GCE

Mathematics

Advanced GCE **A2 7890 - 2**

Advanced Subsidiary GCE **AS 3890 - 2**

Report on the Units

June 2008

3890-2/7890-2/MS/R/08

Oxford Cambridge and RSA Examinations

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Reports should be read in conjunction with the published question papers and mark schemes for the Examination.

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GCE Mathematics and Further Mathematics Certification

From the January 2008 Examination session, there are important changes to the certification rules for GCE Mathematics and Further Mathematics.

1 In previous sessions, GCE Mathematics and Further Mathematics have been aggregated using 'least-best' ie the candidate was awarded the highest possible grade in their GCE Mathematics using the lowest possible number of uniform marks. The intention of this was to allow the greatest number of uniform marks to be available to grade Further Mathematics.

From January 2008 QCA have decided that this will no longer be the case. Candidates certificating for AS and/or GCE Mathematics will be awarded the highest grade with the highest uniform mark. For candidates entering for Further Mathematics, both Mathematics and Further Mathematics will be initially graded using 'least-best' to obtain the best pair of grades available. Allowable combinations of units will then be considered, in order to give the candidate the highest uniform mark possible for the GCE Mathematics that allows this pre-determined pair of grades. See page 2 for an example.

As before, the maximisation process will award a grade combination of AU above, say, BE. Where a candidate's grade combination includes a U grade a request from centres to change to an aggregation will be granted. No other requests to change grading combinations will be accepted. eg A candidate who has been awarded a grade combination of AD cannot request a grading change that would result in BC.

2 In common with other subjects, candidates are no longer permitted to decline AS and GCE grades. Once a grade has been issued for a certification title, the units used in that certification are locked into that qualification. **Candidates wishing to improve their grades by retaking units, or who have aggregated GCE Mathematics or AS Further Mathematics in a previous session should re-enter the certification codes in order to ensure that all units are unlocked and so available for use.** For example, a candidate who has certificated AS Mathematics and AS Further Mathematics at the end of Year 12, and who is certificating for GCE Mathematics at the end of Year 13, should put in certification entries for AS Mathematics and AS Further Mathematics in addition to the GCE Mathematics.

Grading Example

A candidate is entered for Mathematics and Further Mathematics with the following units and uniform marks.

Unit	Uniform marks	Unit	Uniform marks
C1	90	M1	80
C2	90	M2	100
C3	90	M3	90
C4	80	S1	70
FP1	100	S2	70
FP2	80	D1	60

Grading this candidate using least-best gives the following unit combinations:

Mathematics		Further Mathematics	
Unit	Uniform marks	Unit	Uniform marks
C1	90	FP1	100
C2	90	FP2	80
C3	90	M1	80
C4	80	M2	100
S1	70	M3	90
D1	60	S2	70
Total	480 (Grade A)	Total	520 (Grade A)

Under the new system, having fixed the best pair of grades as two As, the mark for the Mathematics would be increased by combining the units in a more advantageous manner. The table below shows the allowable combination of units.

Option	Applied units used for Maths	Total uniform marks for Mathematics	Applied units used for Mathematics	Total uniform marks for Further Mathematics
1	M1, S1	500	M2, M3, S2, D1	500
2	M1, D1	490	M2, M3, S1, S2	510
3	S1, D1	480	M1, M2, M3, S2	520
4	M1, M2	530	M3, S1, S2, D1	470
5	S1, S2	490	M1, M2, M3, D1	510

Option 4 gives the highest uniform mark for Mathematics. However, this would only give a grade B in the Further Mathematics, and so is discarded. Option 1 is the next highest uniform mark for Mathematics and gives an A in Further Mathematics, and so this is the combination of units that would be used.

Chief Examiner's Report – Pure Mathematics

The seven Core and Further Pure Mathematics units at this session were generally at appropriate standards and enabled candidates to produce scripts reflecting their mathematical ability. As usual, there were many excellent scripts, some of which had work presented in an organised and thoughtful manner as well as containing faultless solutions to each question. At the same time, however, examiners were sometimes astonished to note the basic errors made, not always just from candidates of more modest ability. It seemed that the level of concentration needed to grapple with mathematical principles was such that, when faced with routine steps in solutions, a more relaxed approach was temporarily adopted – leading inevitably to careless arithmetical and algebraic errors.

Several examiners commented that many candidates were offering two or more solutions to particular questions, without giving any indication as to which attempt should be assessed. Centres and candidates are reminded of the instruction given to examiners in such cases:

> "If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (full) attempt, and ignore the others."

It is the responsibility of the candidate to make clear which attempt is to be assessed. There were instances of candidates receiving fewer marks than they might otherwise have done because the final – and undeleted – attempt at a particular question was not as good as an earlier attempt.

A similar point about presentation is made in the General Comments for unit 4727. The advice there about keeping solutions to different parts of a question together is appropriate to all units.

4721 Core Mathematics 1

General Comments

This paper proved to be highly accessible to candidates, with the vast majority working through the paper in order and attempting every question. Candidates appeared to have an adequate amount of time to complete the paper.

There were many candidates who displayed an excellent understanding of the techniques required and who were able to produce clear and largely correct solutions to most questions. A number of candidates scored full marks and relatively few scored fewer than 20 marks. It was pleasing to note once again that centres are encouraging candidates to sketch graphs in the answer booklet rather than on graph paper.

However, one very disappointing aspect of the work seen was the number of candidates who failed to work out the most basic calculations correctly. In particular, the evaluation of $36-88$ in Q10(iii) was extremely poorly done but errors were also frequently seen in calculating $4\times 2\times 11$, $14-\left(-\frac{21}{2}\right)$ and $25-20-95$ in other parts of the paper. Candidates should be encouraged both to practise their mental arithmetic skills and to check their answers to seemingly trivial calculations.

Comments on Individual Questions

1) (i) Most candidates gave the correct negative power here although there were a few candidates who thought that the required power was $\frac{1}{4}$.

(ii) This part was answered correctly by almost every candidate.

(iii) Less able candidates found this final part more challenging, not appreciating the link between parts (ii) and (iii). There were a few cases of muddled powers and roots, sometimes leading to a power of $\frac{2}{3}$ rather than $\frac{3}{2}$. However, most candidates were able to gain full marks here.

2) (i) As in previous sessions, candidates' responses reflected their difficulties with this part of the specification. For some candidates, this was the only question where marks were dropped. Conversely, some centres had clearly taught this topic effectively, as almost all candidates scored well, regardless of their total score. In part (i) the equation $y=x^2+2$ was seen more commonly than the correct answer, with $y=x^2-2$ also very frequently given. Candidates who wrote $y=x-2^2$ without the necessary brackets also lost marks here.

(ii) In part (ii) the incorrect answer $y=-x^3-4$ was seen as frequently as the correct answer. Some candidates wrote $-y=x^3-4$, which was not the expected form but which was acceptable.

3) (i) All three parts of this question were extremely well done, with only the very weakest candidates failing to gain at least 3 marks out of 4. Part (i) was almost unanimously correct, with only a few slips like $2\sqrt{10}$ seen.

(ii) A number of candidates gave the answer as $12\left(\sqrt{2}\right)^{-1}$ but most understood that they needed to rationalise the denominator and completed this correctly.

(iii) Although the majority of candidates realised that they needed to convert $\sqrt{8}$ into a multiple of $\sqrt{2}$, which gained them the method mark, a significant number changed $5\sqrt{8}$ into $7\sqrt{2}$ or $20\sqrt{2}$ and lost the final mark.

4) Many candidates dealt with this question well, recognising that a substitution was needed to transform the given equation into a quadratic equation, which they then solved correctly in the vast majority of cases. However, there was some confusion about whether their interim values should then be squared or the square roots taken. Some candidates, often those who wrote $x = x^{\frac{1}{2}}$ as their substitution, failed to do any further working at all to obtain the correct values for x. It should be emphasised to candidates that it is safer to change the letter chosen for the variable in this type of question.

An alternative approach, which was sometimes successful, involved rearranging the original equation to $2x+3=7x^{\frac{1}{2}}$ and then squaring each side. This method was perfectly sound as long as $(2x+3)^2$ was correctly squared, which was unfortunately often not the case.

In contrast to the many good attempts seen, weaker candidates often started by squaring each term individually, leading to an incorrect quadratic, an approach from which there was no recovery. A significant minority of candidates scored zero on this question, with some failing to make any attempt at it.

5) While the majority of candidates realised the need to differentiate, there was a large number who simply substituted $x = 9$ into the given equation, obtaining $y = 33$. Some continued, working out the gradient of the straight line joining (9, 33) to (0, 0). Those who differentiated dealt with the square root well and usually obtained the correct expression for $\frac{dy}{dx}$, although some candidates obtained $4x^{-\frac{1}{2}}$ but omitted the +1 or, less frequently, wrote $+x$. There were a few cases of $8\sqrt{x}$ becoming $x^{\frac{1}{8}}$ but, on the whole, candidates differentiated the expression correctly. A small number thought that $9^{-\frac{1}{2}} = -3$ or $\frac{1}{81}$ but most evaluated the gradient correctly and gained full marks for this question.

6) (i) This question proved to be straightforward for most candidates. The vast majority used a sensible method for multiplying out the given expression and then simplified accurately, earning full marks. However, other incorrect ways of combining terms were seen, often leading to 12 or more terms, none of which was x^3.

(ii) Candidates also executed the graph sketching well and there were very many solutions which gained full marks and only a handful of plotted graphs. The shape of the curve was well known although it was surprising how many candidates gave all 3 x-intercepts but omitted the y-intercept. A small number of candidates plotted the roots as –5, 2 and 5 and a similarly small minority sketched the curve for a negative cubic.

7) (i) There were many perfect solutions to this linear inequality, although there was also a sizeable number of candidates who seemed never to have met such a question before. Many tried to combine all 3 constants resulting in $3x < 21$, while others checked integer values only and gave $x = 4$ as the solution. Unexpectedly, the calculation 11 + 2 = 12 was occasionally seen in working.

(ii) As in previous papers, it appeared that only the strongest candidates knew the method for solving a quadratic inequality. The best candidates factorised to find the critical values, often drew a small graph or number line and then defined the appropriate regions correctly. But a large proportion of the candidates scored only 1 mark (for a correct factorisation) or no marks at all. Many divided through by y and gave the answer $y \geq -2$. Others rearranged to $y^2 \geq -2y$ and then wrote down the ‘square root’ of each side.

8) (i) Many candidates of all abilities scored high marks on this question and differentiation was again seen to be a strength of the candidates sitting this paper. Part (i) was very well done apart from a small number of candidates who integrated the expression. For those who differentiated incorrectly, the main problems stemmed from the last 2 terms, some candidates omitting the +1, others including –3 in their differentiated expression.

(ii) This part was also done extremely well with only a few candidates solving $y = 0$ or, even more occasionally, $\frac{d^2y}{dx^2} = 0$ rather than $\frac{dy}{dx} = 0$. There was a follow-through mark available in this part, so that candidates who had made a slip in part (i) could still earn all 3 marks here.

(iii) In this part, the majority of candidates looked at the sign of the second derivative but some used alternative methods of classifying the stationary point such as finding the gradient for points either side of $x = 1$. These methods were almost always unsuccessful as most candidates failed to carry out the necessary step of establishing the x-coordinate of the other stationary point first.

(iv) This final part proved straightforward for most and even those candidates who had made earlier errors in the expression for $\frac{dy}{dx}$ and/or the value for *k* were usually able to score 2 marks out of 3.

9) (i) This question on circles proved the most challenging on the paper and some candidates barely attempted it. It proved a good discriminator as only the most able candidates were able to score high marks. Many candidates could not recall the formula for the equation of a circle. Those who had some idea often got signs wrong or used 10 instead of 100. A large proportion of candidates who started with a correct equation in the form $(x-2)^2+(y-1)^2=100$ then made errors manipulating it into expanded form. There were also many muddled versions of the equation such as $(x-2)^2-4+(y-1)^2-1=100$. Unfortunately, despite obvious confusion in part (i), candidates persisted in using their incorrect equation in subsequent parts of the question, rather than considering alternative approaches, thus losing more marks.

(ii) In part (ii), candidates who used the fact that the distance between the point (5, *k*) and the centre of the circle had to be 10 were able to score full marks, regardless of their answer to part (i). It was interesting to note that even when candidates had correctly obtained the equation $(k-1)^2=91$, they were much more likely to expand the brackets and to use the quadratic formula than to complete the square to solve it. Although many candidates got as far as $\frac{2+\sqrt{364}}{2}$, simplifying this expression proved too challenging for almost all, $1+\sqrt{364}$ and $2+\sqrt{91}$ being common incorrect answers.

(iii) Part (iii) was very interesting to mark because of the varied approaches seen, many of them valid. The method which proved most efficient was to work out the distance of the point from the centre of the circle. Candidates who approached the problem in this way usually calculated $\sqrt{89}$ correctly and were then easily able to compare this with 10 and establish that the point was inside the circle.

An alternative valid method seen occasionally was to substitute either $x = -3$ or $y = 9$ into their circle equation and solve the resulting quadratic equation, finding the 2 possible values for the other variable. It was then possible to state that the point (–3, 9) was between these two points on the circumference. In contrast, very many candidates substituted both $x = -3$ and $y = 9$ into their (often incorrect) equation from part (i) and even those who obtained the right value were then confused as to what to compare this with and so were unable to decide whether the point was inside or outside the circle.

In a very small number of scripts, candidates attempted solutions by drawing alone and some, but not all, marks were available provided that a high level of accuracy was seen.

(iv) Candidates who remembered that the way to solve this type of question is to find the gradient of the radius to the given point and then consider the perpendicular to it produced the best solutions to part (iv). However, a surprisingly large number of candidates decided to differentiate their equation for the circle, nearly always failing to do this implicitly and almost invariably obtaining an incorrect value.

10) (i) Many candidates scored very well on Q10 as a whole but part (i) caused the greatest loss of marks, even amongst the most able candidates. Although nearly all candidates were able to state that $p = 2$, and a majority that $q = -\frac{3}{2}$, it was much rarer to see a correct value for r. The most common error was to evaluate $11 - q^2$ rather than $11 - 2q^2$ but there was a multitude of alternative wrong answers due to errors in squaring the fraction, doubling the resulting fraction, or adding to rather than subtracting from 11. There was also a significant minority of candidates who wrote $2(x - 3)^2$, although these candidates could still have gained a method mark for evaluating r if they had followed through correctly. It could be pointed out to candidates that, if time allows, an answer in completed square form can be always be checked by multiplying out.

(ii) In this part, both marks were available for candidates who gave coordinates consistent with their expression in part (i). However, some candidates preferred to differentiate to find the minimum point, which gave a generally successful outcome but obviously took slightly longer. Of completely wrong answers, (0, 11) was the most frequent. A few candidates left this question out completely, presumably because they did not recognise the term ‘vertex’.

(iii) Almost all candidates knew the correct formula for the discriminant, although the inability of candidates of all abilities to work out $36 - 4 \times 2 \times 11$ accurately was disappointing. The most common wrong answer was –68 although –54, 58, 36 – 66 and plenty of other incorrect working was also seen.

(iv) The majority of candidates stated correctly that the equation had no real roots, although a few thought that the negative discriminant meant a repeated root. The statement ‘one root because $2x^2 - 6x - 11 = 0$’ was also seen.

(v) This final part was done very well by most, with candidates of all abilities scoring all 5 marks. Most candidates knew how to proceed and obtained the correct quadratic equation. The factorisation was generally correct but there were slips in stating the roots, often with signs but $x = -\frac{2}{3}$ was also quite often seen. Of those with correct x-values, carelessness when calculating the corresponding y-values sometimes led to the loss of the final mark.

4722 Core Mathematics 2

General Comments

This paper was accessible to the majority of candidates, and differentiated well between the weaker candidates and the more able. Some candidates struggled to complete the paper, but they had often used time-consuming methods earlier in the paper. This was particularly noticeable on Q1, where a number of candidates wasted time by attempting more terms than those requested, or even attempted to multiply out brackets, and in Q4, where many attempts involved long division. In topics where a variety of methods can be used, candidates need to appreciate which is the more appropriate for a given situation.

