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Exemplar Candidate Work

MATHEMATICS B (MEI)

H640
For first teaching in 2017

H640/01 – Summer 2019 examination series

Version 1

Contents

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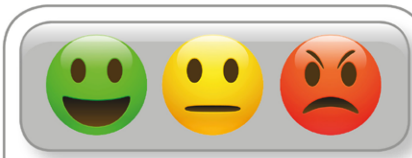

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Introduction

These exemplar answers have been chosen from the summer 2019 examination series.

OCR is open to a wide variety of approaches and all answers are considered on their merits. These exemplars, therefore, should not be seen as the only way to answer questions but do illustrate how the mark scheme has been applied.

Please always refer to the specification https://www.ocr.org.uk/qualifications/as-and-a-level/further-mathematics-b-mei-h635-h645-from-2017/ for full details of the assessment for this qualification. These exemplar answers should also be read in conjunction with the sample assessment materials and the June 2019 Examiners' report or Report to Centres available from Interchange https://interchange.ocr.org.uk/Home.mvc/Index

The question paper, mark scheme and any resource booklet(s) will be available on the OCR website from summer 2020. Until then, they are available on OCR Interchange (school exams officers will have a login for this and are able to set up teachers with specific logins – see the following link for further information http://www.ocr.org.uk/administration/support-and-tools/interchange/managing-user-accounts/).

It is important to note that approaches to question setting and marking will remain consistent. At the same time OCR reviews all its qualifications annually and may make small adjustments to improve the performance of its assessments. We will let you know of any substantive changes.

Question 1

1 In this question you must show detailed reasoning.

Show that $\int_4^9 (2x+\sqrt{x})\mathrm{d}x = \frac{233}{3}$. **[3]**

Exemplar 1 2 marks

1

$\int (2x+\sqrt{x})\ dx$

$= \int 2x + x^{1/2}\ dx$

$= x^2 + \frac{x^{3/2}}{3/2} + c$

$= \left[x^2 + \frac{2x^{3/2}}{3} \cancel{+c}\right]_4^9$

$= \left(81 + \frac{18^{3/2}}{3}\right) - \left(16 + \frac{8^{3/2}}{3}\right)$

$= \frac{233}{3}$, as required.

Examiner commentary

In this exemplar, the indefinite integral is really clear for the first M1, but the use of limits is not accurate. The second M1 awarded in this case, with the candidate given the benefit of the doubt for their intention to substitute the values $x = 9$ and $x = 4$, but the given answer does not follow from their working. Perhaps an intermediate step, showing the substitution into $\frac{2(x)^{\frac{3}{2}}}{3}$ would have avoided the error.

Question 2

2 Show that the line which passes through the points $(2, -4)$ and $(-1, 5)$ does not intersect the line $3x+y=10$. **[3]**

Exemplar 1 — 2 marks

2

$\frac{y_2-y_1}{x_2-x_1} = \frac{5--4}{-1-2}$ Midpoint: $\left(\frac{2+-1}{2}, \frac{-4+5}{2}\right)$

grad $= -3$ $= \left(\frac{1}{2}, \frac{1}{2}\right)$

$y-y_1 = m(x-x_1)$

$y-\frac{1}{2} = -3\left(x-\frac{1}{2}\right)$ $3x+y=10$

$y=-3x+2$ $y=10-3x$

$-3x+2 \neq -3x+10$

$2 \neq 10$

Examiner commentary

This exemplar shows an attempt at the most common method used by candidates. The two method marks were given, but the final mark was not given as there was no conclusion using the evidence that they had found. A comment as simple as "so the lines don't cross" would have been enough.

Question 3 (a)

3 The function $f(x)$ is given by $f(x) = (1 - ax)^{-3}$, where a is a non-zero constant. In the binomial expansion of $f(x)$, the coefficients of x and x^2 are equal.

(a) Find the value of a. **[3]**

Exemplar 1 — 1 mark

3(a)

$1 + nx + \frac{n(n-1)}{2!}x^2 + \dots$

$1 + (-ax)(-3) + \frac{-3(-3-1)}{2}(-ax)^2$

$1 + 3ax - 6a^2x^2 + \dots$

$3ax = -6a^2x^2$

$3 = -6ax$

$-\frac{1}{2} = ax$

$a = -\frac{1}{2}$

Examiner commentary

This exemplar shows both common errors:

- not handling the $(-ax)^2$ correctly.
- equating terms of the binomial expansion rather than the coefficients.

