct this, but did not always amend all of the relevant lines. When a candidate identifies that an error has occurred it can be better to delete this solution and start afresh so as to provide a fully convincing solution.

### Question 5 (b)

(b) Find the equation of the normal to the curve at the point (1, 2). [4]

This part of the question was also very well answered, with the majority of the candidates gaining full credit. The most successful candidates found a numerical value for the gradient of the tangent and from this found the gradient, and hence the equation, of the normal. Attempts to find the negative reciprocal algebraically tended to be less successful, and some candidates even used this algebraic expression in their equation of the normal.

### Question 6

6 Let  $f(x) = 2x^3 + 3x$ . Use differentiation from first principles to show that  $f'(x) = 6x^2 + 3$ . [6]

Most candidates seemed familiar with the process of differentiation from first principles, quoting the formula from the question paper and attempting to apply it to the given function. The best candidates made effective use of brackets and used a new line for each step of their solution rather than attempt to do too much in one go. The expansion of  $(x + h)^3$  was usually correct, although many candidates elected to expand three brackets rather than apply the binomial expansion. When the expansion was multiplied by 2, it was quite common to see some incorrect coefficients, especially in the solutions of those who had expanded three brackets and not then collected like terms. The other common error was due to not using brackets when introducing  $-(2x^2 + 3x)$  in the numerator, resulting in the  $3x$  being of the incorrect sign. Some candidates appeared not to notice and cancelled  $+3x$  with another  $+3x$ , whereas others did realise that there was an error and attempted to correct it, but this was not always applied to every line of working. Candidates who had a correct simplified expression were usually able to complete the proof by explicitly considering the limit as  $h$  tended to 0.

## Exemplar 3

$$f(x) = 2x^3 + 3x$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \rightarrow 0} \frac{2(x+h)^3 + 3(x+h) - 2x^3 - 3x}{h}$$

$$\lim_{h \rightarrow 0} \frac{2(x^2 + 2xh + h^2)(x+h) + 3x + 3h - 2x^3 - 3x}{h}$$

$$\lim_{h \rightarrow 0} \frac{2(x^3 + 3x^2h + 3xh^2 + h^3) + 3x + 3h - 2x^3 - 3x}{h}$$

$$\lim_{h \rightarrow 0} \frac{2x^3 + 6x^2h + 6xh^2 + 2h^3 + 3h - 2x^3}{h}$$

$$\lim_{h \rightarrow 0} \frac{6x^2h + 6xh^2 + 2h^3 + 3h}{h}$$

$$\lim_{h \rightarrow 0} 6x^2 + 6xh + 2h^2 + 3$$

As  $h$  tends to 0

$$6x^2 + 6x(0) + 2(0)^2 + 3$$

$$= 6x^2 + 3$$

$$\text{so } f'(x) = 6x^2 + 3$$

Each step in this proof is clearly detailed, and only one step is made on each line which makes the proof convincing and easy to follow. The candidate uses correct notation throughout, including explicit consideration of the limit as  $h$  tends to 0.

Exemplar 4

$$\begin{aligned}
 f(x) &= 2x^3 + 3x \\
 \frac{f(x+h) - f(x)}{h} &= \frac{2(x+h)^3 + 3(x+h) - (2x^3 + 3x)}{h} \\
 &= \frac{2(x^3 + x^2h + 2xh + 2xh^2 + h^3) + 3x + 3h - 2x^3 - 3x}{h} \\
 &= \frac{2x^3 + 2x^2h + 4xh + 2h^3 + 3x + 3h - 2x^3 - 3x}{h} \\
 &= \frac{2x^2h + 4xh + 2h^3 + 3h}{h} \\
 &= 2x^2 + 4x + 2h^2 + 3
 \end{aligned}$$

The candidate gains the first mark for setting up the correct expressions and gains a second mark for the correct binomial expansion. On the third line it seems that the cancelling by  $h$  has occurred before the numerator has been simplified so no further credit is given. This is an example of trying to do too much in one go.