Scripts were generally well presented, with clear and explicit methods shown, though a number of candidates showed very little detail of the attempts made. It is particularly important that adequate detail is shown when attempting to demonstrate a given answer.

Candidates continue to perform well on questions involving integration and radian measures, but logarithms are still an area of weakness. Whilst candidates seem familiar with the relevant laws, they struggle to apply them in the correct sequence. It was also disappointing to see candidates losing marks through a lack of mastery of basic skills, such as algebraic manipulation, use of indices and solving both linear and quadratic equations. There was also evidence of candidates being unsure of the order of operations – on a number of scripts a correct equation was stated but this was then incorrectly evaluated.

As the use of a calculator is permitted in Core Mathematics 2, it is important that candidates appreciate how to do so effectively. They need to consider whether the calculator should be in degree or radian mode, and be aware of how the calculator will evaluate given expressions, especially with powers of negative numbers, and fractional indices. Once an answer has been obtained, candidates should ensure that they state it to the required degree of accuracy, be this 3 significant figures or an exact answer. When continuing with further calculations, it is important that the value carried through is sufficiently accurate to justify the final answer.

Comments on Individual Questions

1) Most candidates seemed familiar with the binomial expansion, and could make a reasonable attempt at this, though some wasted time by attempting more terms than just the three requested. The $-3x$ term caused problems for a number of candidates, with some just ignoring the sign and others failing to square the entire term. The most successful candidates made effective use of brackets throughout. When using the binomial expansion, a few made errors such as using an incorrect binomial coefficient or using the sum rather than the product of the three components of a term. Others made the question more difficult by attempting to take out a common factor and use the C4 expansion of $(1 + x)^n$. Some candidates attempted to expand all 6 brackets; this was rarely complete and even more rarely correct. The vast majority of candidates gained both method marks on this question, but fully correct solutions were in a minority.

2) (i) This was a straightforward question for many candidates, though some lost marks through calculation errors or not giving exact answers. Some candidates, having found the first term correctly, failed to use the reciprocal in subsequent terms. However, many candidates seemed unfamiliar with recursive sequences and instead treated the question as involving $u_n = 1 - {}^1/_n$ or something similar.

(ii) Candidates are expected to be able to describe the behaviour of sequences, and all that was required in this part was a reference to the fact that it is a repeating sequence, or some equivalent statement. A significant number of candidates thought that it must be either arithmetic or geometric as these are the only two studied in detail. Of those who identified that it was repeating, some then gave additional, incorrect, information such as a period of 4.

3 (i) This question was done very well. Most candidates could recall and use the formula for the area of a sector, though a few omitted the ½. Having obtained an answer of 1.5, some candidates then spoiled this by stating the angle to be 1.5π.

(ii) Again, this was generally well done with most candidates successfully using $\frac{1}{2}r^2\sin\theta$, and subtracting their answer from 48. However, some then failed to give their answer to 3 significant figures and lost the final mark by giving an area of 16. A few candidates were reluctant to work in radians and converted the angle to degrees, usually successfully, before proceeding. Some candidates attempted much more long-winded methods to find the area of the triangle, but these were rarely accurate. A significant minority wasted time by doing extra calculations such as finding the arc length before attempting the required area.

4 (i) Most candidates seemed familiar with the factor theorem and could attempt f(3) and then equate it to zero, though this was not always shown explicitly. A common error was to evaluate 3^3 as 9. Other candidates chose to use much less efficient methods such as division or coefficient matching. Given the algebra involved, these were rarely successful and were very time-consuming.

(ii) Whilst the majority of candidates attempted f(–2) as expected, a surprising number attempted long division even if they had used the factor theorem in part (i). Whilst it is pleasing to see candidates becoming increasingly proficient in using algebraic long division, they should appreciate that this is not the most efficient method when only the remainder is required. Of those who used the remainder theorem, a number of sign errors were seen, particularly with $(-2)^2$ becoming –4. If candidates had made an error in part (i), both marks were available in this part if they correctly used their value of a.

5 (i) Most candidates gained one mark for correctly demonstrating the change of limits, but then failed to get any further credit. Many appreciated the need to change the subject of the equation, but were unable to do so due to poor algebraic skills. A common error was $y^2 = 9 + x + 2$. The final mark was for identifying in some way that the required area is given by $\int x\,dy$, but only the most able candidates gained this mark.

(ii) A few candidates chose to ignore the hint given in part (i) and instead attempted to integrate with respect to x, but the majority could make a good attempt at the given integration and gained all of the 4 marks available. Quite a common error was for the final term to be given as $7x$ not $7y$, and a few candidates lost the 7 when integrating. The majority of candidates could then attempt to use limits correctly, though there were some numerical errors when evaluating the expression. Some candidates lost the final mark by giving a decimal approximation rather than the exact area as requested in the question.

6 (i) The majority of candidates could successfully explain why the given angle was 100°, with the clearest solutions including labelled diagram. Some candidates simply wrote several calculations with no reasons given for the values being used.

(ii) Candidates were generally competent in using the cosine rule, and could state a correct equation, though evaluating this caused a number of difficulties. It was quite common for the calculator to be in radian mode. Other errors included failing to deal with the second term being negative, and treating the entire expression as a coefficient of $\cos x$. Failing to give the answer to the required degree of accuracy cost a number of candidates the final mark. Some of the weaker candidates assumed that it was a right-angled triangle and used Pythagoras' Theorem.

(iii) Most candidates could attempt one of the two unknown angles in the triangle, with angle ABC being the most popular. The sine rule was the most common method, though some elected to use the cosine rule again. A number of candidates lost a mark by failing to use a suitably accurate value from part (ii). Finding the correct bearing was a challenge for all but the most able.

7 (a) This question was done very well, with virtually all candidates appreciating the need to expand the brackets before attempting integration. However, it was disappointing that many candidates could not do the expansion accurately. A common error was for the first term to become x^6 and, in some cases, the second term was then x^3. The integration was usually done correctly, with only a small minority attempting differentiation not integration. A number of candidates failed to gain an easy mark because they omitted the constant of integration.

(b)(i) Most candidates gained one mark for an integral that involved x^{-3}, though a few produced a multiple of x^{-5}. Whilst a number of candidates stated the correct coefficient of –6, some lost the final mark by failing to simplify their fraction and others omitted the negative sign. The constant of integration was often omitted from this part, even by candidates who had previously included it in part (a), but this was not penalised.

(ii) The most able candidates quickly gained both marks by simply stating ¾, but many candidates struggled. Some appreciated that F(∞) = 0, but then made a sign error when attempting to subtract -¾, but many candidates seemed unfamiliar with the topic and left an answer in terms of ∞ . A few candidates stated that $\infty = 0$, and then attempted $-6 \times (0)^{-3}$. This was penalised, even if it resulted in ¾ as a final answer.

8 (i) Most candidates were able to attempt a sketch of an exponential graph, but lack of attention to detail often cost marks. Examiners expected to see the negative *x*-axis as an asymptote but some graphs remained a significant distance away from the axis, or failed to show enough of the graph in the second quadrant. Other errors included the graph either touching or crossing the *x*-axis, particularly when the absence of a ruler meant that the axis bore little resemblance to a straight line. It was disappointing to see that a number of candidates were not familiar with the general shape of the graph and resorted to a table of values which were then plotted. Most candidates could correctly identify the required point of intersection, though (0, 1), (0, 6) and (2, 0) were common errors.

(ii) Whilst most candidates could gain some marks on this question, it was rare to see a fully correct solution. The majority equated the two equations and introduced logarithms, but the 2 was not included in the logarithm of the left hand side. On the subsequent step, $\log\left(2 \times 3^x\right)$ nearly always became the product of two logarithms rather than the sum. However, most candidates then gained a mark for using the power rule, and many also gained another mark for $\log_2 8 = 3$. A number of candidates did not appreciate that the given expression was equal to the *x*-coordinate at *P*, and felt that *x* somehow needed to be eliminated from the equation leading to some incorrect cancelling. As in previous sessions, candidates seem to be familiar with the relevant laws, but only the most able can put them together in a convincing proof. In particular, the rules for adding and subtracting logarithms are often used incorrectly. Candidates also need to make their working clear; in some solutions it was unclear whether $\log 8^x$ or $\log_8 x$ was intended.

9 (a) (i) There were two easy method marks available for correctly using the two relevant identities, and many candidates gained both of these, though $\sin x = 1 - \cos x$ was seen on a number of scripts. However, many then struggled to deal successfully with the denominator of $\cos x$, with the -5 failing to become $-5\cos x$ being the most common error. The more successful candidates moved the -5 to the other side of the equation before multiplying through by $\cos x$.

(ii) The majority of candidates recognised the given equation as a quadratic and attempted an appropriate solution method, though a surprising number chose to use the quadratic formula rather than factorising. Whilst there were a few sign errors, most could obtain the correct two roots but some then struggled with what to do with them. Candidates who had used the substitution $x = \cos x$ often stopped at this point, and others attempted cosine of their root rather than inverse cosine. A number of candidates were not confident working in radians, and it was quite common to see answers in degrees which, in some cases, were subsequently converted to radians. Whilst many candidates could obtain the correct primary solution, it was surprising to see the number that then obtained an incorrect secondary solution, often from adding 1.5π.

(b) Whilst most candidates seemed familiar with the trapezium rule and could attempt the question, many struggled to provide a fully correct solution. There were the usual mistakes of using x-coordinates rather than y-coordinates, using an incorrect number of ordinates and using an incorrect value for h. A surprising number did not evaluate the integral between the requested limits, often starting at $x = 0.25$, or using consecutive integer values for x from 0 to 4. Some omitted brackets from the formula, or placed them incorrectly. If the answer is required to 3 significant figures, candidates must appreciate the need for their working to be more accurate than this – the best solutions gave an expression involving exact values in terms of cosine which was then evaluated. A number of candidates worked in degrees not radians, or used $\cos^{-1}x$. These solutions could still gain some credit, whereas the other common error of attempting integration before applying the trapezium rule scored zero.

10) (i) This was generally done well, with most candidates deducing that it was an arithmetic progression and using the relevant formula, though some used $a + nd$ and others found the sum of the first 15 terms. At this level, it was worrying to see that some candidates could only obtain the correct answer by listing all the terms.

(ii) Most candidates identified this as a geometric progression and attempted the n^{th} term, though some could not identify the value for the common ratio (0.2 was a common error) and others attempted the n^{th} term of an A.P. Whilst many correctly obtained a distance of 12.2 km for Day 20, only the most able appreciated the need to confirm the veracity of the statement by also checking Day 19. Some candidates found $n = 20$ by using logarithms to solve the relevant equation. Some weaker candidates once again resorted to listing terms.

(iii) Some candidates persevered with the belief that this was an A.P. and a surprising number used the formula for the n^{th} term of a G.P. and not the sum. However, most candidates stated a correct equation (or inequality) involving 200 and the sum of the G.P. Solving this equation proved to be beyond many and errors in algebraic manipulation were abundant. It was quite common to see 2×1.1^n becoming 2.2^n and other attempts involved the logarithm of a negative number. Whilst many candidates had an inkling of how to solve an equation involving an index, only the most able could do this accurately.

(iv) This part was generally well done, with a number of fully correct solutions seen, and many more candidates gaining partial credit. Finding the distance swum was an easy mark but there were more slips on the other two components, which included using an incorrect formula or making mistakes when evaluating a correct expression.

4723 Core Mathematics 3

General Comments

Examiners were pleased to see many excellent scripts in response to this paper. A significant number of candidates recorded full marks and presentation of work was often good. The first six questions were answered well by many candidates. Q7 presented problems to many; the given information had to be assessed and an appropriate strategy adopted. Similarly Q9 required some thought to judge how to proceed with a question linking the topics of functions and calculus. Many candidates would have benefited from a moment's pause for thought before launching into their solutions.

Solutions to two of the questions led to simple equations; at this level, finding the roots should have been a routine and brief process. However, faced with $7x^2 - 10x = 0$ in Q1 and by $\tan^2 \alpha = \frac{8}{10}$ in Q5, many candidates adopted unnecessarily protracted methods. The formula for the solution of a quadratic equation was used for the former and, for $\tan^2 \alpha = \frac{8}{10}$, identities were used to find a value for $\sec^2 \alpha$, $\cos^2 \alpha$ or even $\cos 2\alpha$ before a value of α was attempted. The solution of equations is, of course, a vital part of mathematics; recognition that an equation in a particular form is capable of immediate solution is an important skill but it was a skill with which a number of candidates struggled.

Comments on Individual Questions

1) This question enabled most candidates to make a successful start to the paper. Converting the given equation to a quadratic equation was the more popular method and candidates generally completed the solution accurately. There were a few slips in the simplification and, occasionally, the solution $x = 0$ was lost. Candidates adopting a method involving two linear equations were also successful. A few candidates, having correctly found the two solutions, proceeded to conclude with $0 < x < \frac{10}{7}$, thereby losing one mark.

2) This question was not answered so well. In part (i), most candidates produced a graph indicating a reflection in the line $y = x$ although doubts were raised when, as happened in many cases, the intercepts were given as (3, 0) and (0, –2). In part (ii), some candidates had no difficulty, readily recognising the pair of curve transformations involved, and produced the correct graph and correct intercepts. Many other candidates struggled; there were many attempts which involved a reflection in the y-axis and intercepts such as (–6, 0) appeared frequently in other attempts.

3) There was a most encouraging response to this question. All but a small minority of candidates applied the product rule correctly and many candidates proceeded to find the equation of the tangent without difficulty. Candidates seemed comfortable expressing the equation in terms of e and very few resorted to decimal approximations. Some candidates were reluctant to simplify at appropriate stages; a few candidates even produced a final answer in which the gradient was given as $2\mathrm{e}\ln\mathrm{e} + \mathrm{e}$. The one serious error to occur with any frequency was the production of an equation in which the gradient was a function of x. Candidates ought to have known that $y = (2x\ln x + x)(x - \mathrm{e}) + \mathrm{e}^2$ was not the equation of a straight line.

4) Almost all candidates recorded some marks on this question but there were aspects which many candidates found challenging and which meant that the award of full marks was not so common. In part (i), it was common for the factor $4x$ to be omitted from the derivative. Many who had differentiated correctly failed to conclude part (i) convincingly; equating the derivative to 100 and confirming the given result required only a couple of lines but, in many cases, there was no attempt to involve 100 and solutions merely consisted of manipulation of $10x(2x^2+9)^{\frac{3}{2}}$.

   There were various approaches tried in part (ii). A few substituted 0.3 and 0.4 in the expression for y which was incorrect. A very successful approach involved substituting 0.3 and 0.4 in the expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ and comparing the two answers with 100. Also successful were those candidates who substituted in either $x-10(2x^2+9)^{-\frac{3}{2}}$ or $10(2x^2+9)^{-\frac{3}{2}}-x$; the calculations were usually correct and the crucial sign change noted. But the most common approach involved substituting the two values in $10(2x^2+9)^{-\frac{3}{2}}$. The calculations were usually correct but most candidates were perplexed that no sign change was evident; very few could construct a convincing argument based on the values of 0.3595 and 0.3515 obtained.

   Part (iii) was nearly always answered correctly and the 3 marks were easily earned.

5) (a) Most candidates either knew the identity for $\tan 2\alpha$ or could reach it via the identity for $\tan(A+B)$. Dealing with $\frac{2\tan\alpha}{1-\tan^2\alpha}.\tan\alpha=8$ presented various difficulties for many candidates; invalid cancellation or procedures involving a common denominator were not infrequent. Many others did reach $\tan^2\alpha=0.8$ but it was very common for only one possible value of $\tan\alpha$ to follow, with the result that the solution $138°$ was missed.