Question 4

4 Fig. 4 shows a uniform beam of mass 4 kg and length 2.4 m resting on two supports P and Q. P is at one end of the beam and Q is 0.3 m from the other end.
Determine whether a person of mass 50 kg can tip the beam by standing on it. **[3]**

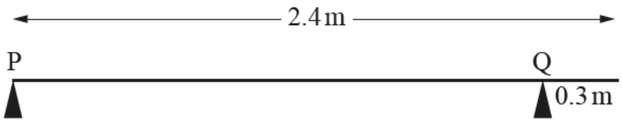
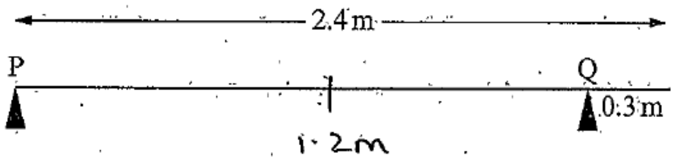


Fig. 4

Exemplar 1 — 2 marks

4



$50 \times 2.1\text{m}$

$50g \times 2.1 = 1029\text{N}$

$50g \times 0.3 = 147\text{N}$

$0.9 \times 4 \times 9.8 = 35.28$

Yes the person can tip the beam

Examiner commentary

This question makes use of the defined command word 'Determine' which indicates that justification should be given for any results found. This candidate has found all the evidence to support their argument but does not clearly argue the case. Neither of the two moments is labelled, leaving the examiner to guess what they are, and there is no comparison of the clockwise and anticlockwise moments about Q found. It would only have taken a few words of explanation to upgrade this to full credit.

Question 6 (a) and 6 (b)

6 (a) Prove that $\dfrac{\sin\theta}{1-\cos\theta} - \dfrac{1}{\sin\theta} = \cot\theta$. **[4]**

(b) Hence find the exact roots of the equation $\dfrac{\sin\theta}{1-\cos\theta} - \dfrac{1}{\sin\theta} = 3\tan\theta$ in the interval $0 \leqslant \theta \leqslant \pi$. **[3]**

Exemplar 1

2, 0 marks

6(a)

$\dfrac{\sin\theta}{1-\cos\theta} - \dfrac{1}{\sin\theta} = \cancel{\dfrac{\sin\theta}{\sqrt{\sin^2\theta}} - \dfrac{1}{\sin\theta}}$ $\sin 2\theta = 2\cos\theta\sin\theta$

$\cancel{= \dfrac{\sin^2\theta - \sqrt{\sin^2\theta}}{\sin\theta\sqrt{\sin^2\theta}}}$ $= \cancel{\dfrac{1-\cos^2\theta - 1+\cos\theta}{\sin^2\theta}}$

$\cancel{= \dfrac{-\cos^2\theta+\cos\theta}{\sin^2\theta}}$ $= \dfrac{-\cos^2\theta}{\sin^2\theta} \cancel{+ \dfrac{\cos\theta}{\sin^2\theta}}$

$= \dfrac{\sin^2\theta - (1-\cos\theta)}{\sin\theta(1-\cos\theta)} = \dfrac{1-\cos^2\theta - 1+\cos\theta}{\sin\theta(1-\cos\theta)} = \dfrac{\cos\theta - \cos^2\theta}{\sin\theta(1-\cos\theta)}$

$= \dfrac{\cos\theta - \cos^2\theta}{\sin\theta - \sin\theta\cos\theta} = \dfrac{\cos}{\sin\theta - \sin\theta\cos\theta} - \dfrac{\cos^2\theta}{\sin\theta - \sin\theta\cos\theta}$

$= \dfrac{\cos\theta}{\sin\theta - \frac{1}{2}\sin 2\theta}$

$\dfrac{\sin\theta}{\sin\theta} - \dfrac{1}{\sin\theta} = \dfrac{1-1}{\sin\theta}$