Question 7

7 In this question you must show detailed reasoning.

A sequence  $u_1, u_2, u_3, \dots$  is defined by  $u_n = 25 \times 0.6^n$ .

Use an algebraic method to find the smallest value of  $N$  such that  $\sum_{n=1}^{\infty} u_n - \sum_{n=1}^N u_n < 10^{-4}$ . [8]

Most candidates were able to gain some credit on this question, but fully correct solutions were in a minority. The most effective method was to start by writing out the first few terms of the sequence, to allow the first term and common ratio to be identified. These values could then be substituted into the relevant formulae, and the inequality rearranged to useable form. Candidates were generally able to make a good attempt at this, but sign errors were relatively common. In this 'detailed reasoning' question, there was an expectation that the use of logarithms to solve the inequality would be explicit and this was nearly always the case. Candidates then had to conclude by stating the correct integer value of  $N$ , a step that some omitted and concluded with a decimal answer. The final answer had to be fully justified by correct use of inequalities throughout (which was often not the case with  $\log 0.6$  involved), or by testing at least one relevant integer value. The most common misconception was to assume that the first term of the sequence was 25, with no check of  $n = 1$ .

## Question 8 (a)

- 8 A cylindrical tank is initially full of water. There is a small hole at the base of the tank out of which the water leaks.

The height of water in the tank is  $x$  m at time  $t$  seconds. The rate of change of the height of water may be modelled by the assumption that it is proportional to the square root of the height of water.

When  $t = 100$ ,  $x = 0.64$  and, at this instant, the height is decreasing at a rate of  $0.0032 \text{ ms}^{-1}$ .

- (a) Show that  $\frac{dx}{dt} = -0.004\sqrt{x}$ . [2]

Most candidates were able to set up an appropriate model, with a fairly even split between those who used  $k$  and those who used  $-k$ , and then attempt to find a value for  $k$ . The more astute candidates appreciated that the height decreasing implied a rate of  $-0.0032$ , and could use this correctly to justify the given differential equation. The most common misconception was to omit the negative sign from  $0.0032$ , with many not noticing that this resulted in value for  $k$  that was not consistent with the given answer.

## Question 8 (b)

- (b) Find an expression for  $x$  in terms of  $t$ . [4]

Most candidates could make an attempt to solve the given differential equation, usually by separating the variables but sometimes by inverting the equation and integrating. The integration was usually correct, and candidates could then use the given initial conditions to find a value for their constant of integration. Candidates were not always successful in finding an expression for  $x$  in terms of  $t$ ; some omitted to attempt this and others either squared term by term or square rooted rather than squaring the function of  $t$ .

## Question 8 (c)

- (c) Hence determine at what time, according to this model, the tank will be empty. [2]

Candidates appreciated that they had to find the value of  $t$  for which  $x = 0$ , and most could attempt to do so. The most efficient method was to use an equation from part (b) that required minimal manipulation as opposed to using the final answer. As this question was in context, units were required in the final answer which a number of candidates overlooked.

## Question 9 (a)

- 9 (a) Express  $3 \cos 3x + 7 \sin 3x$  in the form  $R \cos(3x - \alpha)$ , where  $R > 0$  and  $0 < \alpha < \frac{1}{2}\pi$ . [3]

The majority of candidates were familiar with this standard process, gaining all 3 marks with ease. Some candidates did not gain the final mark through giving their angle in degrees and not radians as specified.

## Question 9 (b)

- (b) Give full details of a sequence of three transformations needed to transform the curve  $y = \cos x$  to the curve  $y = 3 \cos 3x + 7 \sin 3x$ . [4]

This part of the question proved to be much more challenging, and fully correct solutions were in the minority. Barely a third of the candidates could identify the three transformations and describe them using the correct terminology and only the most able could also give them in the correct order. There were some candidates who could use informal language to convey their intention (such as 'shift' and 'compression') and other explanations were ambiguous (for example 'a stretch in the x-axis'). There was however a number of candidates who did not appreciate the link with part (a) and gave descriptions that involved a translation of  $7\sin 3x$ . Candidates should always consider whether earlier parts of a question have been posed to give them assistance with later parts.

