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A LEVEL

*Examiners' report*

# MATHEMATICS B (MEI)

**H640**
For first teaching in 2017

## H640/03 Summer 2019 series

Version 1

# Contents

# Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

# Paper 3 series overview

This is the second series of the reformed linear A Level Maths specification, and the first to be sat by candidates following the standard two year programme. This paper assesses Pure Mathematics and includes a comprehension section. It contributes 27.3% of the total A level.

Many of the candidates appeared well prepared for the new style questions although some struggled with longer, less structured questions.

Some questions contain specific defined 'command words', in particular the instruction '**In this question you must show detailed working**'. In these questions, candidates are required to demonstrate their understanding of the relevant concepts by showing their working, rather than by presenting an answer gained simply by pressing a few buttons. Consequently, candidates should be wary about using their graphical calculators for these questions as the reasoning required to get the solution may not be obvious. Remember that this does not preclude candidates from checking their working using the calculator.

The command words 'Show that' and 'Determine' also indicate that clear working must been seen.

Conversely, some questions have slightly lower mark tariffs than seen in the legacy assessment, where candidates are expected to make efficient use of the full range of functions on their calculator. These are also signposted by the 'command words' used in the question: 'Find', 'Calculate', 'Write down'.

| | | |
|---|---|---|
| i | **OCR support** | A poster detailing the different command words and what they mean is available here: https://teach.ocr.org.uk/italladdsup |

The paper is divided into sections A and B, with the questions placed approximately in order of increasing difficulty in each section. Consequently, some candidates would be well advised to attempt the first few (easier) Pure Maths questions from section A and then attempting the section B comprehension, before attempting the more difficult questions in section A. Another feature of the reformed qualification is the increased emphasis on mathematical modelling: candidates should be encouraged to read the full question since there may be final parts that ask for comments about general limitations or improvements to a given model that may not be totally reliant on the calculations attempted in the preceding parts.

# Section A overview

There were many examples of candidates being well prepared for the skills tested in this section.

## Question 1 (a)

**1** The function $f(x)$ is defined for all real $x$ by

$f(x) = 3x - 2$.

**(a)** Find an expression for $f^{-1}(x)$. **[2]**

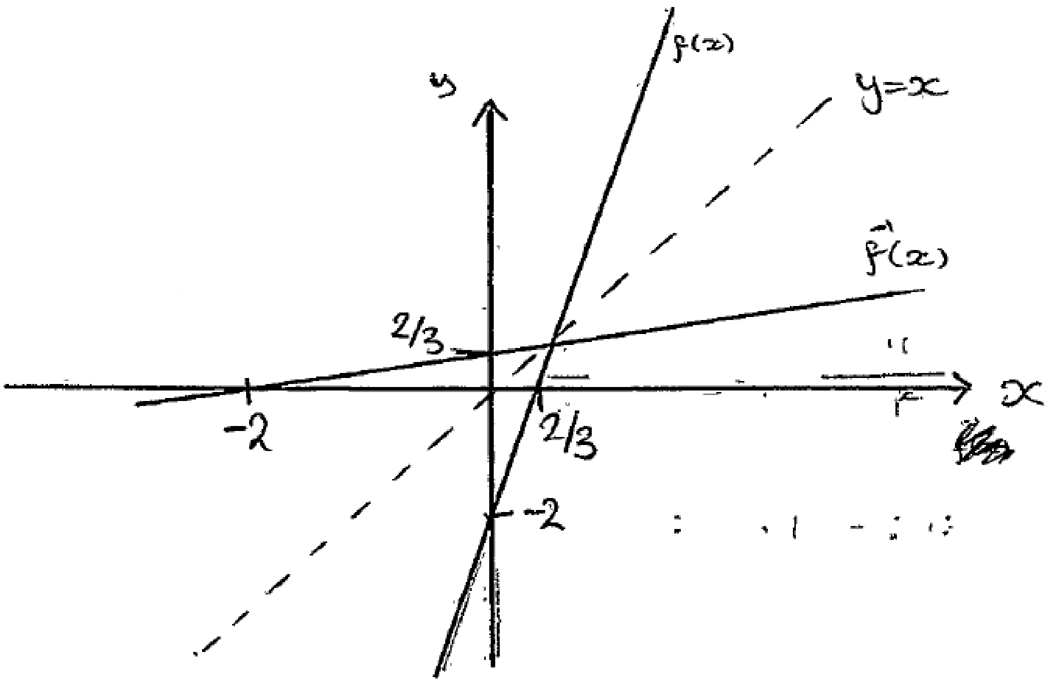
Question 1 provided an accessible start to the paper for most candidates with most scoring all 6 marks. A few candidates did not understand the concept of an inverse function and instead gave the reciprocal of f(*x*) in part (a).

## Question 1 (b)

**(b)** Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same diagram. **[2]**

Most candidates provided good quality sketches and understood the need for the inverse to be a reflection in *y* = *x*; the best solutions added *y* = *x* to their sketches. While a sketch does not need scaled axes, 'important points' should be marked such as the intercepts.

### Exemplar 1




This candidate clearly shows the line of symmetry making it easier for them to draw the graphs and the examiner see their intent. They clearly indicate the y-intercepts and x-intercepts as per the notes in the specification about the level of detail required for a sketch.