   (b) The vast majority of candidates stated the value of $\operatorname{cosec}\beta$ correctly and many candidates succeeded with the value of $\cot^2\beta$ too. The identity $\cot^2\beta \equiv \operatorname{cosec}^2\beta-1$ was not widely known and solutions were commonly based on the lengths of the sides of the appropriate right-angled triangle or on manipulation of $\sin^2\beta$, $\cos^2\beta$ and $\tan^2\beta$.

6) For many candidates this was a straightforward test of integration techniques and they proceeded methodically and accurately, earning full marks without trouble. For other candidates there were several stumbling blocks. Squaring the expressions sometimes led to e^{9x^2} or to $(2x-1)^6$. The integral of $(2x-1)^8$ sometimes became $\frac{2}{9}(2x-1)^9$. There were errors in evaluating the definite integrals. Some adopted wrong processes to find the required volume. The expression $\int[e^{3x}-(2x-1)^4]^2\,\mathrm{d}x$ often occurred; others integrated e^{3x} and $(2x-1)^4$ and perhaps introduced π and squaring at a later stage. The vast majority of candidates did manage to record at least a few marks because some credit was available for the attempts at integration even if these did not involve the correct expressions.

7) Part (i) proved a challenge for many candidates and they struggled to make any significant progress. Typically they presented a page of involved algebra which involved trying to manipulate the two expressions for N, usually without any involvement of $t = 9$. Others misunderstood the scenario and chose a value of 21 for A. These candidates with either incorrect values for A, k and m or no values could still gain some credit in parts (ii) and (iii) for showing the correct procedures.

   Many other candidates – with perhaps some appreciation of the properties of exponential growth – were able to find the values of A, k and m without fuss. They were then usually able to answer parts (ii) and (iii) correctly, although it was disappointing that some used a value of 0.08 for m that was so approximate that the answers obtained in parts (ii) and (iii) were very inaccurate. In answering part (iii), candidates had to choose which expression for N to use and not all made the sensible choice.

8) Most candidates made sensible progress with part (i), using the $\cos(A \pm B)$ identities accurately and simplifying appropriately. There were some errors with the exact values of $\sin 60°$ and $\cos 60°$ and some slips in the simplification. The process needed in part (ii) was well known although, for some, confusion between sine and cosine led to a value for α which was the complement of the correct value.

   Part (iii) was not answered well. Very few candidates seemed to appreciate that the smallest positive value for θ would follow from a value of $\theta + \alpha$ in the third quadrant. The usual method was to reach a value for θ of $-93.1°$ from $\theta + 70.9 = -22.2$ and then to employ some invalid arithmetic to obtain a positive answer.

9) This question revealed that many candidates had a limited understanding of the terminology associated with functions and, in part (i), did not realise what was required in order to find the range of f. Some merely stated $y \geq 0$, a response certainly not in line with the allocation of 6 marks. Others tried the substitution of various numerical values. Some candidates did appreciate the significance of the coordinates of the stationary point and so did proceed to differentiate, usually accurately. However, having correctly found the x-coordinate of the stationary point as $\sqrt{5}$, some claimed the range as $0 \leq x \leq \sqrt{5}$. It was a small minority of candidates who concluded with the correct range of $0 \leq \mathrm{f}(x) \leq \frac{3}{2}\sqrt{5}$.

Part (ii) was seldom answered correctly even by those who had largely succeeded with part (i). The responses $k = 0$ and $k \geq 0$ were common.

The mention of gradient in part (iii) prompted differentiation for those who had not differentiated in part (i); the two marks available for the differentiation of $\frac{15x}{x^2+5}$ were available at any stage of the solution. Many attempts at part (iii) foundered due to incorrect manipulation of $\frac{75-15x^2}{(x^2+5)^2} = -1$ or to uncertainty about how to conclude. The spurious claim that $x^4 - 5x^2 + 100 = 0$ had no roots because it could not be factorised occurred many times. A good number of candidates did conclude with a convincing demonstration, usually appealing to the sign of the determinant or occasionally using completion of the square. At least one candidate found the elegant argument based on $100 = x^2(5 - x^2)$ that, for values of x such that $x > \sqrt{5}$, the right-hand side is always negative and cannot therefore possibly be equal to 100.

4724 Core Mathematics 4

General Comments

Although this paper proved to be a little more challenging than on recent occasions, there was nothing to suggest that the performance of candidates was different. As usual, there was a wide range of responses and, although many candidates produced a nearly correct paper, there was a considerable number obtaining very few marks. Whilst most candidates reached Q9, the impression given was that time was possibly pressing for some of them and partially (but only partially) accounted for a relatively poor performance on the last question.

As has been mentioned frequently in the past – when the answer is given, every aspect of the working is carefully scrutinised. Qs 8(ii) and 9(ii) were key examples of this and, although almost every candidate finished off those parts showing the correct result, relevant working was often omitted -–further details are shown in the reports on the individual questions.
There are certain bad algebraic errors which occur too frequently at this level of examination:

$$(t+1)^2 = t^2 + 1, \quad \sqrt{a+b} = \sqrt{a} + \sqrt{b}, \quad \frac{a+b}{c+d} = \frac{a}{c} + \frac{b}{d}.$$

Mis-reading was a problem; vectors, as in Q6 with some elements being negative, are prime examples but a not inconsiderable number of candidates misread the differential equation in Q7(ii) and wrote the R.H.S. as $-\sin x \tan x \cot x$ which, of course, completely altered the question.

Comments on Individual Questions

1) (a) A good start was made by candidates with only a few multiplying out to produce a cubic numerator and denominator.

(b) The majority used long division and this was the most effective way; the rest used an identity (or some similar, generally ill-explained, method) and often produced the correct result, provided that they realised the remainder was of the form $cx + d$.

2) Almost everyone used integration by parts with the correct split. The split of $u = x^4, \mathrm{d}v = \ln x$ was only accepted provided the candidate either knew or worked out the integral of $\ln x$. Two problems arose, one being at the end of the first stage. The integral of $\frac{1}{x}.\frac{x^5}{5}$ was required and was quite often shown as $\ln x.\frac{x^6}{30}$; others re-wrote as $\frac{x^5}{5x}$ and proceeded to obtain $\frac{\frac{x^6}{6}}{\frac{5x^2}{2}}$. The second problem was a careless use of brackets; $\left(\frac{1}{5}e^5 - \frac{1}{25}e^5\right) - \left(0 - \frac{1}{25}\right)$ was frequently shown as $\frac{4}{25}e^5 - \frac{1}{25}$. On a separate issue, the terms containing e^5 were not always combined and both ln e and ln 1 were left as such – but simplifications were expected.

3) (i) This was almost universally well done. Occasionally "$\frac{dy}{dx} = \ldots$" was written at the very beginning but it was hardly ever used; on the other hand, "= 0" was frequently omitted at the end of the differentiation line though 'recovery' was implied on the second line. Although examiners tend to expect rigorous working when the answer is given, on this occasion the omission was tolerated.

(ii)(a) As might be expected, $y = 2x$ was rapidly produced from the equation $y^2 - 2xy = 0$ but only a handful of candidates had anything to say concerning the division by y and so 1 mark was the mode for this part. The substitution of $y = 2x$ into the expression for $\frac{dy}{dx}$, which produced an answer of 0, was not accepted as this did not prove that $y = 2x$ was the only situation which would provide 0.

(ii)(b) The majority realised they had to solve $y = 2x$ simultaneously with $x^2y - xy^2 = 2$. Apart from careless errors, many of those obtaining $x = -1$ then substituted back into $x^2y - xy^2 = 2$ and produced two solutions $y = 1$ and $y = -2$; most stopped there, a few demonstrated why y could/did not equal 1 and the occasional candidate said that $y = -2$ without giving any reference to the discarding of $y = 1$.

4) (i) Almost the complete candidature knew what was required here but there was some confusion in the labelling; $\overrightarrow{AB} = \ldots,$ the equation of $AB = \ldots$, $AB = \ldots$ and $\mathbf{r} = \ldots$ were all seen.

(ii) Very few managed any coherent attempt at this part. A vectorial effort was required to indicate that $\overline{OP}$ and $\overline{AB}$ were perpendicular but few realised that the direction vector of $\overline{OP}$ was the whole of the RHS of their equation in part (i) and the direction vector of $\overline{AB}$ was the portion after the 't' parameter in its equation; the scalar product of these two, equated to 0, soon gave $t = -\frac{1}{6}$ or $\frac{1}{6}$ or $-\frac{5}{6}$ or $\frac{5}{6}$ [depending on which version of $\mathbf{r} = (a \text{ or } b) + t(b - a \text{ or } a - b)$ was given in part (i)] and this, substituted into the part (i) equation, produced the required position vector.

5) (i) There were various variations on a theme to complete this part. The majority of candidates produced the binomial expansions of $(1-x)^{\frac{1}{2}}$ and $(1+x)^{-\frac{1}{2}}$ and multiplied them, generally showing sufficient working to produce the given quadratic expression. However, some candidates decided that no terms of the second degree were necessary in either expansion and others decided to add the two expansions. The variations seen included $\frac{\sqrt{1-x^2}}{1+x}$, $\frac{1-x}{\sqrt{1-x^2}}$ and $\sqrt{(1-x)(1+x)^{-1}}$.

(ii) Although a direct instruction was given, some candidates failed to see its significance, but the majority gave a clear explanation of the required result.

6) This question was the most productive source of marks on the whole paper – even weak candidates often scored the full 8 marks.

(i) Most candidates obtained t and s correctly and went on to show consistency in a valid way – either by arriving at (9, 12, 11) twice or by substituting into a 'third' (or otherwise appropriate) equation.

(ii) This was done almost as well; some chose the wrong pair of vectors which usually resulted in the loss of the last 2 marks.

7) (i) Many candidates dealt correctly with the differentiation of $\frac{1}{\sin x}$, using either the chain rule or the quotient rule, and then showed sufficient detail in the subsequent manipulation to achieve the given answer. A few, however, said that $\frac{1}{\sin x} = \sin^{-1} x$ and so the derivative was $\frac{1}{\sqrt{1-x^2}}$.

(ii) Some fell at the very first hurdle by not managing the separation of variables – exacerbated by a few misreading cot t as cot x. A substantial number did not recognise the help given in part (i) for integrating cosec x cot x (or did not realise that $\frac{1}{\sin x \tan x}$ was equivalent to this). Because the integral of cot x appears in the formula booklet, integrating the R.H.S. of the equation was far more successful (though some candidates retained the variable x from the booklet!). Those candidates who got to this stage usually managed the final step of evaluating the constant of integration.

A poor understanding of algebra and calculus was sometimes exposed in this question with statements such as
$\int -\frac{1}{\sin x \tan x}\,dx = \int -\frac{1}{\sin x}\,dx \times \int \frac{1}{\tan x}\,dx = -\ln \sin x \times \ln \tan x$.

8) (i) Although many candidates were successful in this part, it was disappointing to note that a significant minority failed to produce the correct identity $2t = A(t+1) + B$. By using various methods on an incorrect identity, the correct values of A and B were determined but, as this was more by luck than judgement, marks were not awarded. As the phrase 'Partial Fractions' was not mentioned, there was scarcely any use of the cover-up rule.

(ii) Weak algebraic and differentiation skills were often in evidence when transforming dx to dt (often in error by a factor of 2) and $x + \sqrt{2x-1}$ to $\frac{1}{2}(t+1)^2$. It was disappointing to see statements such as $\frac{1}{x+\sqrt{2x-1}} = \frac{1}{x} + \frac{1}{\sqrt{2x-1}}$.

8) (iii) Again, many candidates failed to recognise the help offered in parts (i) and (ii), despite the use of the word "Hence". Significant numbers simply integrated $2t$ and $(t+1)^2$ completely separately. The use of 'integration by parts' was seen on many occasions and this offered a path to the answer equally as straightforward as using the result from part (i). It is, however, pleasing to report that the majority of candidates made an attempt to change the limits.

9) This somewhat unusual question seemed to disconcert many candidates – but it did act as a good discriminator of those who really understood parametric equations. This question was, in fact, done poorly by most candidates. An additional factor was that time was running short for some candidates who had possibly spent too long on Qs 7 and 8.

(i) There seemed to be some confusion in the minds of candidates between values of x and y and the parameter θ. Significant numbers failed to realise, by looking at the graph, that they needed the third solution (from 0, $\pi, 2\pi$ …) given by solving the equation $\sin\theta = 0$.

(ii) Mistakes made in differentiating $2\theta + \sin 2\theta$ were seen fairly frequently, with factors of 2 often omitted. Very often candidates failed to show a convincing series of steps leading to the given answer, and mistakes in the substitution of the relevant identity and subsequent simplification were sadly commonplace.

(iii) Most candidates attempted to solve $\sec\theta = 2$ but, as in part (i), failed to realise they needed negative values of x and y and hence of θ. Consequently the last 2 marks were rarely scored.

4725 Further Pure Mathematics 1

General Comments

Most of the candidates showed that they could cope well with a good proportion of the specification. Candidates generally answered the questions sequentially and there was no evidence of candidates being short of time.As has been mentioned in previous reports, when answers are given in the question, candidates must show sufficient working to justify their answer but many failed to do this.

Comments on Individual Questions

1) (i) This was answered correctly by most candidates with the only significant error being from candidates who thought that $\mathbf{I} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

(ii) Most knew how to deal with the diagonals to find the inverse matrix, omission of the determinant being the most frequent error. Some candidates wrote down a general matrix for the inverse and then solved, correctly, two pairs of simultaneous equations to find the elements of the inverse. This required more working than the candidates who simply wrote down the correct answer.

2) (i) The modulus and argument were found correctly by the majority of candidates.

(ii) The quality of the sketches was often not of the required standard. Many did not indicate the coordinates of the centre clearly and many had the centre not in the first quadrant. Many did not take enough care to draw the circle passing through the origin, while a small but significant minority thought that the locus in (a) was a straight line. In (b), the locus being a half line was often not appreciated, and there were many candidates who did not indicate clearly the slope of their line.

3) (i) This was generally answered correctly, but those candidates who used a denominator of $r!(r + 1)!$ did not then show sufficient working to justify the given answer.

(ii) Almost all candidates used part (i) correctly to find the required sum.

4) The most common error was to try to establish that $\mathbf{A}^{n+1} = \mathbf{A}^n + \mathbf{A}$. Many omitted sufficient working to show either that the result is true for $n = 1$ or that the result of the matrix multiplication gives the element $\frac{1}{2}(3^{n+1} - 1)$. A few candidates gave no statement of the Induction conclusion.

5) A common error was to use $\sum r^2 \times \sum (r-1)$, while a small number multiplied out to get $r^3 - r$. Some who used the correct standard formulae then expanded both terms, before trying to factorise their expression, frequently making a mistake in the process. Candidates should be encouraged to look for common factors, thus shortening their solution and minimising the risk of an algebraic error.

6) (i) Most candidates wrote down the conjugate root.

(ii) Those who used the sum and product of roots approach usually scored heavily, with the only error being the omission of the negative values required for *a* and *c*. Those who expanded three linear brackets were also usually successful, provided that the two brackets with the complex conjugates were expanded first. Those who substituted two (or three) roots and set up 3 simultaneous equations for *a*, *b* and *c* generally failed to complete the question.

7) (i) The enlargement was usually described correctly.

(ii) Most recognised a reflection, but gave an incorrect mirror line. A few candidates thought the transformation was "no change".

(iii) Many thought that this was a shear, rather than a stretch.

(iv) In this part, and occasionally in previous parts, candidates did not give a **single** transformation. The most common error was to give the wrong direction of rotation.

8) Most wrote down the values of $\alpha + \beta$ and $\alpha\beta$ and found correctly the sum and product of the new roots. Many then failed to write down a quadratic **equation**, omitting "= 0". Those who used a substitution approach either made little progress or derived the correct equation, often by an ingenious method.

9) (i) A significant number of candidates showed no working and simply wrote down the answers, presumably from a graphical calculator. The most common errors were to give only one square root, or to omit to write the answers as complex numbers.

(ii) Very few errors were made in this part.

(iii) Most candidates solved a quadratic equation, but many then failed to appreciate that a quartic equation has four roots.