## Question 9 (c)

- (c) Determine the **greatest** value of  $3 \cos 3x + 7 \sin 3x$  as  $x$  varies and give the smallest positive value of  $x$  for which it occurs. [2]

Candidates were marginally more successful in identifying the greatest value of the function as opposed to the value of  $x$  where it occurred. While just writing down the greatest value was condoned, the 'determine' instruction meant that some justification was required for the value of  $x$ . A surprising number of candidates did not make the link between this part of the question and the form of the function found in part (a), with some resorting to calculus, use of their calculator or simply considering the function term by term.

## Exemplar 5

the maximum value of  $\cos(\theta) = 1$   
 $\therefore$  maximum value of  $3\cos 3x + 7\sin 3x = \sqrt{58}$   
 ~~$\cos(3x - 1.17) = 1$~~   
 $\cos(3x - 1.17) = 1$        $3x = 1.17$   
 $\cos^{-1} 1 = 0$        $x = \underline{0.389}$

A good example of sufficient justification being given in response to the command word of 'determine' in the question.

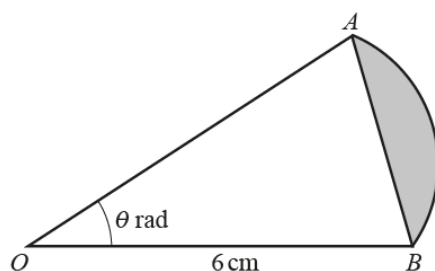
## Question 9 (d)

- (d) Determine the **least** value of  $3 \cos 3x + 7 \sin 3x$  as  $x$  varies and give the smallest positive value of  $x$  for which it occurs. [2]

This part of the question was less well answered, with the most common error being to assume that the least value of the function was 0 and then giving the associated value of  $x$ .

## Question 10 (a)

10



The diagram shows a sector  $AOB$  of a circle with centre  $O$  and radius 6 cm.  
The angle  $AOB$  is  $\theta$  radians.  
The area of the segment bounded by the chord  $AB$  and the arc  $AB$  is  $7.2 \text{ cm}^2$ .

- (a) Show that  $\theta = 0.4 + \sin \theta$ . [3]

This part of the question was very well answered, with the majority of the candidates able to quote the relevant formulae, set up an equation and manipulate this to obtain the given answer.

## Question 10 (b)

- (b) Let  $F(\theta) = 0.4 + \sin \theta$ .

By considering the value of  $F'(\theta)$  where  $\theta = 1.2$ , explain why using an iterative method based on the equation in part (a) will converge to the root, assuming that 1.2 is sufficiently close to the root. [2]

To gain the first mark, candidates simply had to find the value of  $F'(1.2)$ , yet the majority did not follow this instruction. A further few found the derivative but did not substitute 1.2. Of those who did carry out the evaluation correctly, some did not understand the relevance of this value with references to tangents and Newton-Raphson being the most common error. Others clearly had an inkling that the gradient related to the convergence of the iterative formula but their explanation was insufficiently specific to gain the mark, most typically just stating that it was less than 1. The most able candidates were able to give a suitably detailed explanation, referring to the modulus of the gradient being less than 1, and some even illustrated this with a staircase diagram.

### Question 10 (c)

- (c) Use the iterative formula  $\theta_{n+1} = 0.4 + \sin \theta_n$  with a starting value of 1.2 to find the value of  $\theta$  correct to 4 significant figures.  
You should show the result of each iteration. [3]

By contrast, this part of the question was very well attempted with the vast majority of candidates gaining full credit. Some candidates did not heed the instruction to give the answer to 4 significant figures, thus losing the final mark, and others worked in degrees and not radians. A small minority decided to use the Newton-Raphson method rather than the given iterative formula.