## Question 1 (c)

**(c)** Find the set of values of $x$ for which $f(x) > f^{-1}(x)$. **[2]**

A good sketch made it easier for candidates to answer part (c) although most could handle the necessary algebra.

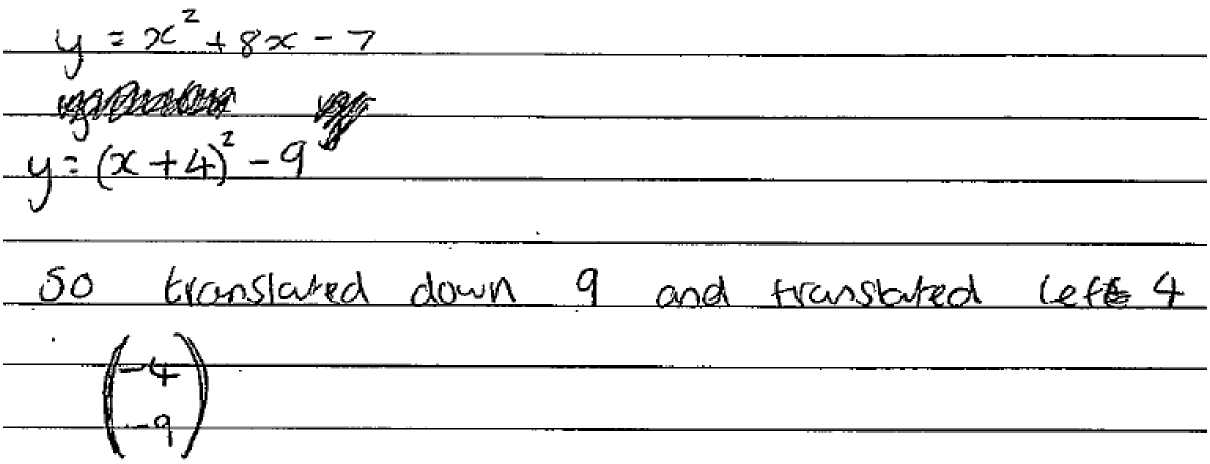
## Question 2 (a)

**2** **(a)** Find the transformation which maps the curve $y = x^2$ to the curve $y = x^2 + 8x - 7$. **[4]**

Most candidates provided some working to justify their answer; most candidates successfully completed the square while a few used differentiation.

In giving their answer a few confused 'transformation' and 'translation' and some gave more than one transformation. Some less able candidates thought the 8*x* term led to a stretch of factor 8 or 1/8 a few even talked about shears or got the *x* – component of the translation as 4 rather than -4. Examiners were disappointed to see pre-GCSE language such as 'shift' and components given as so much 'left' or 'up' rather than using vector notation.

### Exemplar 2




This candidate earns M1 for starting to complete the square with $(x+4)^2$ and they can also earn B1 for stating 'translated' (note that saying translated twice does not count as two transformations.).

## Question 2 (b)

**(b)** Write down the coordinates of the turning point of $y = x^2 + 8x - 7$. **[1]**

Part (b) was generally correct, unsurprisingly as the answer could be got straight from their calculators. Weaker candidates, however, did not see the link between parts (a) and (b) and did not use mismatched answers as a prompt to go back and revisit part (a).

## Question 3 (a)

**3** **(a)** Express $\dfrac{1}{(x+2)(x+3)}$ in partial fractions. **[3]**

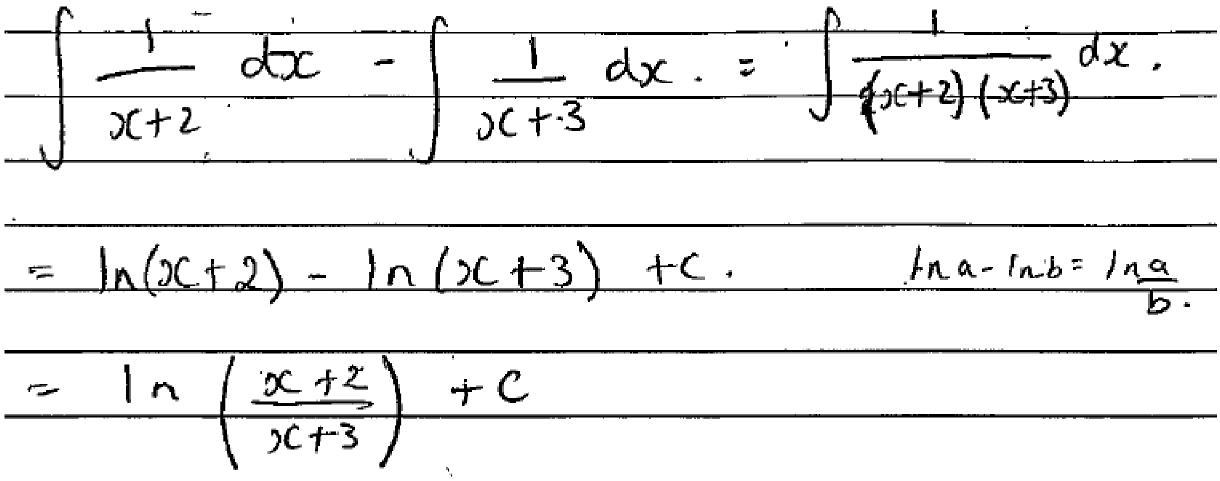
Practically all candidates found the correct partial fractions in (a) with only a handful missing the – sign.