10) (i) The majority of candidates found the determinant correctly and hence showed that **AB** is non-singular.

(ii) A good proportion showed that they could deal with all the aspects of finding the inverse matrix, with occasional errors in the elements in the bottom row.

(iii) Many candidates thought that $(\mathbf{AB})^{-1} = \mathbf{A}^{-1}\mathbf{B}^{-1}$. Those who knew the correct result did not always appreciate that matrix multiplication is not commutative and evaluated $\mathbf{A} \times (\mathbf{AB})^{-1}$.

4726 Further Pure Mathematics 2

General Comments

Most candidates found the examination accessible, answering the questions in the order set. There appeared to be no great problem with timing, although there was some evidence that a poor choice of method or indifferent algebraic manipulation caused some candidates to rush the later questions. Nevertheless, the majority of candidates finished the paper. The early questions gave most candidates a sound start to the paper, and it was not until Q5(i) was reached that real problems arose. Candidates appeared to be well-prepared for the range of questions asked, and no question proved particularly difficult. However, the general standard of integration in Qs 3, 5 and 9 was poor, and it was surprising to find basic errors in differentiation in Qs 6 and 7.

There was evidence that some candidates were not sufficiently aware of the importance of full and detailed responses to questions in which the answer to be proved is given. A full justification is expected in such questions, with nothing seen as “obvious”. Candidates are encouraged also to explain fully in Qs such as 4(iii) and 9(ii).

Nevertheless, most candidates were able to demonstrate their knowledge and there were more well-presented scripts than usual. It was particularly pleasing to see thoughtful efforts, especially in Q7, and to find candidates able to find their own ways through questions.

Comments on Individual Questions

1) Most candidates produced the correct partial fractions at the start of the question, with only a minority using Bx (or even just B) for the fraction associated with $(x^2 + a^2)$. The better candidates then used $x = 2a$ and equated coefficients of x^2 to produce two coefficients at once. It was obvious that some candidates knew only one method and either equated coefficients to produce three equations in three unknowns or chose three values of x to substitute. Even so, many candidates scored well, although some time was lost, particularly by those candidates who substituted numerical values of x.

2) This question was generally well answered, with most candidates picking up at least three marks. Marks were lost in giving the incorrect turning point but more widely in not showing that the x-axis was crossed at 90°. Candidates are encouraged to exaggerate this to ensure that it is clear to examiners or to state it in commentary.

3) Most candidates gained at least half marks. The substitution in terms of t was well known, although some candidates again wasted time by deriving expressions for $\cos x$ and dx in terms of t. Standard results can be quoted even if they are not found in the Formulae Booklet. Candidates often produced the correct expression to integrate and most recognised a $\tan^{-1}$ answer. The final two marks depended on how accurate they were with the coefficients involved. Many attempted to use the standard result in the Formulae Booklet, often successfully, whilst others used a substitution to simplify $(3t^2 + 1)$.

4) (i) This question as a whole was very well answered. Candidates had a good knowledge of sech x, both in terms of the graph and its exponential definition. Odd marks were lost in not giving (0,1), but most candidates gained full marks in part (i). A small minority did not know $y = x^2$ and dropped the easiest mark on the paper.

(ii) Again, this part was well answered. This is an example of an answer given in the question, and it was expected that some algebraic manipulation whould be used to produce the result. In this case, candidates usually did enough to gain the marks.

(iii) The majority of candidates knew that this was a cobweb diagram, but many expressed themselves badly when attempting the explanation. Statements such as "values go up and down" were not enough to gain the mark. Clear statements were infrequent, and candidates should take more care in expressing themselves, even if that takes more than a few words or a single sentence.

5) (i) This part provided the first difficulty for most candidates, despite the clue being given in the question. Many candidates got nowhere with this part as they started by using parts on $1\tan^n x$ or on $\tan x.\ \tan^{n-1}x$. Others are to be commended for using $\tan^2 x.\tan^{n-2}x$, replacing $\tan^2 x$ with $\sec^2 x$-1 and getting to the same point as those who used the clue in the question. The integration of $\sec^2 x.\tan^{n-2}x$ then provided a challenge to many, with few able to write down the answer at once. However, it was pleasing to see some candidates having the confidence to use parts on this and eventually arriving at the correct answer.

(ii) It was surprising that many candidates did not attempt this part as it did not depend upon part (i). Candidates starting this part were often successful, with only numerical errors seen. Some candidates used I_2 instead of finding the easier I_0, but the integral of $\tan^2 x$ was generally well done.

6) (i) This question as a whole was well answered, with many candidates gaining at least five marks. The main error in part (i) was in differentiating $1\text{-}7/x^2$ incorrectly, which is surprising at this level. The application of Newton-Raphson was usually accurate, although time was lost by those candidates not using their calculator facilities but starting again for the next approximation.

(ii) The word "exact" was missed by many candidates who continued to use their calculators until "equal" successive approximations were seen. This did not affect the marks which could be gained in part (iii).

(iii) Candidates who produced e_3 as 0.00008 and stated that this was the same as $e_2{}^3/e_1{}^2$ without evidence were penalised one mark. Otherwise, this part was well answered.

7) (i) Many candidates failed to use the chain rule on $\tanh^{-1}((1-x)/(2+x))$ and gained one mark only for $f'(x)$, for quoting the result for $\tanh^{-1}$. Candidates who used the chain rule were often successful, with only algebraic errors spoiling their solutions. A significant minority of candidates are to be commended for rewriting $f(x)$ as a logarithmic function, even simplifying it using the logarithm laws. This led to a straightforward differentiation and to a more direct means of producing the Maclaurin series in part (ii). $f''(x) = -\frac{1}{2}\ln(1+2x)$ was surprisingly common.

(ii) Candidates were generally more successful on this part. Three marks were available to those with the incorrect $f''(x)$, and the majority of candidates used the Formulae Booklet for the logarithmic equivalent of $\tanh^{-1}\frac{1}{2}$ rather than resorting to the exponential definition of $\tanh x$.

8) (i) There were many correct answers to this part of the question. Candidates made odd errors in solving $r = 0$, giving answers for α which clearly did not reflect the given diagram. Full marks were gained for the use of θ in place of α and for giving more answers than just $\frac{1}{4}\pi$. No credit was given for differentiation.

(ii)(a) This part proved difficult for candidates. The first problem lay in an inability to work out $f(\frac{1}{2}(2k+1)\pi - \theta)$ accurately, with many candidates failing to double the θ, whilst others appeared to believe that they had to solve $\sin((2k+1)\pi-2\theta) = \sin 2\theta$. Even those candidates who got the first stage correct then believed that they had done sufficient work for the three marks available. The most successful solutions went on to expand $\sin(A+B)$ and discuss the values of sin and cos of $(2k+1)\pi$ for integer k. Candidates who discussed the periodicity of sin and cos were also successful, although precision was often lacking in dealing with the general k.

(ii)(b) This part also proved difficult with relatively few candidates specifying the four lines of symmetry in $0 \le \theta < 2\pi$. A minority could quote the answers using part (ii)(a), but candidates knowing the symmetry from the $\sin 2\theta$ term also gained marks. Candidates just giving the angles rather than the equations of the lines were not penalised in this case. It was surprising to see lines of symmetry specified which then did not appear in the diagram.

(iii) A number of candidates thought that the given part of the diagram indicated that the curve had symmetry in $\theta = 0$ and π, and even in $\theta = \frac{1}{2}\pi$. This then led to maximum $r = 1$ and no marks. Diagrams varied greatly in quality and some allowance was made for the nature of freehand diagrams. Two roughly symmetrical loops in the correct quadrants gained one mark, but some evidence was required for the approaches along the lines of symmetry. A large number of candidates lost one mark by not giving both values of θ for maximum r, despite accurate diagrams.

9) (i) Many candidates gained only one mark because they could not deal with the integral of $x/(1+x)$. The more successful candidates used a simple substitution or divided out (many being able to write the answer down at once). The question was asked as a proof, and candidates attempting to quote the integral of $\ln x$ and then generalising to the given function were given only limited credit. Similarly, candidates differentiating the "given" answer were given some credit. Other candidates who quoted a result which then led to the given answer were given no credit.

(ii)(a) It was pleasing that many candidates were better prepared for the explanation required. It has been reported before that candidates should expect to answer in terms of areas (and not, for example, integrals), and should explain the areas of the rectangles in relation to the area under the curve. It should be clear that the area of the rectangles has been used, and not just the height. The limits in this part could be omitted as they were clearly given in the question.

(ii)(b) There was a lack of precision in this part by many candidates. The best answers involved either a diagram, clearly showing the first and last rectangles used (and hence giving the limits), or a description of a translation by one unit of the rectangles in part (ii)(a). Candidates who began with "these rectangles…" or who merely reiterated what was in the inequality without any explanation gained no marks.

(c) Most candidates used the earlier parts of the question, but it was rare to see an explanation of where ln(70!) came from. Only a small number of candidates were able to round the answers correctly to the given accuracy. The result overall meant that many candidates gained one mark only.

4727 Further Pure Mathematics 3

General Comments

This paper was done well by a large proportion of the candidates. Several questions tested topics in ways which were familiar, and these were answered accurately. The last three questions were more demanding, although some parts of each were very straightforward. Although some candidates submitted well-written answers, the presentation of many others was poor: some of their writing was difficult to read, especially figures and signs, and on occasions they miscopied their own work; they did not display their answers clearly; they continued questions on later pages of the answer booklet, or on supplementary sheets or booklets, often without any indication that they were doing so, and parts of questions were mixed up with other questions. Candidates would help themselves (and the examiners) if, having temporarily abandoned a question, they left themselves space to return to it, rather than having to copy work from one page to a later one, with the likelihood of making mistakes. Mathematics is about clarity and precision, and poor presentation does not help. There did not appear to be any major problems about the length of the paper, although a small number of candidates ran out of time part way through Q8. In most cases this would have been because some earlier answers, in particular Q7, had been rather longer than necessary.

Comments on Individual Questions

1) (a)(i) This should have been an easy starting question, but many candidates appeared not to understand exactly what was being asked. In some answers there was confusion between the order of the subgroup and the order of elements within the subgroup. This may have arisen from a misreading of the question as "the elements of *G* which are of order 4". The most common incorrect answers were the sets of four elements $\{e, r, r^2, r^3\}$ and $\{e, r^4, r^8, r^{12}\}$.

(a)(ii) Credit was given here for answers which, in some way, stated that if two elements were *e* and *r*, then the subgroup would have to contain all the elements of *G*, and thus would not be a proper subgroup. The phrase "*r* is a generator" was often used correctly to describe the situation. A significant number of candidates misread the question, believing that it asked why the set consisting of only the two elements *e* and *r* was not a subgroup, and giving the answer that it was not closed.

(b) Answers to this part were about equally divided between those who gave all six possible orders, those who gave only *m*, *n* and *p*, and those who gave only *mn*, *np* and *pm*.

2) This was a straightforward vector question which was answered well by a large majority of candidates. They progressed confidently through the stages of finding the normal and then using a scalar product to find the angle. The most common error was at the end, where many omitted to subtract their answer from 90° if they had used $\cos\theta$ in their calculation. Some arithmetical errors were also made. The second method shown in the mark scheme was seen very infrequently. A small number of candidates found instead the angle between the given line and one of the direction vectors in the plane: this was a serious misunderstanding of the geometry and could score a maximum of one mark.

3) (i) The substitution method for solving certain differential equations was well known, and most had no difficulty in establishing the given result. A few candidates took a round-about route by differentiating the substitution with respect to y instead of x, but they got there in the end.

(ii) Most answers started off confidently by separating the variables correctly, although a few had the fraction the wrong way up. But it was only the better candidates who realised that division was necessary, or perhaps a simple substitution, in order to integrate. Those who did so almost always went on to complete the solution correctly, including the arbitrary constant and the replacement of z by $x + y$. The most popular approach was to try integration by parts, but most soon abandoned the attempt, having found another unfamiliar integral. Very occasionally integration by parts was made to work, deriving or quoting $\int \ln(z+1)\,dz$ on the way. Others tried to find some sort of integrating factor, but fortunately did not pursue the method very far.

4) (i) This fairly standard result was proved confidently by a large proportion of candidates. Those solutions which used z in place of $e^{i\theta}$ were among the neatest. Examiners were surprised to see that a comparatively large number of answers laboriously multiplied out $\left(e^{i\theta} + e^{-i\theta}\right)^5$ in stages, without using the binomial theorem. This took far longer, although it was generally done accurately.

(ii) The application of the first part to the solution of the trigonometrical equation was well understood. There were some who solved $\cos\theta = \cos^5\theta$ instead of the correct equation. A fair number of those who solved the right equation obtained all three values of θ, but many omitted $\frac{1}{2}\pi$ because they had divided through by $\cos\theta$, or omitted $\frac{2}{3}\pi$ because they had lost $\cos\theta = -\frac{1}{2}$.

5) (i) Nearly all candidates used the parametric forms of the equations of the lines, writing them down and solving accurately for λ and μ (or whatever parameters they used). Only a few weaker candidates used the same parameter for both lines. However, the last two marks were not so easily gained. Many omitted to check that their values of λ and μ also satisfied the equation which had not already been used, or else they re-used an equation to check. Some observed that k cancelled in the first equation and thought that this meant that the lines intersected. The final request, for the point of intersection, was overlooked in quite a number of answers. A few candidates solved the cartesian equations without using parameters (Method 3 of the mark scheme), which was fine, although usually it was rather longer as solutions tended not to be done systematically. Method 2 was rarely seen, sensibly in this case, because although it shows neatly that the lines intersect, the parametric method was still required for the point of intersection.

(ii) This part caused few problems, with most using the vector product correctly and inserting one of the possible points to find the value of the constant term in the equation. Such mistakes as there were, were usually errors in calculation.

6) (i) Many candidates believed that, just because $a|b|$ appears to be different from $b|a|$, the operation was not commutative. The only acceptable answers involved consideration of the signs of a and b, either in general or in a particular case. Of those who scored both marks, the majority used a pair of numbers, one positive and one negative, to give an example of non-commutativity. Others showed that when $a > 0$ and $b < 0$, or vice versa, different values were obtained. Some tried to do this, but muddled their notation, for example by using $-a$ to represent a negative number without explanation.

(ii) This part was very easy and full marks were frequently earned. Examiners were pleased that almost all understood what associativity meant and that very few candidates used either particular numbers or repeated letters.

(iii) This part was very demanding. Two misconceptions about groups which were seen were that, presumably because the operation involved multiplication, the identity was 1, and that, because the operation had been shown to be non-commutative, the set could not form a group. The fundamental reason why this rather unusual operation does not form a group is that there is no unique identity, and to make statements about inverses in disproving the group properties is therefore meaningless. Candidates who stated that there was no identity (for whatever reason) or that the identities would have to be +1 and –1 scored the first mark, and if they then deduced that the set was not a group they scored another mark. But the other two marks were awarded only to those who proved algebraically from the definition of an identity that $e = \pm 1$, or who found that $e = +1$ for positive numbers and $e = -1$ for negative numbers. There were some excellent answers from the best candidates, but many failed to score any marks at all in this part.

7) (i) Most diagrams were awarded the single mark, but it was not uncommon for them to be drawn inaccurately and unlabelled.

(ii) All the methods shown in the mark scheme were seen, the most popular being to express ω and ω^2 in cartesian form. The neatest method is to state that the coefficient of z^2 in the cubic equation is 0, but this was not seen as often as had been expected.

(iii) The evaluation of both expressions was carried out correctly by many candidates, but methods which involved the values of ω and ω^2 tended to be lengthy. A surprisingly large number left their answer to the first part in the form $2+\omega^3$, not realising that this simplified to 3. The second part was usually done by combining the fractions and, again, some answers included ω^3.