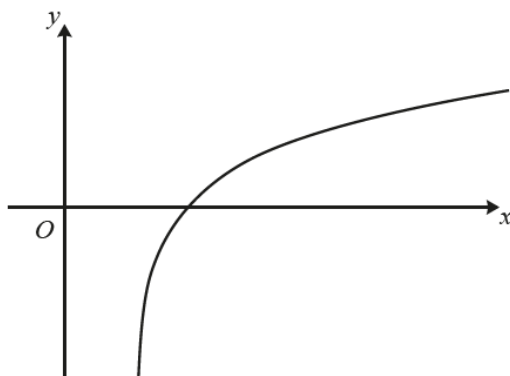
### Question 10 (d)

- (d) Use a change of sign method to show that the value of  $\theta$  found in part (c) is correct to 4 significant figures. [3]

This topic is new to the specification, and many candidates seem unsure as to how to proceed. The most common error was to use 1.381 and 1.383 in their sign change attempt. Others did show the two relevant values, 1.3815 and 1.3825, but then just stated that there was a sign change with no empirical evidence for this. The most successful candidates first rearranged the equation to make it equal to 0, substituted in two correct values, noted the sign change and gave a final conclusion that explained why this justified the root to 4 significant figures. Other candidates substituted into the equation in its given format; this is an acceptable alternative method but candidates need to be aware that they need to substitute into both sides of the equation and look for a change in the direction of the inequality sign as most gave up when they did not obtain the sign change that they were expecting.

## Question 11 (a)

11



The diagram shows part of the curve  $y = \ln(x-4)$ .

(a) Use integration by parts to show that  $\int \ln(x-4) dx = (x-4)\ln|x-4| - x + c$ . [5]

Many candidates could attempt integration by parts and gain the first two marks available, but were then unsure as to how to deal with the improper fraction and struggled to make any further progress. The most efficient method was to carry out a division attempt, either formal algebraic division or an informal balancing method, and then integrate the resulting expression. The other common method was to use a substitution that resulted in the denominator becoming a single term, thus facilitating the subsequent division. The latter method did cause problems for some candidates as it resulted in an extra term of +4 which they had to justify being absorbed into the constant of integration; in some cases it just conveniently disappeared. Some candidates used a substitution as their first step; this approach was condoned as integration by parts was still required in this method. Candidates were expected to produce the given result and a lack of clear modulus signs in the final answer was penalised. There was also evidence that a few candidates initially used the correct method but then spoiled this by attempting to adjust their working to give the given answer straightaway.





Question 11 (b)

- (b) State the equation of the vertical asymptote to the curve  $y = \ln(x - 4)$ . [1]

This question was generally very well answered, with most candidates able to state the correct equation.

Question 11 (c)

- (c) Find the total area enclosed by the curve  $y = \ln(x - 4)$ , the  $x$ -axis and the lines  $x = 4.5$  and  $x = 7$ . Give your answer in the form  $a \ln 3 + b \ln 2 + c$  where  $a$ ,  $b$  and  $c$  are constants to be found. [4]

Only the most able candidates appreciated that the requested area existed both above and below the  $x$ -axis and were able to devise an appropriate strategy. Other candidates did calculate the two areas separately, but then simply added them together rather changing the relevant signs before combining the two integrals. However the majority of candidates simply used the two limits in the integral; many of these were still able to gain a mark for rewriting  $\ln 0.5$  as  $-\ln 2$ .

Question 12 (a)

12 A curve has equation  $y = a^{3x^2}$ , where  $a$  is a constant greater than 1.

- (a) Show that  $\frac{dy}{dx} = 6xa^{3x^2} \ln a$ . [3]

The derivative of  $a^x$  is a new specification item, and candidates were expected to use this in the chain rule to show the given answer. Some were able to provide a convincing justification using this method, and others either introduced logarithms and used implicit differentiation or rewrote the function in the form  $e^{f(x)}$ . These different approaches tended to be equally successful. Some candidates were unable to devise an appropriate strategy and others seemed to be working back from the given answer in an attempt to justify it.