## Question 3 (b)

**(b)** Find $\displaystyle\int \frac{1}{(x+2)(x+3)}\,dx$ in the form $\ln(f(x))+c$, where $c$ is the constant of integration and $f(x)$ is a function to be determined. **[3]**

In (b) most could integrate the reciprocals correctly to get natural logarithms but only the strongest realised that modulus should be used as negatives are not part of the domain for log functions.

### Exemplar 3



$\int \frac{1}{x+2}\,dx - \int \frac{1}{x+3}\,dx = \int \frac{1}{(x+2)(x+3)}\,dx$

$= \ln(x+2) - \ln(x+3) + c$ $\quad \ln a - \ln b = \ln\frac{a}{b}$

$= \ln\left(\frac{x+2}{x+3}\right) + c$

This was probably the most common response we saw to this part. The candidate has correctly integrated each fraction to get ln and also uses a log law correctly. If they had used modulus bars instead of round brackets they would have earned the last mark as well.

## Question 4

**4 In this question you must show detailed reasoning.**

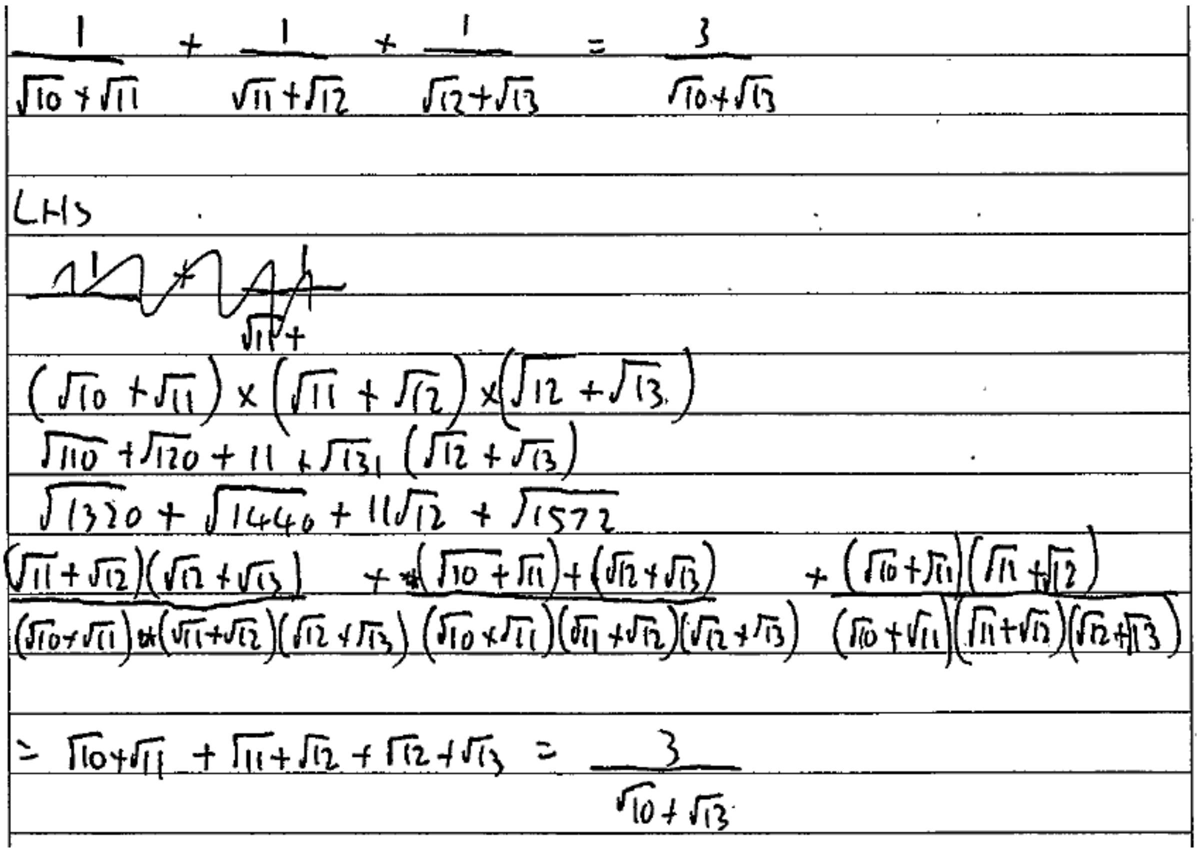
Show that $\frac{1}{\sqrt{10}+\sqrt{11}}+\frac{1}{\sqrt{11}+\sqrt{12}}+\frac{1}{\sqrt{12}+\sqrt{13}}=\frac{3}{\sqrt{10}+\sqrt{13}}$. **[3]**

This question has included both the 'Show that' and the 'detailed reasoning' prompt to reinforce the requirement that some written mathematical logic needed to be seen and that a solution obtained directly from the calculator would not be sufficient.

Many did and recognised the form of the first fraction as something they knew how to rationalise and went on to multiply by $\frac{\sqrt{10}-\sqrt{11}}{\sqrt{10}-\sqrt{11}}$, this then giving them the confidence to do similarly to the next two fractions and simplifying to $\sqrt{13}-\sqrt{10}$. Examiners then saw two correct ways of finishing off either by going to the right hand side and rationalising again to show that both sides equal $\sqrt{13}-\sqrt{10}$, or by 'unrationalising' the denominator and multiplying by $\frac{\sqrt{13}+\sqrt{10}}{\sqrt{13}+\sqrt{10}}$. Weaker candidates tried to add all three fractions together, often leading to pages of arithmetic in supplementary booklets which rarely earned any credit.