(iv) Some very neat solutions were seen to this part, but others were long and quite involved. About a third of the answers used Method 1 of the mark scheme, with the rest multiplying out the factors. The first method was usually shorter, but those who first multiplied out the two complex factors and used the results of part (ii) quickly arrived at $(z-2)\left(z^2-z+\frac{1}{3}\right)=0$, which was easy to multiply out and then the fractions could be cleared. Equivalently, occasional answers used the sum and product of the two complex roots to give the quadratic factor. Candidates who used this approach are to be commended on their ability to apply what they know in a non-standard manner. A common slip was to calculate the value of $\alpha\beta\gamma$ as 2×3 instead of $\frac{2}{3}$. Some of those who multiplied everything out in terms of ω before simplifying got lost in the algebra. It was fairly common for the equation to be left in fractional form, or without an " = 0 ", neither of which was accepted for the final mark.

8) (i) This part should have been done easily by all candidates, but it was not. Most wrote down the correct auxiliary equation, though even here there were some errors. But the solutions to the equation were often incomplete or wrong: i only, and ±1 were seen frequently. The complementary function was often given incorrectly, even when the correct solutions ± i had been given. In this part acceptable forms were $A\cos x + B\sin x$, $A\cos(x+\varepsilon)$ and $A\sin(x+\varepsilon)$.

(ii)(a) The given form of the particular integral was likely to have been unfamiliar to almost all candidates, but examiners were most impressed by their ability to carry out the required differentiation, substitution and rearrangement correctly. Many obtained full marks for this part. When mistakes were made, they were often in the first differentiation when one term of $\frac{\mathrm{d}}{\mathrm{d}x}p\left(\ln\sin x\right)\sin x$ was given as $p\frac{\sin x}{\sin x}$. Those who obtained the first two derivatives correctly usually went on to obtain the given identity accurately. Even at this stage of the paper, when time may have been running out, there were comparatively few slips in signs and other essential details.

8) (ii)(b) Many wrote down the correct values of p and q, sometimes without showing any working. All that was necessary was to equate the constant and $\sin^2 x$ terms in the identity. Others floundered, trying to solve the identity for x or p.

(iii) Many obtained full marks for this final part. The mark for the general solution was often awarded, as the acceptable forms included a follow-through for the C.F. from part (i) (provided this had two arbitrary constants), and the given form of the P.I. if the values of p and q had not been found in part (ii) (b). In a pleasing number of answers there was a realisation that $\sin x$ had to be positive in order for $\ln \sin x$ to be valid, or that $\operatorname{cosec} x$ was not defined when x was a multiple of π. The range of values of x had to be exactly correct, with < signs at both ends, for the final mark to be awarded.

Chief Examiner Report - Mechanics

The overall standard of work seen at this session was good, though a small minority of papers fell well below the minimum standard expected for a grade E. Across the modules there were some common problems; not having a calculator set to the correct calculator mode was one, the confusion in interpreting negative quantities was another.

In general the presentation of work was sufficiently clear for appropriate credit to be given to partly correct solutions. This became more difficult where candidates apparently worked on diagrams in their question paper, making no copy in their answer booklet, or compressed their solutions to seven or eight questions onto two pages of the booklet.

Where the quality of a diagram was particularly important (centres of mass in M2, relative velocity in M4), it was felt that many candidates produced sketches too ambiguous to be helpful in answering the questions set.

4728 Mechanics 1

General Comments

Candidates were well prepared for the examination, and showed not only a good knowledge of the content of the specification, but also how to present mathematics clearly. One unusual feature of the answers seen this session was the influence earlier questions had on solutions seen to later ones, in Q2/Q4 and Q3/Q6.

Where numerical answers were given in the paper, candidates generally took care to show how the values arose. However, there were many examples of intricate, but worthless, methods for deriving values from given data. This gained no credit, and wasted time.

The principal general weakness lay in the understanding of the significance of negative quantities, and their relationship (in the context of a problem) to direction and magnitude. In the last two questions, many scripts indicated that the narrative drive of the problems was not fully appreciated, and best use was not made of the assistance given. There were also many instances of only one answer being given in a part of a question requesting two. In this way marks were needlessly lost in Qs 1(ii), 4 (i), 5 (i), 6 (iv), and 7 (ii).

A few unfortunate candidates had their calculator set in radian mode for the entire examination. Errors in selecting the use of sine or cosine when resolving were rare, except in Q4 where the directions of the tensions were referred to the vertical, not the horizontal as has been more common in recent papers.

Comments on Individual Questions

1) (i) Very rarely was any difficulty apparent, though a few "solutions" arose from 240/600 = 0.4, the answer given in the paper.

(ii) The time and distance were generally calculated independently, to avoid a wrong solution for one creating a wrong value for the other. The most common loss of marks arose from finding only one of these quantities.

2) (i) Fully correct solutions were frequently seen, though a significant number of candidates made no progress. Both vector addition/cosine rule and resolving/Pythagoras theorem were popular, but scale drawing was seen. This last method lies outside the syllabus specification, and candidates should be aware that using it brings no credit. The given answer could be achieved in several meaningless ways, the least imaginative being 12cos15+14cos15, for which no marks were given.

The similarity of the diagram for this question with that for Q4 may have prompted the many attempts to start the solution to that question by calculating the resultant of the two 15N tensions.

(ii) The only common error was not giving the bearing angle as requested, but finishing the solution once any relevant angle had been found. A smaller number of solutions were based on a triangle assumed to be right-angled, though this was inconsistent with the lengths of its sides.

3) (i) This was almost always accurate.

(ii) The solution to this part of the question was usually correct, though the solutions given were sometimes too long. No numerical answer had been supplied in the paper, and finding the area of one part of the diagram and multiplying it by 2 or by 4 as appropriate was all that was expected. Solutions showing a detailed calculation of four different areas and their subsequent summation gained no extra credit.

(iii) Though the correct negative value of the velocity was usually obtained, often the calculation led to positive 3.6. The approach by candidates was usually based on using $v = u + at$. Errors arose from using 0 or 6 as the value for u, –1.2 as a, or 17 for t. A significant number of scripts concluded (after correctly finding –3.6) with the statement "velocity = 3.6 ms^{-1}.

4) (i) Most candidates approached the problem in the expected way, though those who approximated 15sin50 to 11.5 subsequently obtained 4N as the magnitude lost an accuracy mark. However, there were a significant number of other errors:
- Using cos30 and cos50
- Obtaining a negative value for *F* but not reconciling it correctly with the context, or having a friction force which was not horizontal
- Finding the magnitude and direction of the resultant of the tensions (with or without the weight)
- Confusing *F* meaning force or friction, and *R* - reaction or resultant.

(ii) Again there were many correct solutions, but candidates who had become confused in part (i) seldom recovered. The commonest errors were:
- The resultant calculated in (i) might be used as *F* or *R*
- Using 30 or 30g as the normal reaction
- Substituting negative values into $F = \mu R$
- Using sin50 and sin30 in calculating the vertical components of the tensions.

5) (i) Nearly all candidates scored well, though a few obtained (correctly) a negative value for their v and failed to convert it to speed.

(ii) Again this part of the question was well answered, though in part (ii)(b) the commonest value for the change in momentum was 9600 $kgms^{-1}$, through disregarding the vector nature of momentum.

6) (i) Most candidates scored all 3 marks, using differentiation and showing numerical expression whose evaluation gave 1.28 ms^{-1}.

(ii) Again the differentiation was completed accurately, but candidates who began to manipulate their expression for a before substituting zero usually lost a mark.

(iii) It was at this stage that many candidates began to lose a significant number of marks, not appreciating that solving the quadratic equation given at the end of part (ii) gave the values of t at which the maximum and minimum values of v would occur. Alternative methods based on the factorisation of the formula for $v(t)$ were tried, with varying degrees of success.

(iv) Smooth curves were rarely attempted, and simple diagrams resembling the graph in Q3 were drawn. The most common sketch consisted of a scalene triangle with its base between 0 and 6 on the t-axis, and its vertex at (2, 1.28). Often there was no portion of the graph beyond $t = 6$ s, and the direction of motion of the train on leaving B was equally likely to be omitted/correct/wrong .

(v) Very often candidates based their answer on finding the area of their triangular or polygonal graph. Other candidates evaluated $x(7)$, even though their train had come to rest when $t = 6$. There were however many candidates who answered this part correctly as it did not require total accuracy in earlier work.

7) (i) Though many valid derivations of the stated value of the frictional force were seen, often the problem seemed to be tackled as if it were an isolated particle at rest in limiting friction on an inclined plane, and simply quoted $F = 0.2\text{gsin}45$ without any reference to the coefficient of friction.

(ii) Newton's Second Law was frequently used correctly, either for the two particles together, or for the individual particles. The former approach required the appearance of both the component of the total weight and friction for the derivation of the given answer to be regarded as valid. In many scripts there was no attempt to find the tension.

(iii) This was almost always done correctly, using the given value of a.

(iv) Rigorous solutions showing a=0 through the use of a single use of Newton's Second Law were seen. However verbal "explanations" were often inadequate, and there were some circular arguments based on the speed at the foot of the slope being unchanged, and zero acceleration being deduced from use of constant acceleration formulae.

Though many correct solutions were seen, the change in the acceleration of particle P was often overlooked.

4729 Mechanics 2

General Comments

The paper demonstrated a wide spread of ability. There were many excellent candidates who showed thorough understanding and some of whom scored full marks. However, a significant number of candidates showed a lack of knowledge of the techniques required and appeared to have had little preparation for the examination.

As is often the case, the clarity of diagrams was often poor and this frequently led to misunderstanding, particularly with Qs 5, 6 and 8. On the positive side, there was very little confusion between speed and angular speed in Q6, and projectile motion was well done in Q7(ii). However, many candidates could improve their performance by taking greater care over presentation, notation and their diagrams. It is, of course, possible that candidates use the diagram on the question paper, but this is of little much use to the examiner if it is desirable to understand an error. Most candidates appeared to have had sufficient time to complete the paper.

Comments on Individual Questions

1) The majority of candidates scored three marks. However, a few lost marks through simple calculator errors such as multiplying by cos35° instead of dividing, and in some cases calculators were set to radian measure.

2) Well answered, although unit errors for cm and grams were not uncommon. The energy method was more commonly used than the equation of motion. As in previous papers, another disappointing observation was the frequency with which $\frac{1}{2}mv^2$ was written down but the v was not squared when numbers were substituted.

3) (i) This was well done, although the given answer may have helped.

(ii) There were a large number of very good solutions. Most candidates derived the appropriate quadratic equation and many went on to solve it correctly.

(iii) In general this was well answered although some candidates included a weight component and some omitted the resistance.

4) (i) There was often a lack of clarity between the half time and complete time of flight. A few candidates quoted the formula for the range of a projectile. This was not good as the question asked for a derivation via the time of flight. The double angle formula was generally well known.

(ii) Most candidates gained three of the five marks available. Marks were commonly lost as only one angle was often found when the trigonometric equation was solved.

5) (i) This was well answered, usually through taking moments about the end of the rod. The majority of candidates used the correct formula for the centre of mass of a solid hemisphere.

(ii) The proportion of correct solutions was small. Candidates who gained four marks drew clear diagrams and expressed clear logic in their descriptions of the position of the centre of mass relative to the points of contact with the ground. Several candidates used the unexpected phrase "the toy toppled to an upright position". If this was accompanied by correct work, full marks could be awarded. In calculating a relevant distance, the position of the right angle in a used triangle was often incorrect.

6) (i) A significant number of candidates would have benefitted from learning to understand the difference between an attached object and a smooth ring in circular motion problems such as this one. Unfortunately, a large number of candidates assumed that the tension was the same in both sections of the string.

(ii) The majority of candidates realised that the tension in the section *BP* became zero. However, many did not resolve correctly in either the vertical or horizontal directions.

7) (i) This was well answered with the majority of momentum and restitution equations being consistent in their sense of direction.

(ii) In the majority of cases scripts contained good solutions.

8) (i) The location of the centre of mass of a triangular lamina was generally correctly found and this usually led to correct solutions to finding the coordinates of the centre of mass of the complete lamina. However, some candidates confuse the 1/3 for 2/3 and had the position of the centre of mass of the triangular section closer to *C* and to *D*.

(ii) A small number of candidates were successful with this last part. Common errors were the omission of a weight component, incorrect trigonometry and incomplete moments equations.

4730 Mechanics 3

General Comments

Candidates found the first five questions fairly straightforward and there is evidence that the maximum mark was the modal mark in each case. However in Qs 2 and 5 there was a much more significant number of candidates who failed to score any marks, than was the case in Qs 1, 3 or 4. Appropriately Qs 6 and 7 were found to be more searching, and weaker candidates found particular difficulty with Qs 6(iii) and 7(iv).

An unwelcome feature was evidence that a significant number of candidates did not on occasions read the question carefully. A significant minority had the string vertical in Q1 and similarly the surface struck by the particle was taken to be vertical in Q2.

Work was generally well presented.

Comments on Individual Questions

1) This question was very well attempted. Some candidates introduced weight into part (i) and potential energy into part (ii) as a result of having the string vertical.

2) This was the most taxing question of the first five, but was nevertheless generally well attempted.

   In part (i) the most common mistake was to obtain the vertical component of velocity without reference to the coefficient of restitution; this was a result of having a wall struck by the particle rather than a table.

   In part (ii) the most common mistake was to calculate the difference, instead of the sum, of the magnitudes of the vertical component of impulse before and after impact.

3) There were many excellent solutions to this question.

4) The most common errors in this well attempted question were the omission of the minus sign from the logarithmic term on integration, and failing to include a constant of integration.

5) Each of the three parts of the question was targeted first by a significant number of candidates.

   Those who started by taking moments about X for XA used the given value of R_A to find F_A. They usually followed this by finding F_B and R_B by resolving forces horizontally and vertically, respectively, on the complete structure. However most candidates using this strategy failed to tidy up in part (i) by showing that $R_A = 125$.

   Starting by taking moments about A and about B for AX and BX respectively, proved a popular and successful strategy. The method yields simultaneous equations for the components required in part (iii). Thereafter it was a straightforward matter of resolving vertically on each rod to yield the answers required in (i), and horizontally to yield the answers required in part (ii).

6) In part (i) of this question most candidates successfully obtained $\ddot{x} = -1.44y$ but failed to make the observation that $\ddot{x} = \ddot{y}$. In many cases this didn't matter because a satisfactory explanation of why the equation represents SHM was given. Most candidates obtained the period satisfactorily.

In part (ii) it was very common to see the amplitude given as 3, the equation leading to the required values for x then being given as $0.48^2 = 1.2^2(3^2 - x^2)$ instead of $0.48^2 = 1.2^2(0.5^2 - y^2)$.

Part (iii) of the question was poorly attempted for two main reasons. Firstly candidates failed to appreciate the role of y in facilitating correct answers, thus muddling x and y. Secondly candidates failed to appreciate that 'P is moving towards O' in the question implies here that P is moving in the same direction on the first two occasions for which its speed is 0.48 ms^{-1}, and is moving in opposite directions on the second and third occasions. A further complication might have been that on sketching a sine or cosine curve for displacement, candidates might have been looking for same (or different) signs for two displacements, instead of the same (or different) signs for the slopes.

7) The $a\omega^2$ of the printed answer in part (i) often triggered the use of Newton's second law. However on realising its need in part (ii) such candidates usually revisited part (i) with success. Both parts were generally well attempted.

Candidates who used Newton's second law or quoted a formula for transverse acceleration fared rather better than those who used a more fundamental approach, such as differentiating the equation printed in part (i) with respect to t.

Part (iv) was expected to provide a difficult test even for the best candidates, and it was pleasing to see how well it was attempted.

4731 Mechanics 4

General Comments

Most candidates for this paper appeared to be well prepared and were able to demonstrate a sound understanding of almost all the topics examined. The only exception was the application of energy to small oscillations in Q7. The candidates seemed to have sufficient time to complete the paper, and generally they presented their work clearly. The marks were considerably higher than last year; almost half the candidates scored 60 marks or more (out of 72), and under 10% scored fewer than 30 marks.

Comments on Individual Questions

1) *(Angular momentum)*
This question was answered correctly by nearly all of the candidates. A few used conservation of energy instead of angular momentum.