$\dfrac{\sin\theta - 1}{\sin\theta}$

6(b)

$\dfrac{\sin\theta}{1-\cos\theta} - \dfrac{1}{\sin\theta} = 3\left(\dfrac{1-\cos\theta}{\sin\theta} - \sin\theta\right)$

$\cancel{\dfrac{\sin\theta}{1-\cos\theta} - \dfrac{1}{\sin\theta}} = \sin\theta - 1 = 3\left(1-\cos\theta - \sin^2\theta\right)$

$\sin\theta - 1 = 3 - 3\cos\theta - 3\sin^2\theta$

$3\sin^2\theta + \sin\theta - 4 = -3\cos\theta$

$= -3(1-\sin\theta)$

$= -3 + 3\sin\theta$

$= 3\sin^2\theta - 2\sin\theta - 1 = 0$

$\sin\theta = 1, -1, \frac{1}{3}$

$\therefore \theta = \frac{1}{2}\pi, \frac{3}{2}\pi$ ~~5.94~~ (3sf), 2.80 (3sf), 0.3398 (4sf), 3.481 (4sf)

Examiner commentary

This exemplar shows the development of ideas the candidate had and the crossing out near the beginning was very helpful to indicate to the examiner where a restart has been made. The first method mark for combining fractions and the B mark for *using* the trig identity were very clear. The subsequent work gets no nearer to a complete proof but shows how the intended method is often much simpler than candidates might think. This candidate may have used more time on part (a) than they should have done.

Part (**b**) shows the candidate not taking on board the "*Hence*…." in the question and the time taken here was not profitable.

Question 7

7 The velocity $v\,\text{m}\,\text{s}^{-1}$ of a particle at time t s is given by

$$v = 0.5t(7-t).$$

Determine whether the **speed** of the particle is increasing or decreasing when $t = 8$. **[4]**

Exemplar 1 — 3 marks

7 $V = 0.5t(7-t)$ $\quad t=8$

~~ans~~ $v = \frac{7}{2}t - 0.5t^2$

differennate to get accelleration

$a = \frac{7}{2} - t$

$a = \frac{7}{2} - 8 = -\frac{9}{2}\ \text{ms}^{-2}$

negative accelleration therefore the particle is slowing down when $t = 8$

$t=8$ $V = 0.5 \times 8(7-8) = -4\text{ms}^{-1}$

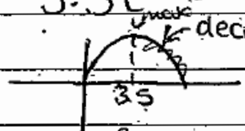
Examiner commentary

The first exemplar shows the most common answer – arguing from a negative value of acceleration that the speed is decreasing (it would be a valid argument had velocity been required). This candidate also finds the negative value for velocity but is still not able to complete the argument correctly.

Exemplar 2 — 2 marks

7 $0.5(8)(7-8)$ $\quad V = 3.5t - 0.5t^2$

$t=8$ $0.5(8)(-1) = -4$



~~$t=7$ $0.5(7)(7)$~~

~~$t=6$~~ $=$

$V = 3.5t - 0.5t^2$

$\frac{dv}{dt} = 3.5 - t$

$3.5 - t = 0$

$3.5 = t$

Examiner commentary

The second exemplar demonstrates the value of a sketch graph here – had the graph shown the velocity, a complete argument could have been made without evaluating the acceleration.

Question 8 (b)

8 An arithmetic series has first term 9300 and 10th term 3900.

(b) The sum of the first n terms is denoted by S. Find the greatest value of S as n varies. **[4]**

Exemplar 1 4 marks

8(b)

(1) 9300, (2) 8700, (3) 8100, (4) 7500, (5) 6900, (6) 6300, (7) 5700,
(8) 5100, (9) 4500, (10) 3900, (11) 3300 (12) 2700, (13) 2100, (14) 1500, (15) 900,
(16) 300, (17) -300

greatest value of S is at S_{16}.