Candidates should be wary about spending too much time on any one question, particularly near the beginning of the exam, and the awareness of when to move on could be a useful skill to nurture.

### Exemplar 4




This candidate went down the common denominator route before giving up and trying to fudge the answer. Many less able candidates persevered much longer but usually earning no more marks.

## Question 5

5 A student's attempt to prove by contradiction that there is no largest prime number is shown below.

> If there is a largest prime, list all the primes.
>
> Multiply all the primes and add 1.
>
> The new number is not divisible by any of the primes in the list and so it must be a new prime.

The proof is incorrect and incomplete.
Write a correct version of the proof. **[3]**

Many candidates struggled with constructing a proof by contradiction.

The first step is to assume the opposite of what you are trying to prove, then show how this leads to a contradiction and importantly to complete the proof explain the contradiction and what this then proves.

Many candidates were familiar with proof started in the question and could provide the above structure to complete the proof although weaker ones tried to change it by adding 2 or subtracting 1, etc. A few of the more confident candidates knew the full proof in that the constructed number must either be prime or have a prime factor larger than the previously assumed 'largest prime'. However examiners accepted either or both of these points as adequate contradictions and were more interested in seeing the 3-step structure outlined above.

### Exemplar 5

~~List the~~ Assume there is a finite number of primes.
To find the next largest prime number, multiply all the primes and add 1.

The new number is not divisible by any of the primes in the list.
Therefore it must be a prime number, which is a contradiction,

This means that there is an infinite ~~number~~ amount of prime numbers.

Possibly one of the most concise full mark answers seen. This illustrates that that fancy language and post A Level skills are not necessary.

## Question 6 (a)

**6** A circle has centre C (10, 4). The $x$-axis is a tangent to the circle, as shown in Fig. 6.

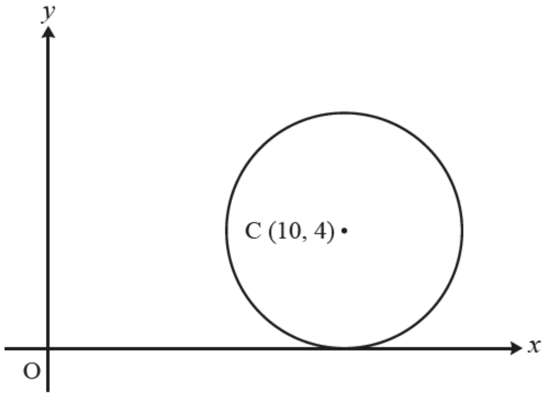



**Fig. 6**

**(a)** Find the equation of the circle. **[2]**

Nearly all candidates earned both marks in part(a) with only a few using other than 4 for the radius (2 and 10 were both seen) and a few candidates who missed the squares or did minus between the brackets or no sign at all.

## Question 6 (b)

**(b)** Show that the line $y = x$ is not a tangent to the circle. **[4]**

The common method for part (b) was to solve their equation from (a) simultaneously with $y = x$ and the method was generally competently shown the difficulty coming with finishing the question off with many leaving their answer as discriminant < 0, or 'no real roots' or giving the complex roots and not completing by stating 'hence it is not a tangent'.

## Question 6 (c)

**(c)** Write down the position vector of the midpoint of OC. **[1]**

In part (c) nearly all could find the midpoint but it was often left as the coordinate (5, 2) rather than given in vector form.

## Question 7 (a)

7 **In this question you must show detailed reasoning.**

(a) Express $\ln 3 \times \ln 9 \times \ln 27$ in terms of $\ln 3$. [2]

With this being a detailed reasoning question, it was expected that candidates show enough steps in their answers to provide detail of why each step follows. In part (a), many went straight to ln3x2ln3x3ln3 without showing the intermediate step. This meant that those who got to $6(\ln 3)^3$ only scored 1 mark. Other fairly common mistakes were in misplacing the brackets in the final answer.

## Question 7 (b)

(b) Hence show that $\ln 3 \times \ln 9 \times \ln 27 > 6$. [2]

Not very many candidates were successful in part (b), with only a small proportion even using the fact that e < 3. Many tried finding numerical answers via their calculators rather than working with e.

## Question 8 (a)

**8 In this question you must show detailed reasoning.**

A is the point (1, 0), B is the point (1, 1) and D is the point where the tangent to the curve $y = x^3$ at B crosses the $x$-axis, as shown in Fig. 8. The tangent meets the $y$-axis at E.