2) *(Constant angular acceleration)*
The constant acceleration formulae were well understood, and generally applied accurately; About three-quarters of the candidates scored full marks on this question. A fairly common error was the use of 16 radians instead of 16 revolutions.

3) *(Centre of mass)*
The methods for finding the centre of mass of a lamina were very well known, and over half of the candidates scored full marks on this question. Some lost a factor ½ from the y coordinate, and some tried to find the y coordinate by substituting the x coordinate into the equation of the curve.

4) *(Relative velocity)*
Candidates now seem to be much more confident about finding and applying relative velocities, and many of them scored full marks on this question. Many others lost just one mark, usually as a result of using the sine rule to find an angle which was in fact $91°$ but was invariably assumed to be $89°$. This problem should have been avoided by finding the other angle, which is known to be acute.

5) *(Moment of inertia, and compound pendulum)*
In part (i) most candidates were able to derive the given moment of inertia satisfactorily. Some attempts were flawed, but there were no commonly recurring errors.

   In part (ii), many candidates used the moment of inertia about the x-axis instead of the axis of rotation of the pendulum. A fair number seemed to misunderstand the scenario, and calculated the position of a centre of mass to use in the compound pendulum formula.

6) *(Rotation, and force acting at the axis)*
In part (i), almost all candidates derived the moment of inertia successfully, and in part (ii) most could use conservation of energy to obtain the given result.

   In part (iii) sign errors occurred quite frequently in the equations of motion, and some candidates did not appreciate the need to find the angular acceleration.

7) *(Equilibrium, and small oscillations)*
This was the worst answered question, but even so, nearly a quarter of the candidates scored full marks.

In part (i) the methods were well understood, although there were often algebraic slips in the working.

In part (ii) a very large number of candidates tried to proceed without considering the kinetic energy, and they could score at most 1 mark out of the 9. Those who did include kinetic energy usually made substantial progress, although minor slips often led to an incorrect final answer.

Chief Examiner's Report - Statistics

As usual there was much good work seen on all statistics units. It is pleasing to record that the majority of Centres have acted on the notice given in previous Reports concerning statements of hypotheses and that over-assertive conclusions to hypothesis tests (for instance, "aneroid readings do not overestimate blood pressure") would be penalised. (Preferable is "there is insufficient evidence that aneroid readings overestimate blood pressure".) However, verbal answers continue to need improvement in many respects.

In all statistics units, emphasis needs to be given to the correct use of the formulae in MF1. As mentioned in the Report on S1, some candidates use different, and generally inferior or confusing, formulae. In all units, weaker candidates tend to be over-reliant on the booklet by attempting to find a plausible-looking formula when they do not know what to do. This practice is invariably unsuccessful and should be discouraged.

There was evidence of a decline in candidates' capacity to answer questions on routine hypothesis tests. Vague or lazy methods were more apparent than in the past, especially for those questions that involve discrete distributions. Some centres in particular would seem to need to pay especial attention in this area.

The following advice on good practice is repeated from last year's Report:

> Statements of hypotheses should include a statement of the meaning of the symbol used, for instance:
> "$H_0 : \mu = 0$, $H_1 : \mu > 0$, where μ is the population mean difference in blood pressure measurements."
> In the immediate future the absence of such a definition will not be penalised unless it is explicitly requested, but it is intended that in due course such statements should be made as a matter of course. It is certainly good practice, and it may well also help candidates to focus on the key difference in the roles of population parameter and sample statistic in hypothesis tests, which is at present a widespread weakness.

4732 Probability & Statistics 1

General Comments

Many candidates showed a good understanding of most of the mathematics in this paper. There were some very good scripts, although very few candidates gained full marks. There were several questions that required an interpretation to be given in words, and these were not always answered well. Some answers were too vague, while others contained "parrot-fashion" responses that were not appropriate in the context of the particular question. Even some of the most able candidates did not score full marks on these questions. Those who used algebra in Q5(iii) sometimes showed a weakness in this area. There were no questions that made a significant call upon candidates' knowledge of Pure Mathematics.

This year again it was pleasing to note that very few candidates ignored the instruction on page 1 and rounded their answers to fewer than three significant figures, thereby losing marks. However, in a few cases marks were lost through premature rounding of intermediate answers. Hardly any candidates appeared to run out of time. Most candidates failed to fill in the question numbers on the front page of their answer booklet.

In order to understand more thoroughly the kinds of answers which are acceptable in the examination context, centres should refer to the published mark scheme.

Use of statistical formulae

The formula booklet, MF1, was useful in Qs 1(ii), 4(ii) and 8(i) & (iii) (for formulae) and 3(i) and 7(iv) (for binomial tables). However, a few candidates appeared to be unaware of the existence of MF1. Other candidates tried to use the given formulae, but clearly did not understand how to do so properly (eg Σd^2 was sometimes misinterpreted as $(\Sigma d)^2$ in Q1). A few candidates found Σxp correctly in Q4(ii) but then divided by 3. In Q8(i) a few candidates quoted their own (usually incorrect) formulae for r_s, rather than using the one in MF1. Some thought that, eg, $S_{xy} = \Sigma xy$. In Q8(iii) a few candidates used the less convenient version, $b = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$ from MF1. This meant that they were unable to use the values of S_{xy} etc that they had found in part (i). Also most of those who used this formula did not understand it at all, interpreting it as $\frac{(\Sigma x-\bar{x})(\Sigma y-\bar{y})}{(\Sigma x-\bar{x})^2}$. Some candidates' use of the binomial tables showed that they understood the entries to be individual, rather than cumulative, probabilities. Others did not know how to use the tables to handle, eg, $P(X < 7)$. In Qs 3(i)(a) and (c) some candidates used the binomial formula rather than the tables, although the latter are clearly more appropriate and give fewer opportunity for arithmetical errors. Perhaps some centres advise students to use the formula for all binomial calculations, since it is always applicable whereas the tables can only be used for those values that are included therein. This is bad advice.

It is worth noting yet again that candidates would benefit from direct teaching on the proper use of the formula booklet, particularly in view of the fact that text books give statistical formulae in a huge variety of versions. Much confusion could be avoided if candidates were taught to use the versions given in MF1. They need to understand which formulae are the simplest to use, where they can be found in MF1 and also how to use them.

Comments on Individual Questions

1) (i) This question was well answered. A few candidates answered “approximately” –1 and 0. These were accepted. The most common incorrect answers were (a) 1 or –0.75 and (b) –1 or 0.5. A few candidates gave an answer in words, such as (i) “Perfect negative correlation”. This does not answer the question.

(ii) Many candidates failed to rank the data, but found differences between the original pairs and then substituted into the formula for Spearman’s coefficient. They were apparently unmoved by arriving at an answer of –3.79. Others found the product-moment correlation coefficient. A few of those who did find ranks made arithmetical errors. A few candidates mis-quoted the formula, eg $\frac{1-6\times\Sigma d^2}{4(4^2-1)}$ and $1-\frac{\Sigma d^2}{4(4^2-1)}$. Some interpreted Σd^2 as $(\Sigma d)^2$.

2) (i) This part proved to be surprisingly difficult. Many candidates used combinations, although some of these found only ${}^7C_2/{}^{15}C_5$, while others found the relevant pair of combinations for the numerator, but added them. Division by 5! was common. A few candidates found the number of combinations but not the probability. Some candidates preferred to use the product of probabilities rather than combinations. Some of these stopped at $^7/_{15} \times {}^6/_{14}$. Others found the correct product of five fractions but failed to multiply by ${}^5C_2$. Other examples of incorrect methods were $^2/_7 \times {}^3/_8$, $10\times(^7/_{15})^2 \times (^8/_{15})^3$ and $(^1/_7)^2 + (^1/_8)^3$.

(ii) Many candidates wrote BABAB or similar, which gained a mark. But some then wrote “Therefore 1 order”, while others gave incorrect working such as ${}^5C_2$ or ${}^5P_3$ or just 5!. Others found $^{4!}/_{2!}$ which leads to the correct answer, but gained no marks. Some found 3!, which is correct, but failed to multiply by 2!.

3) (i)(a) This straightforward question elicited all sorts of complicated calculations, many of them incorrect. Many of those who used the binomial tables found $P(X \le 7)$ or $P(X \le 7) - P(X \le 6)$ or $1 - P(X \le 7)$. A minority used the formula, although very few of these succeeded. Some just found $P(X = 6)$ or $1- P(X = 2)$

(i)(b) Use of the formula was generally more successful than use of the tables in this part. A common error by those using the tables was to find $P(X \le 6) - P(X \le 5)$.

(i)(c) Those candidates using the tables commonly had one or both ends of the interval incorrect, using $P(X \le 6)$ or $P(X \le 3)$ or both. Others correctly found $P(X \le 2)$ but went on to calculate $P(X \le 5) - (1 - P(X \le 2)$ or similar. A few candidates used the formula and added the correct terms, although sometimes with one extra term included or one omitted. Many of these candidates lost a mark through premature rounding. Some showed several decimals which they added, but did not show the calculations from which these decimals came. These risked losing marks unnecessarily.

(ii)(a) This part was answered well on the whole. A few candidates omitted the combination or just found $^5/_{12} \times {}^7/_{12}$.

(ii)(b) Some candidates were confused as to the appropriate formula and found $\frac{1}{\frac{5}{12}}$ (mean of geometric distribution), $\frac{(1-\frac{5}{12})}{(\frac{5}{12})^2}$ (variance of geometric distribution) or $\frac{\frac{5}{12}(1-\frac{5}{12})}{10}$ (variance of sample proportion). Searching the formulae book for a formula that might be relevant is mere desperation.

4) (i) Perhaps half the candidates correctly found $^1/_{10} \times {}^1/_{20}$ but failed to mutiply by 2. A few added $^1/_{10} + {}^1/_{20}$ and some of these went on to find $(^1/_{10}+{}^1/_{20})\times(^1/_{10}+{}^1/_{20})$.

(ii) Many candidates tackled this question by "common sense" methods, such as considering 20 turns. These candidates were often successful, but a common error with this approach was to add £0.50 + £5.00 instead of 2×£0.50 + £5.00. Those who used Σxp often reached the correct answer, although a few divided Σxp by 3. Many candidididates found the variance. Some tried to use it; others just ignored their result.

5) (i) This part was often answered correctly, almost always by multiplication of probabilities rather than by combinations. Some candidates found $^1/_{22}\times {}^1/_{21}$. Others cancelled the first probability ($^{12}/_{22}$) correctly to $^6/_{11}$, but then for the second probability gave $^5/_{10}$ instead of $^{11}/_{21}$.

(ii) Many candidates correctly found the route GGB, giving $^7/_{15} \times {}^6/_{14} \times {}^8/_{13}$, but some stopped here. Others combined two or four routes rather than three. Some just found P(GG), while others did P(GG) + P(GBG) + P(BGG).

A small minority tried to use the formula for conditional probability, obtaining expressions such as $\frac{\frac{8}{15}\times\frac{7}{14}\times\frac{7}{15}}{\frac{8}{15}\times\frac{7}{14}\times\frac{7}{15}+\frac{8}{15}\times\frac{8}{15}\times\frac{7}{14}}$. Centres should note that this formula is not required in this module. In fact it only appears in the specification for module S4.

(iii) Trial and Improvement was popular here and was often successful. Some candidates attempted an algebraic approach, but made errors such as $\frac{x}{45}\times\frac{x}{44}=\frac{1}{15}$ or $\frac{x}{45}\times\frac{x}{45}=\frac{1}{15}$ or $\frac{x}{45}\times\frac{x-1}{45}=\frac{1}{15}$. These candidates arrived at answers such as 11.6, which they then rounded to the nearest integer, thus arriving at the correct answer of 12 by an incorrect method. This scored no marks. However, if they went on to verify that $\frac{12}{45}\times\frac{11}{44}=\frac{1}{15}$, they were rewarded.
Some other quite different (incorrect) methods were also seen:
$P(YY) = {}^1/_{15} = {}^1/_3 \times {}^1/_5 \Rightarrow P(Y) = {}^1/_3 \Rightarrow$ No. of $Y = {}^1/_3 \times 45 = 15$
$Y^2 = {}^1/_{15} \Rightarrow Y = \sqrt{1/15} = 0.258.$ $0.258 \times 45 = 11.6 \Rightarrow 12$ Yellow discs

A very rare but ingenious correct method was as follows:
$\frac{{}^xC_2}{{}^{45}C_2}=\frac{1}{15}\Rightarrow {}^xC_2={}^{45}C_2\times\frac{1}{15}=66\Rightarrow x=12$.

6) Centres are reminded that the published mark scheme gives full details of acceptable and unacceptable answers for parts (i)(b) and (ii)(a, b and c).

(i)(a) Errors such as $128 \times {}^{360}/_{120}$ and ${}^{120}/_{128} \times 240$ were common. Many candidates were unable to see the simple piece of arithmetic required.

(i)(b) Many candidates answered correctly, discussing either the total for the year or the fact that pie charts show only proportions. Some gave as their reason that the number of girls might be smaller. This was not accepted unless it was clearly explained that this referred to the same cohort having moved from Y12 to Y13, with a larger proportion of girls than boys having left.

(ii)(a) This was usually answered correctly.

(ii)(b) There were many good answers to this question, although some candidates claimed that the "mean" of the females was higher than that of the males. Some candidates gave two reasons why it could be seen that the females generally did better than the males, but gave no comment about spread.

(ii)(c) Answers which stated that it is easier either to draw or to interpret a box-and-whisker plot, or to compare box-and-whisker plots, were not accepted. Some candidates gave a disadvatage of a box-and-whisker plot that applies equally to a histogram, for example "A box-and-whisker plot does not show all the data" or "A box-and-whisker plot does not show the total number of people." Others gave an incorrect answer such as "A box-and-whisker plot does not show the number of people who got each mark, whereas a histogram does." Others claimed, incorrectly, that a box-and-whisker plot "shows the spread of the data more clearly" or "shows the skew of the data more clearly." Some wrote about a histogram being "more accurate" than a box-and-whisker plot.
The simplest way to score both marks, without writing an essay, was to give an answer such as "A box-and-whisker plot shows the median but does not show frequency densities."

(iii) Most candidates answered this part correctly, with a few making the obvious error of ${}^{1}/_{2}(51 + 59)$.

7) In parts (i) to (iii), some candidates used the formula for binomial probabilities. This was incorrect in almost every case.

(i) This was well answered on the whole, although some candidates found $0.7^4 \times 0.3$. A few found ${}^4C_1 \times 0.3^1 \times 0.7^3$.

(ii) Many candidates realised that 0.7^6 was involved, but either found $1 - 0.7^6$ or 0.3×0.7^6. Others used the long method, $1 - P(X = 1 \text{ or } 2 \text{ or } \ldots \text{ or } 6)$, but some of these omitted a term or added an extra one ($0.7^6 \times 0.3$ or even $0.7^{-1} \times 0.3$!). A very few used ${}^6C_0 \times 0.3^0 \times 0.7^6$ which is correct.

(iii) Common errors were $1 - 0.3\times0.7^9$, $1 - 0.7^{10}$, $(1 - 0.7)^9$ and just 0.7^9. Some candidates used the binomial distribution incorrectly, finding P(1 success in 9 trials). However, a very small number of candidates found P(at least 1 success in 9 trials), which is correct. Some candidates used the very long method of adding the first 9 terms of Geo(0.3).

(iv) Not many candidates realised that a binomial distribution is appropriate here. However, some worked from first principles, evaluating $0.3 \times 0.3 \times 0.7 \times 0.7 \times 0.7$. Many stopped here, although a few multiplied by $^5C_2$, or just by 5.

8) (i) Most candidates substituted correctly in the correct formula, although a few made errors as noted in the introduction above. Premature rounding was a problem in a few cases.

(ii) Most candidates made comments purely based on looking at the data in the table (eg "The values of x go up regularly"). Others gave supposed causation as a reason, eg "y is dependent on x". Some candidates, however, saw the point that the values of x were externally constrained.