$\cancel{S_{16} = \frac{n}{2}(2a(n-1)d)}$

$\cancel{S_{16} = \frac{16}{2}(2)(9300)(15)(-600)}$

~~S_{16} = [illegible]~~

after U_{16} it starts to decrease

$S_n = \frac{n}{2}(2a+(n-1)d)$

$\cancel{S_{16} = 8(2(5000)}$

$S_{16} = 8(2(9300)+(15)(-600))$

$= 8(18600 - 9000)$

$= 8(9600)$

$S_{16} = 76800$

Examiner commentary

This exemplar shows a mix-and-match method with a calculator generated list to find the last positive term and an algebraic method to evaluate the sum. To get full marks as this did, it is really important to show all the supporting evidence – so a_{16} and a_{17} both clearly labelled, some reasoning given and the correct final answer.

Methods which exploit the ability of the calculator to create lists need to be very clearly argued to be sure of getting full marks. We would hope to see why the biggest value found is the biggest value globally with some understanding of the structure of the mathematics.

Question 9 (b)

9 A cannonball is fired from a point on horizontal ground at $100\,\text{m}\,\text{s}^{-1}$ at an angle of 25° above the horizontal. Ignoring air resistance, calculate

(b) the range of the cannonball. **[4]**

Exemplar 1

4 marks

9(b)

t @ max height.

V

$V = U + at$

$0 = 100\sin(25) + (-9.8)t$

$\frac{-100\sin(25)}{-9.8} = t$ @ max $= t\,\frac{1}{2}$ Journey $= 4.3$

total time $= 8.6$ Seconds

H

$S =$

$U = 100\cos(25)$ $\quad S = Ut + \frac{1}{2}at^2$

$V = 100\cos(25)$ $\quad S = Ut$

$A = 0$ $\quad S = 100\cos(25) \times 8.6$

$t = 8.6$ $\quad S = 779.4$ m

Examiner commentary

This candidate chooses a three-stage method for this question – finding the time to travel to the top, double it to find the total time of flight and lastly to find the range. This is inefficient as a method, so valuable time can be lost by candidates. This candidate rounds the time values to 2 significant figures, so the final answer does not appear to match the given answer, and often accuracy marks are lost in cases like this. As it happens, this answer and the given answer both round to 780 to 2 significant figures, so this candidate is fortunate that their solution is worth full marks.

Question 10 (a) and 10 (b)

10 **(a)** Express $7\cos x - 2\sin x$ in the form $R\cos(x+\alpha)$ where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving the exact value of R and the value of α correct to 3 significant figures. **[4]**

(b) Give details of a sequence of two transformations which maps the curve $y = \sec x$ onto the curve $y = \dfrac{1}{7\cos x - 2\sin x}$. **[3]**

Exemplar 1 — 4, 1 marks

10(a)

$7\cos x - 2\sin x$ $\quad$ ~~$R(\cos(x+\alpha))$~~ $R(\cos(x+\alpha$

~~$\cos x\cos\alpha - \sin\alpha\sin x$~~

$\cos x\cos\alpha - \sin x\sin\alpha$

$\cos\alpha = 7 \quad \sin\alpha = 2$

$R = \sqrt{2^2 + 7^2} = \sqrt{53}$

$\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$

$\tan\alpha = \dfrac{2}{7}$

~~Arctan(2~~ $\quad \text{Arctan}\left(\dfrac{2}{7}\right) = 0.278299\ldots$

$\alpha = 0.278$ (3 sig fig)

$7\cos x - 2\sin x = \sqrt{53}\cos(x + 0.278)$

10(b)

$y = \sec x = \dfrac{1}{\cos x}$

$y = \dfrac{1}{7\cos x - 2\sin x}$

Stretch scale factor $\frac{1}{7}$ parallel to the y axis

translation $\begin{pmatrix} 0 \\ -\dfrac{1}{2\sin x} \end{pmatrix}$

Examiner commentary

Part (**a**) here shows the fully correct answer given from not quite correct working. The mark scheme on this occasion allowed us to award full marks without seeing the equations explicitly, so a mark was not taken off in this case.

The candidate has made no explicit link between part (**a**) and part (**b**) which may have helped secure more marks. Candidates should expect the parts to be potentially linked in some way.

Question 11 (b)

11 In this question, the unit vector **i** is horizontal and the unit vector **j** is vertically upwards.

A particle of mass 0.8 kg moves under the action of its weight and two forces given by $(k\mathbf{i}+5\mathbf{j})$ N and $(4\mathbf{i}+3\mathbf{j})$ N. The acceleration of the particle is vertically upwards.