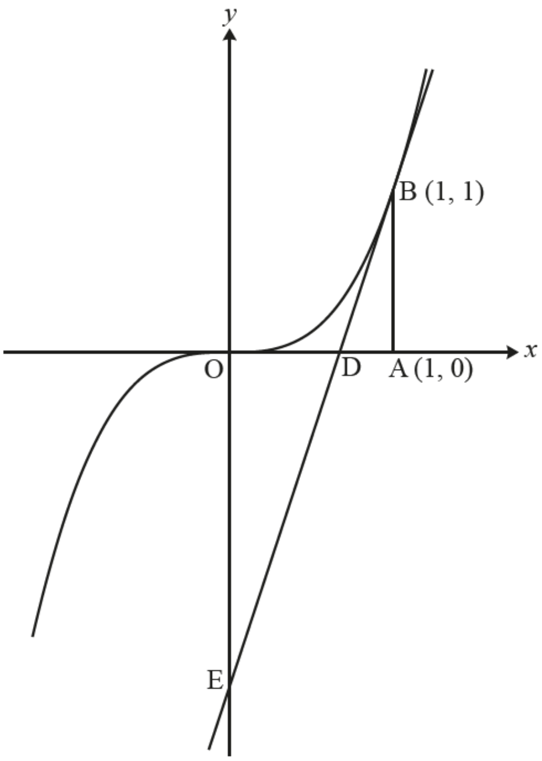



**Fig. 8**

**(a)** Find the area of triangle ODE. **[6]**

This question was very well answered with many students gaining full marks. They clearly understood the aim of the question and the methods needed to solve. The majority of errors seen were simple slips in arithmetic.

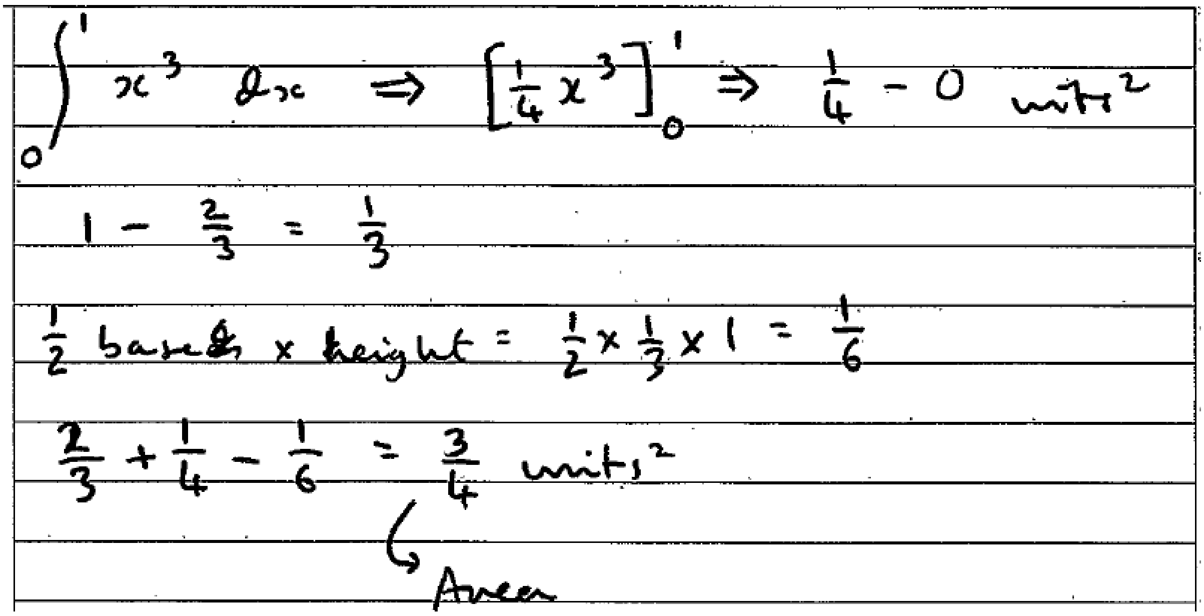
The great majority of candidates realised that they needed to find the equation of the straight line in order to find the lengths of OE and OD. Those who differentiated the equation of the curve correctly were generally successful in the whole question.

## Question 8 (b)

**(b)** Find the area of the region bounded by the curve $y = x^3$, the tangent at B and the $y$-axis. **[4]**

A reasonable number of complete answers, but many candidates did not know what area they were trying to find and ended up with only part marks. Some tried integrating between the curve and the y-axis but only very few were successful doing it this way.

Exemplar 6



$\int_0^1 x^3 \, dx \Rightarrow \left[\frac{1}{4}x^3\right]_0^1 \Rightarrow \frac{1}{4} - 0$ units$^2$

$1 - \frac{2}{3} = \frac{1}{3}$

$\frac{1}{2}$ base x height $= \frac{1}{2} \times \frac{1}{3} \times 1 = \frac{1}{6}$

$\frac{2}{3} + \frac{1}{4} - \frac{1}{6} = \frac{3}{4}$ units$^2$

Area

At first glance, with the correct answer, ¾, it would appear that this would score full marks. However examiners need to make sure that the answer does not come from wrong working. Notice that their initial integration is wrong so they do not get the first M1. They do get the M1 for the triangle area of $\frac{1}{6}$ and also the M1 for knowing how to combine their areas. Since their initial integration was not accurate they do not score the final accuracy mark (i.e. overall they get M0M1M1A0.)

## Question 9

**9 In this question you must show detailed reasoning.**

The curve $xy + y^2 = 8$ is shown in Fig. 9.