(iii) This part was well answered by many candidates, but some made one of the errors mentioned above or made arithmetical slips. In particular the formula for b given in MF1 was, as usual, misunderstood by most who used it.

(iv)(a) The majority of candidates used a correct method but were apparently unconcerned that their estimate of the average number of cars entering the city was 1.4.

(iv)(b) Errors in part (iii), leading to an unrealistic answer to part (iv)(a), prevented some candidates from seeing the point of this question. However, many candidates quoted the fact that the estimate is an extrapolation, and gained a mark.

The second mark, which was rarely scored, was for commenting on the high correlation shown by the answer to part (i). Disappointingly, a significant number of candidates wrote that the estimate was unreliable because "r is negative" or because "r is close to –1".

Answers such as "It is reliable because it follows the trend" were not accepted. Many candidates referred to issues such as rounding errors as reasons for unreliability.

This is an example of a question requiring interpretation in the actual context, rather than simply a standard comment which has been learnt and is reproduced parrot-fashion. Acceptable answers were such as these:
"The value of r shows that the correlation is very good, and 20 is only just outside the original data range, so this estimate may be reliable."
"Although the value of r shows that the correlation is very good, 20 is outside the original data range, so this estimate may not be reliable."

(v) Most candidates stated, incorrectly, that the regression line of x on y should be used, since a value of x is to be estimated from a value of y. This would have been correct if neither variable were controlled. However, in this case x is controlled and so the y on x line must be used even when estimating a value of x.

4733 Probability & Statistics 2

General Comments

This was basically a straightforward paper on which most competent candidates could score high marks. Full marks were not very unusual. However, more than in previous sessions there seemed to be a strong distinction between centres who had taken notice of previous Reports and often, clearly, of Inset courses, and those whose candidates did not seem to have benefited from such information. There also seemed to be a large proportion of under-prepared candidates.

This paper contained, by design, a strong emphasis on hypothesis testing, a topic on which comments have been made in most previous Reports. None of these questions was particularly difficult but many candidates lost a large number of marks unnecessarily.

As announced in previous Reports, examiners expected that the conclusions to hypothesis tests would acknowledge the uncertainty involved. Thus an answer such as [Do not reject H_0 as] "the mean time has not changed" is too assertive and would lose a mark compared with "there is insufficient evidence that the mean time has changed". In fact a majority of candidates who lost marks in this way did not lose them in all three questions, indicating that laziness rather than lack of knowledge was to blame. On the other hand, it is pleasing that most candidates were able to state hypotheses correctly without prompting.

Other mistakes were as common as usual. In some centres a majority of candidates seemed unable to deal with hypothesis tests for discrete random variables and attempted to convert everything to the normal distribution. They lost a large number of marks.

It is distressing to find at this level a number of candidates who think that probabilities work proportionately, for example that $P(R = 3)$ from the distribution Po(19/8) is the same as $P(R = 24)$ from Po(19).

As mentioned often in the past, candidates would be well advised to avoid mentioning the word "random" in answers to verbal questions, unless they refer to "random numbers". Otherwise the only meaning that can be given to the word "random" is "not precisely predictable" and this adds nothing to an answer.

One or two centres have so devotedly followed previous reports as to encourage their candidates to give verbal answers in "inverted commas". This shows excessive reverence.

Comments on Individual Questions

1) (i) To obtain full marks it was necessary to give a reason (such as "only those with strong opinions would volunteer") and to say why this would matter (such as "it would be biased"). The method is not random, but this is irrelevant.

(ii) Those who took trouble over this answer usually gained full marks, but answers that merely said "obtain a list of the candidates, number them and select using random numbers" did not gain full credit. It is important to specify a method of numbering (say, 1 to n), because a random numbering (as stated by some candidates) would then be chosen in a different way. Also the expression "choose numbers randomly" begs the question. A systematic sample was acceptable and would gain full marks if the starting point was chosen randomly. Use of a hat, or equivalent, does not gain full marks; candidates are expected to mention the use of random numbers.

2) (i) Generally well done. Marks were lost by wrong handling of the tails (typically 0.7976 – 0.3085), by spurious continuity corrections, or by rounding 0.8333… to 0.83. Those who made mistakes with the tails would probably have benefited from drawing a diagram.

(ii) Both a reason and a conclusion were required. Most appreciated that the distribution would be asymmetric (skewed) and therefore the normal distribution was inapplicable. Some said that the information was consistent with the normal distribution, and this received partial credit, but it is most unlikely that there would be a corresponding number of salaries the same distance below the mean.

3) Most knew generally what to do here. The standard mistakes were: failing to realise that 37.05 had to be multiplied by 40/39; using a one-tail test; and omitting the $\sqrt{40}$ factor. It is a serious mistake to state hypotheses using the sample mean 26.44. Some candidates did not state the conclusion in context, limiting themselves to something like "there is insufficient evidence that the mean has increased", which did not gain the final mark.

4) (i) This was generally well done, apart from the handling of the inequality sign. Some attempted to substitute 4.08 into the expression $(53 - 50)/(\sigma/\sqrt{10})$; this "trial and improvement" method needs 4.075 to be checked as well if the required inequality is to be properly established.

(ii) For some this application of a geometric distribution was very easy; others could not see how to start, or tried to use a binomial distribution. Many tried to recalculate the probability of a Type I error.

5) (i) Year on year the question on continuous random variables is often found the easiest, but this entirely straightforward one was perhaps less well done than usual. Some forgot to consider the mean at all. Those who found it by integration often did so wrongly, making sign errors when substituting the limits. Some automatically wrote down a formula for the variance in terms of two integrals, which wasted time and occasionally lost a mark when the integral for the mean was not shown squared.

5) (ii)(a) Some candidates ignored the restriction $-3 \le x < 3$ and drew a parabola that went beneath the axes. Such candidates seem to think that the "0 otherwise" part of the definition has no purpose.

(b) Better candidates realised what mattered here: the areas under the two graphs had to be equal (to 1). A good answer was "both graphs must have area 1, *W* has greater width so it must have less height."

(c) A very simple request, it might be thought. But many said "the SD is the same as the graphs are the same shape" or "*X* varies more than *W*" (focussing on the wrong axis).

6) (a) This was probably the easiest question and many scored full marks on all three parts. However, some rounded $19 \div 8$ down to 2, either because 2.375 was not in the tables or because they thought the mean number had to be an integer. Others misunderstood the question and left the two probabilities as separate answers. Some "scaled up" and found the probability of 24 or 32 using Po(19), which is a gross misunderstanding of the way that probability works.

(b) (i) Candidates could give either verbal answers such as "*n* is large, *p* is small" or inequalities. But if inequalities were quoted they had to be the ones given in the specification, namely $n > 50$ and $np < 5$.

(ii) Apart from those candidates who did not know the probability of getting a double 6, most answered this confidently and accurately. Those who used the exact distribution B(108, 1/36) were not following the instructions in the question.

7) (i) This sort of question appears in most papers. Some candidates hit it with a blunt instrument and wrote things like "dropped catches must happen randomly, singly, independently and at constant rate", which just about got both marks on this occasion, even though the "randomly" and "singly" conditions are completely irrelevant here (how can more than one catch happen at the same time?) and it should be "constant *average* rate". The context must, as always, be mentioned, and it is wrong to say that "the probability of a dropped catch must stay the same" (which sounds like a condition for a binomial distribution rather than a Poisson).

7) (ii), (iii) This question was very poorly answered, and indeed examiners often had difficulty in following what candidates were trying to do. First, as this is a discrete distribution, it is not true that the probability of a Type I error equals the significance level of 0.05. Candidates needed to appreciate that they were looking at the upper tail of the distribution and that the relevant distribution was Po(10), rather than Po(2). They also had to avoid using a normal approximation, which is not valid. Some weaker candidates attempted to use Po(2) and scale 14 down to 2.8, which is not only completely wrong but impossible on a Poisson.

Many had considerable difficulty with the details of a test involving the right-hand tail, both here and in Q8(i). It is necessary either to find the probability of $R \geq 14$ and compare it with 0.05, or to find the critical region in the form $R \geq a$ where $P(\geq a) < 0.05$. Those who compared probabilities from tables with 0.95 usually left unclear the status of the particular value of x they were considering, and lost several marks. Thus the answer "$P(\leq 14) = 0.9165 < 0.95$ so do not reject H_0" is <u>wrong</u>; we need the probability of a result as bad as the test statistic <u>or worse</u>, not as bad as the test statistic <u>or better</u>. In this type of question candidates are strongly recommended to convert probabilities compared with 0.95 (and their associated regions) into the complementary probabilities for comparison with 0.05, and the complementary regions – for example, "$P(\leq 15) = 0.9513$, so $P(\geq 16) = 0.0487$".

8) (i) Many of the problems identified in Q7 reappeared here, though this question was slightly better done, partly no doubt because there was no problem of changing the mean. However, the hypotheses were often poorly stated, for instance "$H_0 : \mu = 0.4$" etc. The many weaker candidates who used the distribution B(12, 0.75) would seem to have no grasp of the concept of a hypothesis test.

(ii) This was by contrast generally well done. Most used the distribution N(160, 96). There were some mistakes with signs, and as usual a spurious factor of $\sqrt{400}$ was often seen, but the most common error was a small one: omission of the continuity correction.

4734 Probability & Statistics 3

General Comments

There were many excellent scripts seen and most candidates were able to answer a significant number of questions accurately and confidently.

There were rather more verbal explanations and comments required than last year and these are not always satisfactory. These usually should be in the context of the question, and terse statements – *normal*, *equal variances* – will not gain any credit.

Marks may be lost by lack of accuracy. The rubric requires that answers which are not exact should be given to 3 significant figures. However, a test statistic is not an answer and if this is compared with a critical value correct to 4 significant figures then it is inconsistent to round the test statistic.

The message about not expressing test conclusions in an assertive manner seems to have been received by most candidates.

Comments on Individual Questions

1) This was found to be an easy start and only in part (iii) was there a problem with the condition.

2) A significant number of candidates did not understand what was required by the critical region of a test. Those that did obtained a good score even if the used z rather than t.

3) This question turned out to be more difficult than intended, and although most knew the required null hypothesis, many miscalculated the significance level even if they had found that $v = 4$.

4) This was attacked joyfully and there was a large number of 12 marks seen. In part (i), some thought that $P(X < 2)$ meant $P(X \leq 1)$, but this was rare, and most could negotiate the required integrals and in all parts. There were several ways of answering Part (ii) but using $\int_0^m \frac{4}{3}x^3 \mathrm{d}x = \frac{1}{2}$ and finding $m > 1$ was not one of them unless a suitable explanation was given.

   Part (iii) caused no difficulty and the infinity obtained in the expression for the variance in part (iv) seemed well-understood.

5) (i) Although most candidates could obtain the mean correctly full credit could be obtained if the two Poisson approximations were justified and the addition justified. The former was sometimes seen but the latter very rarely.

   (ii) This was often found correctly but $P(> 4)$ was often interpreted as $1 - P(\leq 3)$.

6) (i) The procedure for finding the CI was well-known and was usually well done.

(ii) Those who had seen similar had little difficulty with this part.

7) (i) That the samples were random was given in the question, so neither this nor independence should have been mentioned. The standard requirements are for normal distributions with a common variance, which should have been given in context, but often were not.

(ii) The fact that the two sample variances were unequal was usually thought to imply that there could not be a common variance. Candidates should be aware that sampling errors can cause this amount of difference.

(iii) Despite part (i), many used a z-test, which did gain some credit. Those using the t-test could usually carry it out correctly.

8) (i) This was an easy 3 marks, not always earned.

(ii) As in Q7(ii) it was often thought that the different sample mean and variance meant that a Poisson distribution was inappropriate.

(iii) Both marks for this were earned by most candidates.

(iv) The two points of concern were the combining of the two last cells and the calculation of the required value of v. Very many managed the former, but very few the latter.

(v) It was hoped that the change in expected values and the change in calculating the value of v would be mentioned. The former was often seen, but not the latter.

4735 Probability & Statistics 4

General Comments

There was a small entry and most of the candidates were well-prepared for the paper. However, some were unable to cope with the trickier demands.

The best answered questions were Qs 2 and 4, where the procedures for non-parametric tests were well known and many high marks were achieved. Q6 proved to be discriminating, partly through theory and partly through technique.

Comments on Individual Questions

1) (i) Many knew the principles but could not always apply them convincingly.

(ii) The demonstrations were often unconvincing and $x \neq 0$ was not always seen.

(iii) This was very well done, with only a few making a sign error in the expansion of $P(A \cup B \cup C)$.

2) (i) The condition that the underlying distribution needs to be symmetric seems to be well-known and it was usually seen that the data did not support this.

(ii) The sign test was mostly carried out well. Candidates sometimes lost marks by referring to median rather than <u>population</u> median when they expressed their hypotheses in words. The accepted letter for this is m, which most use.

3) The general response to the question was good, although the conditional probability in part (iii) was not always found correctly.

In part (iv), many candidates were aware that X and Y being independent implies a zero covariance, but that the converse is not true. Some demonstrated that X and Y were not independent.

4) (i) The question clearly required comment on the implication of the sample variances, but should not have been too assertive. Several candidates referred to the normality of the data.

(ii) The procedure for the test is given fully on the formula list and so deviations were penalised. However, it was generally well-answered.

5) (i) Most candidates could obtain and solve the relevant equations. Some just found one equation and substituted $a = \frac{1}{8}$ to find b. This scored very little.

(ii) The required variance formula is given and the differentiation simple, so this scored high marks.

(iii) This was intended to be discriminating, but it was often answered successfully.

6) (i) $S > s$ requires all of Y_1, Y_2 and Y_3 to be $> s$ and this was recognised by the best

candidates, but many tried to guess from the given answer. The p.d.f. of S requires $F(s) = 1 - (a/s)^9$, but many used $(a/s)^9$ and this led to a negative $f(s)$ which was usually accepted by the candidate.

(ii) Although the principles were often known there were sometimes errors in the integration.

(iii) The requirements for greater efficiency were usually known and there were good scores for this part.

(iv) Both $a \leq 4.5$ and $t_2 > 4.5$ were required for the last 2 marks, but usually only 1 was earned.

7) The first three parts were often well done with only algebraic errors seen in part (ii).

In part (iv), only two candidates realised that it was necessary to show how H(–1) and H(1) gave the required probability. Most candidates merely obtained the value of the given probability without showing that it was the probability of obtaining an even number.

4736 Decision Mathematics 1

General Comments

The presentation of candidates' answers was significantly poorer than in previous sessions; some candidates' work was so untidy that it was impossible to read. Many candidates had written in coloured inks other than black, and some continue to use highlighter pens and correcting fluid. Far more candidates than usual struggled with very basic mathematics, and some were not really able to access the paper because they had not learnt the terminology involved in Decision Mathematics or did not understand when and how to apply the standard algorithms.
There was little evidence to suggest that candidates did not have enough time to complete the paper.

Comments on Individual Questions

1) (i) Most candidates realised that the largest number would be correctly positioned at the end of the list after the first pass. Some candidates answered the question for the specific case example that followed part (i), rather than answering about the general result.

(ii) Many candidates answered this correctly and counted the comparisons and swaps used, a few only gave tally marks for the counts. The candidates who did not succeed in the first pass through bubble sort fell into three groups: the largest group were those who did not understand the difference between a pass and a comparison, and so only carried out the first comparison and its swap; the next largest group were those who tried to use shuttle sort instead of bubble sort; the smallest group were those who did not know seem to know what to do at all.

(iii) The majority of candidates identified that after the second pass the list would be sorted. Some thought that because the list was now sorted no further passes were needed and some did not appreciate that with bubble sort, unlike shuttle sort, it can terminate early if there is a pass in which no swaps occur.

(iv) The majority of candidates were able to calculate the time as being 7.2 seconds.

2) (i) Most, but certainly not all, candidates were able to draw an appropriate graph with the required properties. The explanations offered were often poor, although the majority of candidates managed to describe how their graph was not simple, and hence not simply connected. There was considerable confusion between arcs and vertices. The word 'loop' was used by some candidates to mean 'tour' or 'cycle' rather than a single arc that connects a vertex to itself.