Exemplar 7

12(a)	$y = a^{3x^2}$
	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$a^x$ $\ln a \cdot a^x$	$= a^u \ln a \times 6xc$
	$= 6xc a^u \ln a$
	$= 6xc a^{3x^2} \ln a$

Handwritten notes on the right side of the table:

- $u = 3x^2$
- $\frac{du}{dx} = 6xc$
- $y = a^u$
- $\frac{dy}{du} = a^u \ln a$

This candidate gains full credit for making clear use of the derivative of  $a^u$  in the chain rule.

Exemplar 8

$y = a^{3x^2}$   
 ~~$y = a^{3x^2}$~~   
 ~~$\frac{dy}{dx} = 6x \ln a a^{3x^2}$~~   
 ~~$y = e^{\ln(a^{3x^2})}$~~   
 ~~$y = e^{3x^2 \ln a}$~~   
 ~~$\frac{dy}{dx} = 6x \ln a e^{3x^2 \ln a}$~~   
 ~~$e^{3x^2 \ln a} = a^{3x^2}$~~   
 $\frac{dy}{dx} = 6x \ln a e^{3x^2 \ln a}$   
 $\frac{dy}{dx} = 6x (a^{3x^2}) \ln a$

This candidate cannot recall the required derivative so instead writes it in the form  $e^{f(x)}$  which they are able to differentiate. Each step is clear in showing this given answer.

Exemplar 9

$y = a^{3x^2}$       $\ln y = \ln a^{3x^2}$   
 ~~$\frac{dy}{dx} = 6x \ln a a^{3x^2}$~~       $\frac{dy}{dx} \left( \frac{1}{y} \right) = 6x \ln a a^{3x^2}$   
 $\ln y = 3x^2 \ln a$       $\frac{d}{dx} \text{ every term: } \frac{dy}{dx} \left( \frac{1}{y} \right) = 6x \ln a$   
 $\frac{dy}{dx} = \frac{6x \ln a \times y}{y}$       $\frac{dy}{dx} y = a^{3x^2}$   
 $\therefore \frac{dy}{dx} = 6x a^{3x^2} \ln a$

Another candidate who cannot recall the required derivative so takes logarithms on both sides and differentiates implicitly. This is a good example of a candidate being able to recall and apply a relevant part of the specification.

## Question 12 (b)

- (b) The tangent at the point  $(1, a^3)$  passes through the point  $(\frac{1}{2}, 0)$ .

Find the value of  $a$ , giving your answer in an exact form.

[4]

A number of candidates were able to make a good attempt at this question, producing fully correct solutions. The most common approach was to find a general equation for the tangent and then substitute in the given points, sometimes to give an equation that could be then solved and other times to set up two simultaneous equations. The less common, but more efficient, method was to use the two given points to find an expression for the gradient that was then equated to the derivative. Some candidates did not read the question carefully and used the wrong point when finding the gradient of the tangent, and others attempted to use both points in the derivative. A common misconception was to use the derivative in terms of  $x$  as the gradient rather than appreciating that a straight line must have a constant gradient.

## Question 12 (c)

- (c) By considering  $\frac{d^2y}{dx^2}$  show that the curve is convex for all values of  $x$ .

[5]

This proved to be a challenging finish to the paper and only the most able candidates were able to make much progress. The majority continued to work with the function in terms of  $a$ , with only a few using the numerical value for  $a$  from part (b); either approach could gain full credit. Candidates appreciated the need to use the product rule, but one part of the product was often seen as  $\ln a$  rather than splitting it into two functions of  $x$ . Some did then attempt a further split but an all too common error was for  $\ln a$  to differentiate to  $a^{-1}$  rather than 0. Candidates were then expected to consider the sign of each component of each term to show that the second derivative was always positive and hence the curve always convex. Many candidates were clearly familiar with the definition of a convex curve and could make some attempt at a relevant method, but statements were not always suitably specific; it was common to see 'x<sup>2</sup> is always positive' and candidates should be precise with their reasoning.

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