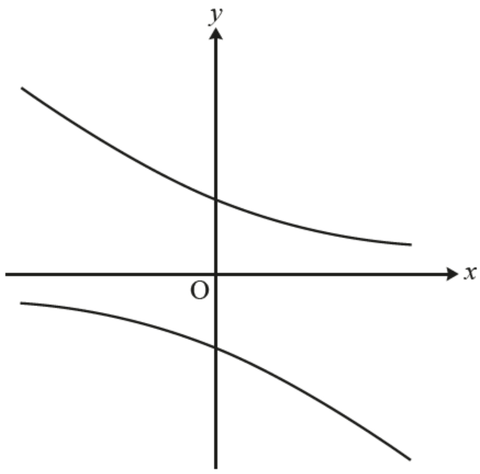



**Fig. 9**

Find the coordinates of the points on the curve at which the normal has gradient 2. **[6]**

There seemed to be a good understanding of the need for implicit differentiation, and many fully correct solutions were seen. The marking of this question allowed for those with the correct concepts, but poor algebraic manipulative skills to achieve a reasonable number of marks. A few candidate tried to rearrange to get a function of the form y = g(x), but this then lead to greater difficulty in completing it correctly. Many got through the whole question but did not get the last mark for only giving one solution to $y^2 = 8$.

## Question 10

**10** Show that $\mathrm{f}(x) = \dfrac{\mathrm{e}^x}{1+\mathrm{e}^x}$ is an increasing function for all values of $x$. **[4]**

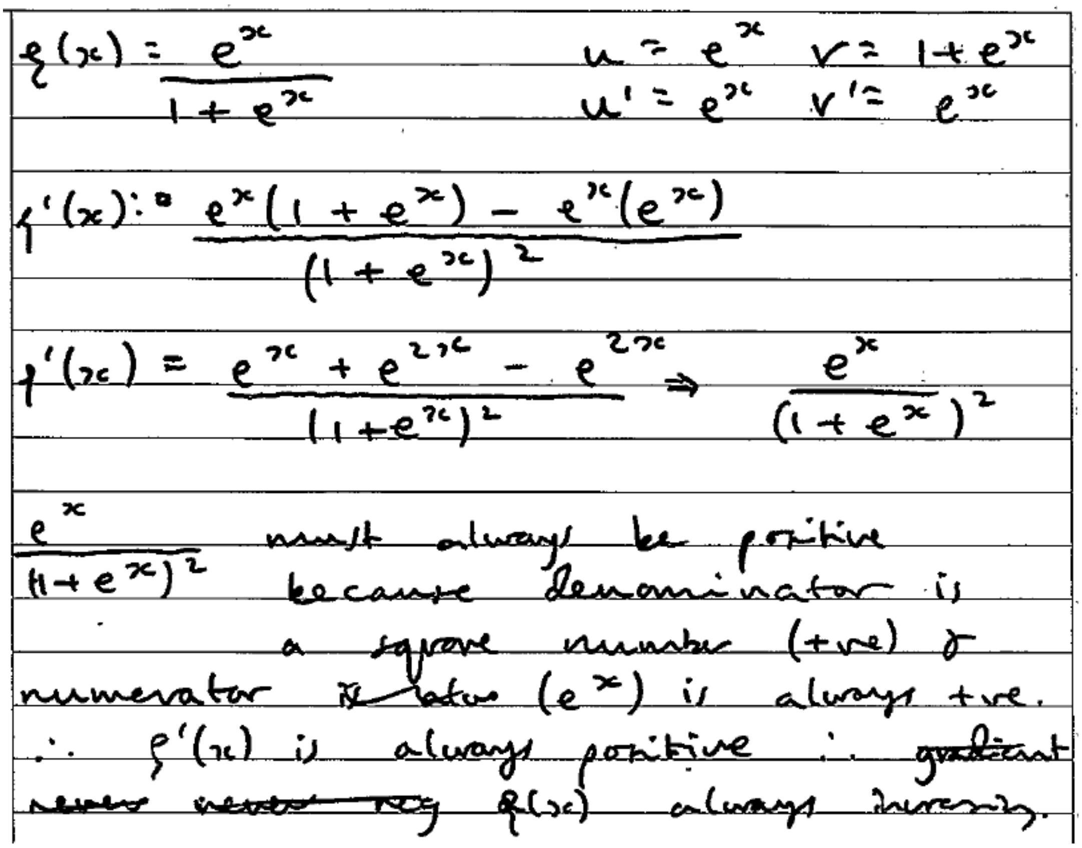
Most candidates realised that they needed to find d*y*/d*x* . Those who used the quotient rule were generally more successful than those who re-wrote the function as $(\mathrm{e}^x)(1+\mathrm{e}^x)^{-1}$ and used the product rule. In the latter case candidates often missed the $\mathrm{e}^x$ which was the 'differential' of the bracket and gave $\mathrm{d}y/\mathrm{d}x = (\mathrm{e}^x)(-1)(1+\mathrm{e}^x)^{-2} + (\mathrm{e}^x)(1+\mathrm{e}^x)^{-1}$.

Those using the quotient rule occasionally forgot to square the denominator or wrote $(\mathrm{e}^x)^2$ as the denominator.

Simplifying the numerator was usually successful but some candidates wrote $\mathrm{e}^{x\,2}$ when calculating $(\mathrm{e}^x)(\mathrm{e}^x)$.

By far the most common fault was in completion where candidates merely wrote $\mathrm{e}^x > 0$ as justification for d*y*/d*x* being positive and made no mention of the denominator.