(ii) Several candidates drew an alternative answer to part (i), or sometimes repeated their graph from part (i), and then tried to claim that it was non-Eulerian. The graph could only be non-Eulerian by failing to be connected.

3) (i) It was disappointing to find large numbers of candidates who could not give the equations of the boundary lines, let alone deal with the inequalities.

(ii) Many candidates ignored the instruction to calculate the coordinates of the vertices of the feasible region and just read them, rather inaccurately, from the diagram.

A substantial number of candidates claimed that the vertex on the x-axis was the point (0, 6).

(iii) Candidates who attempted to calculate the value of the objective at their vertices usually scored well in this part, although several did not actually state the values of x and y at the optimal point or did not explicitly state the corresponding value of the objective, usually leaving one or the other to be deduced from previous working.

(iv) Several candidates just left this part out. Those who attempted it often tried to compare the expressions for the new objective at the vertices with the previous maximum value, rather than comparing the expressions with each other. Most candidates who attempted this part used the expressions for the objective, although a few compared the gradient of the objective with the gradients of the boundaries.

4) (i) For some candidates, this was the only question on which they scored any marks. Very few candidates were not able to make a reasonable attempt at carrying out Dijkstra's algorithm. Some wrote down all the temporary label values, rather than only updating when the calculated value is an improvement on the current value.

Most candidates were able to find the shortest route from A to K and its length.

(ii) The question asked candidates to 'use your answer to part (i)', several chose instead to start again, using their diagram from part (i) to do working for part (ii), and in the process making it impossible to identify which values belonged to the answer to part (i).

(iii) Some candidates did not appreciate that the stem referring to the locked door applied to both part (ii) and part (iii), even though the wording of part (iii) repeated the fact that CJ was not to be used.

Most candidates identified that the next shortest route goes from G to J using $GHKJ$ and that the two routes would be of equal length if FJ were increased by 2 metres. Several candidates then treated the lengths as only being able to be complete numbers of metres and gave an answer of 3, rather than the correct answer of 'more than 2'.

Some candidates tried to use $GHKFJ$ as an alternative route, forgetting that changes to FJ would affect the length of this route too.

5) (i) The majority of candidates were not able to complete the table correctly, often assuming that if there were x buses to Easton from company A and y buses to Easton from company B then there must be z from company C, or that the number of buses to Weston was the same as the number to Easton from each company.

The candidates who were able to complete the table were usually able to also show how the total cost had been calculated. Several students gave no answer at all to the first three parts of this question.

(ii) There were very few attempts at this part, even though both the algebraic expression for the total cost and the actual maximum amount available were both given in the question.

(iii) There were very few attempts at this part, the impression given was that candidates thought that they could get marks more easily by skipping the setting up of the LP problem and going straight to the Simplex algorithm.

(iv) Several candidates rewrote the problem using slack variables and then actually gave the initial Simplex tableau in their answer to part (v).

Some candidates did not understand why the initial objective becomes $P - x - 2y = 0$, and so either gave x and y positive values in the objective row or gave P a negative value as well.

Some candidates either omitted or misused the slack variable columns, negative entries for the slack variables (slack variables being subtracted ie surplus variables) sometimes being seen.

(v) A significantly larger number of candidates than usual were not able to proceed past the setting up of the initial Simplex tableau. A common error was candidates trying to pivot on the 1 from the y column, resulting in negative entries in the column corresponding to the right-hand side of the equations, and candidates then continuing without apparently having registered that their tableau was now invalid.

Some candidates did not perform valid pivoting operations, resulting in columns that contained two 0s with a value other than 1, and sometimes columns consisting entirely of 0s.

Candidates' attempts at the Simplex algorithm were noticeably more muddled than in previous sessions.

6) (a) (i) Many candidates thought that this was a travelling salesperson problem, despite the repetition of the information about covering every arc.

(ii) Several candidates identified the odd nodes and found the length of the shortest route, although often units were omitted. Many candidates did not show the three pairings and their totals, $AB + CD = 450$, $AC + BD = 350$, $AD + BC = 550$; and some just tried to write out a route that they hoped fulfilled the criteria.

6) (iii) The shortest of AB, AD and BD is $AD = 250$m, so this is the only arc that now needs to be repeated, giving a route of length 3200m that starts at C and ends at B.

(b) (i) It was disappointing to see how many candidates were not able to apply the nearest neighbour method accurately, many made an error right at the start by not choosing DG as the first arc, and often one of the nodes was revisited before the route terminated at D.

The explanation of why the nearest neighbour method fails if you start at A hinges on the fact that the route reaches G and then you are stuck. The majority of candidates either left G out completely or else referred to travelling $ACDGC$ and then being stuck.

(ii) It was common to see node A included in the tree and list of arcs, even though the question asked candidates to construct a minimum spanning tree on the reduced network formed by deleting node A and all the arcs that are directly joined to node A.

The question asked candidates to use Prim's algorithm and to list the order in which nodes were added to their tree. Several candidates just drew the tree and gave its weight or explicitly used Kruskal's algorithm, in both cases incurring a loss of easy marks.

(iii) The candidates who appreciated that part (b)(ii) had asked for a minimum spanning tree on a reduced network usually used their answer to find a lower bound for the travelling salesperson problem. Some candidates also identified their nearest neighbour tour from part (b)(i) as giving an upper bound.

Although some candidates referred to the context in their answers to this final part, there was no evidence that the context had in any way prevented them from understanding which mathematical processes were being tested.

4737 Decision Mathematics 2

General Comments

Most candidates achieved good marks on this paper. The candidates were, in general, well prepared and were able to show what they knew.

Comments on Individual Questions

1) (a)(i) Almost all the candidates were able to draw a correctly labelled bipartite graph.

(ii) Nearly all the candidates drew a second bipartite graph showing the incomplete matching. A few candidates included their alternating path on this diagram and it was not always possible to identify which arcs belonged to the incomplete matching and which were part of the alternating path for part (iii).

(iii) Some candidates did not show their alternating path clearly, a listing is preferred to trying to show the path on a diagram. Some candidates did not give the shortest alternating path, this usually resulted in them obtaining the matching needed for part (iv). It is preferable to list the allocation rather than show it on a diagram.

(iv) Most candidates found the matching with the required property.

(b)(i) There were some confused explanations to this part. Some candidates said that the Hungarian algorithm solves minimisation problems, rather than that it finds the minimum cost matching or minimum cost allocation, although most recognised in some way that the subtraction was needed to convert a maximising problem into a minimising problem. Most candidates knew that column X is a dummy column and that it is needed to make the number of rows and columns match. A few candidates rather strangely described column X as a dummy row.

(ii) Many candidates were able to apply the Hungarian algorithm accurately and efficiently. Some candidates reduced rows first, rather than columns as asked in the question, and some candidates achieved the correct reduced cost matrix but then tried to augment further in various ways. Even when candidates had made numerical slips they were usually able to write down the appropriate allocation. Some candidates did not go back to the original values to find the total score.

A small number of candidates applied the Hungarian algorithm to the original values and found the matching using the least suitable students.

(iii) Generally done well. Most candidates wrote down the correct 4 by 4 matrix and reduced its rows and columns. A few candidates only reduced one way and then tried to augment. Any reasonable attempt, even with small errors, usually led to the correct matching.

Some candidates only showed the matching by indicating the entries in the table. Centres should encourage candidates to list their final matching.

2) (i) Most candidates were able to say that Collete scores –2, or that she loses 2. In a zero-sum games the pay-off matrix shows the winnings for the player on rows, and the winnings for the player on columns are the negatives of the entries.

(ii) Many candidates identified that *W* is dominated by *Y* and explained why by comparing appropriate values. Some candidates did only referred to 'the values for *W* being greater than the values for *Y*', rather than specifically identifying that they are greater than the corresponding values for *Y*. Several candidates gave good written explanations, describing what happened with Collete's strategies *W* and *Y* when Rowena played each of her strategies.

A few candidates claimed that *W* dominated *Y*, rather than the other way round. This seemed to be due to misunderstanding that Collete's scores are the negatives of the entries in the table, rather than poor English.

(iii) Some candidates showed no working at all, and some found the row maximin and then the column maximin as well. Even the candidates who calculate the row minima and column maxima correctly sometimes then chose *Q* rather than *P* or *Z* rather than *Y*. The most successful candidates were those who stated the row maximin and column minimax before identifying the play safe strategies as *P* and *Y*.

(iv) Most candidates were able to find appropriate expressions for the probabilities, although there were some slips with minus signs.

(v) Some candidates made sign errors in transferring their equations to the graph, and a few did not plot the correct lines. There were fewer instances of graphs with excessive scales, most candidates had a p-axis that only went from 0 to 1. A few candidates stopped their lines short of $p = 1$; the lines need to extend across the full width of the graph.

Many candidates knew that the boundary of the feasible region is defined by the lowest line at each value of p and could solve the appropriate pair of equations to find $p = 0.5$ with an expected pay-off of –0.5, in the worst case.

(vi) Several candidates knew that 4 needed to be added throughout the table. Some correctly said that this was to make all the values non-negative, although some confused this with making all the values positive. Candidates then needed to explicitly identify the expressions $4p_1 + 3p_2 + 6p_3$ with column *Y* and $7p_1 + 2p_3$ with column *Z*. Finally, because we are solving a maximin problem, m must be less than or equal to each of these values for every valid set of values for p_1, p_2, p_3. It was not sufficient to only describe the behaviour at the maximin solution.

(vii) Most candidates stated that $p_1 + p_2 + p_3 \leq 1$ is needed because the sum of the probabilities cannot exceed 1, or because the probabilities sum to 1. Whilst this is true, the reason for the inequality is that without it the Simplex algorithm (as in module 4736) would never pivot on a row that allowed the value to increase.

(viii) Most candidates were able to state the optimal value as $p_3 = \frac{3}{7}$, although some said $p_3 \leq \frac{3}{7}$. The majority of the candidates then calculated the corresponding value of m (or M) as $4\frac{6}{7}$ but only a few went on to refer back to the original situation and realise that the minimum expected pay-off for Rowena was $\frac{6}{7}$.

3) (i) Many candidates thought that the arc *GE* was flowing from *T* to *S* instead of from *S* to *T* and either omitted it from their calculation or included it as a negative value.

(ii) Several candidates seemed to have difficulty explaining the problem here. Often they would give answers like 'only 2 into it', rather than the more correct answer that the maximum that can flow into *GE* is 2 litres per second (along *DG*). A few candidates just described what 'not full to capacity' means and some gave vertex *E*.

(iii) Although superficially the diagram might suggest that 10 litres per second can flow through vertex *E*, we already know that at most 2 litres per second can flow in *GE* which reduces the maximum flow through *E* to 8 litres per second.

Some candidates then used a labelling procedure instead of marking the flow in each arc on the arrows given.

(iv) The use of the labelling procedure to indicate this initial solution was generally done quite well.

Some candidates reversed the arrows, showing the excess capacity in the backwards direction and the potential backflow in the forwards direction. The question had explained that the excess capacities should be in the original flow directions and the potential backflows in the opposite directions.

Several candidates did not label the arcs that were not included in the initial solution.

(v) Most candidates identified that an additional 2 litres per second could flow along the given route and many were able to augment the labels appropriately.

(vi) Some candidates tried to split the flow into two parts, contrary to what the question had requested. Those who identified the flow augmenting route as *SBCET* were usually also able to augment the labels appropriately.

(vii) Even candidates who had made a mess of the labelling procedure were usually able to show the flow resulting from parts (iv), (v) and (vi).

(viii) Several candidates identified an appropriate cut, although not always in the form requested in the question, a common wrong answer was to use the cut through arcs *FT*, *ET* and *GT* which has a capacity of 5 + 6 + 6 = 17 litres per second although only 11 litres per second is flowing through it.

4) (a) Many candidates were able to complete the dynamic programming tabulation correctly. Some omitted the suboptimal maximum values and rather more did not transfer the values correctly from one stage to the next.

A large number of candidates with correct dynamic programming tabulations were, nevertheless, not able to trace back to find the route of the longest path. The action label corresponding to each suboptimal choice is the state label for the previous stage. For example, stage 0 state 0 has a suboptimal maximum of 13 corresponding to the action 1, this means that we reach this point from state 1 of stage 1. Even the candidates who identified (0; 0) – (1; 1) – (2; 2) often omitted the 'end' vertex, which had to be (3; 0) because of the action labels for the states in stage 2.

(b)(i) Some candidates drew neat activity networks, others used an excessive number of dummy activities to ensure that the precedences were correct. This is strictly wrong and is likely to be penalised in future papers.

(ii) Most candidates were able to carry out a correct forward pass on their activity network. There were some errors on the backwards pass, usually through choosing the largest value instead of the smallest value when there was a choice. Some candidates recorded the earliest possible start time and latest possible finish time for each activity instead of the early event times and late event times.

(iii) Candidates whose original network contained several dummy activities were often not able to unpick the precedences here, and those who did frequently omitted the directions. All arcs in an activity network should be directed, but it is especially important for the dummy activities.

Grade Thresholds

Advanced GCE Mathematics (3890-2, 7890-2)
June 2008 Examination Series

Unit Threshold Marks

7892		Maximum Mark	A	B	C	D	E	U
4721	Raw	72	63	55	47	39	32	0
	UMS	100	80	70	60	50	40	0
4722	Raw	72	56	49	42	35	29	0
	UMS	100	80	70	60	50	40	0
4723	Raw	72	55	47	40	33	26	0
	UMS	100	80	70	60	50	40	0
4724	Raw	72	56	49	43	37	31	0
	UMS	100	80	70	60	50	40	0
4725	Raw	72	57	49	41	34	27	0
	UMS	100	80	70	60	50	40	0
4726	Raw	72	49	43	37	31	25	0
	UMS	100	80	70	60	50	40	0
4727	Raw	72	54	47	41	35	29	0
	UMS	100	80	70	60	50	40	0
4728	Raw	72	61	53	45	37	29	0
	UMS	100	80	70	60	50	40	0
4729	Raw	72	56	47	38	29	20	0
	UMS	100	80	70	60	50	40	0
4730	Raw	72	56	47	38	29	21	0
	UMS	100	80	70	60	50	40	0
4731	Raw	72	59	50	42	34	26	0
	UMS	100	80	70	60	50	40	0
4732	Raw	72	60	52	45	38	31	0
	UMS	100	80	70	60	50	40	0
4733	Raw	72	56	48	41	34	27	0
	UMS	100	80	70	60	50	40	0
4734	Raw	72	55	48	41	34	28	0
	UMS	100	80	70	60	50	40	0
4735	Raw	72	56	49	42	35	28	0
	UMS	100	80	70	60	50	40	0
4736	Raw	72	53	46	39	32	26	0
	UMS	100	80	70	60	50	40	0
4737	Raw	72	61	54	47	40	34	0
	UMS	100	80	70	60	50	40	0

Specification Aggregation Results

Overall threshold marks in UMS (ie after conversion of raw marks to uniform marks)

	Maximum Mark	A	B	C	D	E	U
3890	300	240	210	180	150	120	0
3891	300	240	210	180	150	120	0
3892	300	240	210	180	150	120	0
7890	600	480	420	360	300	240	0
7891	600	480	420	360	300	240	0
7892	600	480	420	360	300	240	0

The cumulative percentage of candidates awarded each grade was as follows:

	A	B	C	D	E	U	Total Number of Candidates
3890	33.3	50.4	65.4	77.0	86.6	100	14679
3891	100	100	100	100	100	100	1
3892	57.2	76.7	88.2	94.1	97.6	100	1647
7890	45.4	67.3	82.4	92.1	97.8	100	10512
7891	33.3	66.7	100	100	100	100	6
7892	56.5	77.9	90.0	95.4	98.2	100	1660

For a description of how UMS marks are calculated see:
http://www.ocr.org.uk/learners/ums_results.html

Statistics are correct at the time of publication.

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