### Exemplar 7



$f(x) = \dfrac{e^x}{1+e^x}$ $u = e^x$ $v = 1+e^x$ $u' = e^x$ $v' = e^x$

$f'(x) = \dfrac{e^x(1+e^x) - e^x(e^x)}{(1+e^x)^2}$

$f'(x) = \dfrac{e^x + e^{2x} - e^{2x}}{(1+e^x)^2} \Rightarrow \dfrac{e^x}{(1+e^x)^2}$

$\dfrac{e^x}{(1+e^x)^2}$ must always be positive because denominator is a square number (+ve) & numerator ~~is~~ value $(e^x)$ is always +ve.

$\therefore f'(x)$ is always positive $\therefore$ ~~gradient never never neg~~ $f(x)$ always increasing.

This is a good example of a candidate who clearly explains how they know that both the numerator and denominator of their final fraction for their derivative are positive.

## Question 11

**11** By using the substitution $u = 1 + \sqrt{x}$, find $\int \frac{x}{1+\sqrt{x}}\,dx$. **[6]**

The more able candidates often produced fully correct work but some lost the factor of 2 after integrating. The expansion of $(u-1)^3$ was often inaccurately done.

Some candidates wasted time expanding brackets and attempting to factorise at the end – no particular form was requested so candidates could leave their answer unsimplified.

Some candidates attempted to use integration by parts rather than simplifying the integral by expanding brackets and dividing by *u*.

# Section B overview

Quite a number of candidates left all parts of the comprehension blank without attempting any of the question and it is worth highlighting that it is 20% of the marks for this paper. Many of the questions were accessible within the time available to them.

## Question 12

**12** Show that the equation of the line in Fig. C2 is $ry + hx = hr$, as given in line 24. **[2]**

Many correctly found the gradient was -h/r and successfully found the equation. Some tried to work backwards from the given answer which did not score. Those who used the gradient as h/r tried to fudge the required answer but scored zero.

## Question 13 (a) (i)

**13** **(a)** **(i)** Show that the cross-sectional area in Fig. C3.2 is $\pi x(2r - x)$. **[2]**

Part (a)(i) saw many correct answers, although some messy presentation made it difficult to follow at times. With the expression given there was the tendency to see some set up an incorrect equation and then try to fudge the answer.

## Question 13 (a) (ii)

**(ii)** Hence show that the cross-sectional area is $\frac{\pi r^2}{h^2}(h^2 - y^2)$, as given in line 37. **[2]**

Part (a)(ii) saw a lot of correct attempts, but few seem to see the simplest route through the algebra. Some tend to make their expressions more complicated than is necessary, with there then being more room for errors, as with those who are messy in their presentation.

## Question 13 (b) (i)

**(b)** Verify that the formula $\frac{\pi r^2}{h^2}(h^2 - y^2)$ for the cross-sectional area is also valid for

**(i)** Fig. C3.1, **[1]**

For those who attempted part (b)(i), the most common error seemed to be substituting $y = 0$ and $h = 1$.

## Question 13 (b) (ii)

**(ii)** Fig. C3.3. **[1]**

Part (b)(ii) was similar to above, with candidates not realising that all that was required was to use $y = h$.

## Question 14 (a)

**14 (a)** Express $\lim_{\delta y \to 0} \sum_{0}^{h} (h^2 - y^2)\delta y$ as an integral. **[1]**

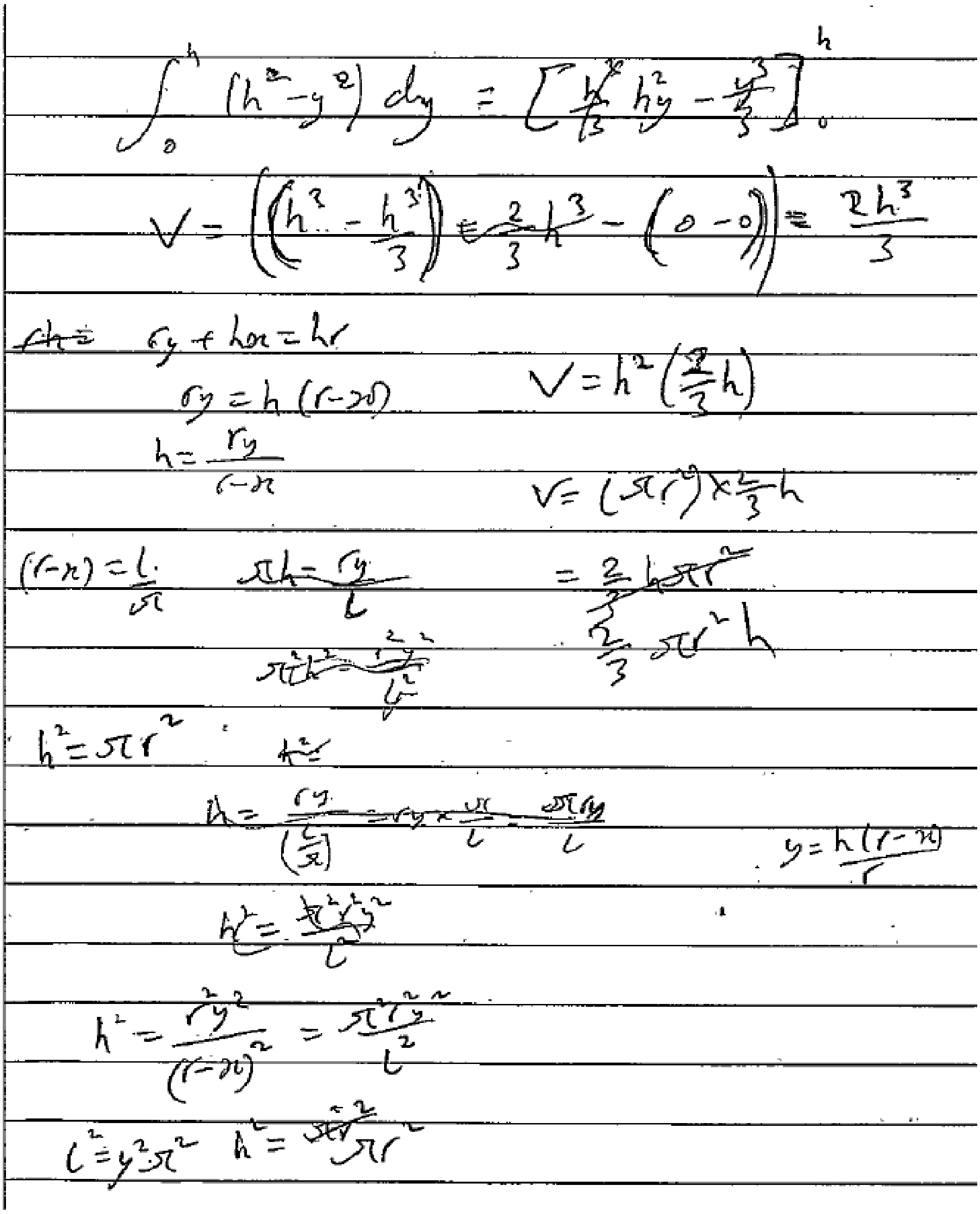
Only few candidates answered part (a) correctly, recognising that the given summation was the 'definition' of integration. Some did not change δy to dy, some put both and others omitted the limits.

## Question 14 (b)

**(b)** Hence show that $V = \frac{2}{3}\pi r^2 h$, as given in line 41. **[3]**

Only the better candidates made it through part (b). Errors included integrating $h^2$ as $h^3/3$, forgetting to include the multiplying term or just getting into a mess with the algebra.

## Exemplar 8


$$\int_0^h (h^2 - y^2)\,dy = \left[h^2 y - \frac{y^3}{3}\right]_0^h$$

$$V = \left(\left(h^3 - \frac{h^3}{3}\right) = \frac{2}{3}h^3 - (0 - 0)\right) = \frac{2h^3}{3}$$

$ry + hx = hr$

$ry = h(r - x)$ $\qquad V = h^2\left(\frac{2}{3}h\right)$

$h = \frac{ry}{r - x}$ $\qquad V = (\pi r^2) \times \frac{2}{3}h$

$(r - x) = \frac{l}{\pi}$ $\qquad \pi h = \frac{ry}{l}$ $\qquad = \frac{2}{3}\pi r^2 h$

$h^2 = \pi r^2$

$h = \frac{ry}{\left(\frac{l}{\pi}\right)} = ry \times \frac{\pi}{l} = \frac{\pi r y}{l}$ $\qquad y = \frac{h(r - x)}{r}$

$h^2 = \frac{r^2 y^2}{(r - x)^2} = \frac{\pi^2 r^2 y^2}{l^2}$

$l^2 = y^2 \pi^2$ $\qquad h^2 = \pi r^2$

This candidate made a good start on this part with correct integration earning M1. Notice that they avoided the potential pitfall of integrating $h^2$ to get $\frac{h^3}{3}$.

However, they did not multiply their answer by $\frac{\pi r^2}{h^2}$ missing out on the second method mark.

## Question 15

**15** A typical tube of toothpaste measures 5.4 cm across the straight edge at the top and is 12 cm high. It contains 75 ml of toothpaste so it needs to have an internal volume of 75 cm$^3$.

Comment on the accuracy of the formula $V = \frac{2}{3}\pi r^2 h$, as given in line 41, for the volume in this case. **[3]**

Very often done well, with a simple minimal comment about being accurate sufficing for full marks. The most common approach was to find the volume when $r = \frac{5.4}{\pi}$, although some did find the value of *r* from volume=75 and proceeded to correctly compare with the decimal version of $\frac{5.4}{\pi}$. The most common error seen was the use of $r = \frac{5.4}{2}$.

### Exemplar 9

$V = \frac{2}{3} \times \pi \times \left(\frac{5.4\times10^{-2}}{\pi}\right)^2 \times 12\times10^{-2}$

$= 7.42\times10^{-5}\ m^3$

$= 74.3\ cm^3$

the model predicted 74.3 cm$^3$ when we needed 75cm$^3$ and therefore was less than 1%. out. That is a significance level that allows me to say the model is sufficiently accurate. However, because the model underpredicts the accuracy needs to be changed because mathematically its good however to work in the real world would need to overestimate

I included this example as the candidate had only scored 1 mark so far in the comprehension and given 'No response' to half of the 8 previous questions. But notice that, with presumably a very sketchy understanding of the whole situation, the candidate could still score full marks on this part – the final question on the paper. The comprehension is definitely an area where resilience is to be encouraged however difficult previous questions have been.

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