(b) Find the velocity of the particle 10 seconds later. **[4]**

Exemplar 1 — 2 marks

11(b)

~~u = 4i + 7j~~ $u = 4i + 7j$

$u = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$ $t = 10$ $v = ?$

$F = ma$

$\begin{pmatrix} -4i \\ 5 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \end{pmatrix} = 0.8a$

$\begin{pmatrix} 0 \\ 8 \end{pmatrix} = 0.8a$

$a = \begin{pmatrix} 0 \\ 10 \end{pmatrix} ms^{-2}$

$v = u + at$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \end{pmatrix} + \begin{pmatrix} 0 \\ 10 \end{pmatrix} t$ $t = 10$

$y = 7 + 10 \times 10$

$y = 107$

$x = 4$

$v = \begin{pmatrix} 4 \\ 107 \end{pmatrix} ms^{-1}$

$v = (4i + 107j)\ ms^{-1}$

Examiner commentary

These contrasting exemplars show two possible approaches to motion in two dimensions and both methods could have been awarded full marks. This first exemplar uses vector notation well throughout. However, the candidate omits weight altogether. This was awarded both method marks for Newton's second law and using the vector form of the *suvat* equations.

Exemplar 2 1 mark

11(b)

↑	→
s :	s :
u : 7	u : 4
v :	v :
a : ~~9.8~~ 9.8	a : 0
T : 10	T : 10
v = u + at	v = u + at
v = 7 + (9.8 × 10	v = 4
v = 105	
(4i + 105j)	

Examiner commentary

In this second exemplar the candidate has worked in the vertical direction only and then combines their answer back into a vector answer. However, the "upwards" weight was awarded B0 and no attempt at Newton's second law. However the vertical motion was subsequently used to give a vector answer so awarded the second of the method marks.

Question 13 (a), 13 (b) and 13 (c)

13 A 15 kg box is suspended in the air by a rope which makes an angle of 30° with the vertical. The box is held in place by a string which is horizontal.

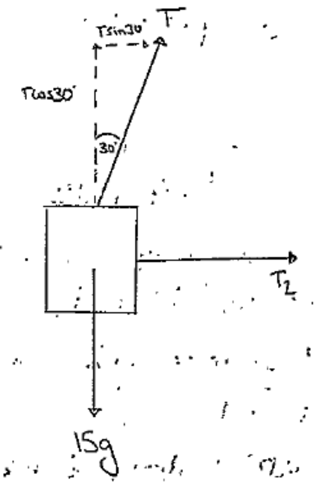
(a) Draw a diagram showing the forces acting on the box. **[1]**

(b) Calculate the tension in the rope. **[2]**

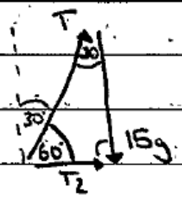
(c) Calculate the tension in the string. **[2]**

Exemplar 1 — 0, 2, 2 marks

13(a)



13(b)



$\sin 60° = \dfrac{15g}{T}$

$T = \dfrac{15g}{\sin 60}$

$\cos 30 = \dfrac{15g}{T}$

$T = 98\sqrt{3}$

$T = 169.7\text{ N (1dp)}$

13(c)

$\tan 60° = \dfrac{15g}{T_2}$

$T_2 = \dfrac{15g}{\tan 60}$

$= 49\sqrt{3}$

$= 84.9\text{N (3sf)}$

Examiner commentary

For part (**a**), this exemplar shows great practice in adding components to a force diagram in such a way that it is clear they are not additional forces. Making the tensions distinct is also good practice and necessary here. Unfortunately the direction of the horizontal force makes no sense in an equilibrium question, so this diagram was not awarded full marks.

The candidate has not been penalised a second time for this initial mistake, so although their equation in part (**c**) does not follow from the diagram they have scored full marks in both part (**b**) and part (**c**).

Question 14 (a), 14 (b) and 14 (c)

14 Fig. 14 shows a circle with centre O and radius r cm. The chord AB is such that angle AOB $= x$ radians. The area of the shaded segment formed by AB is 5% of the area of the circle.

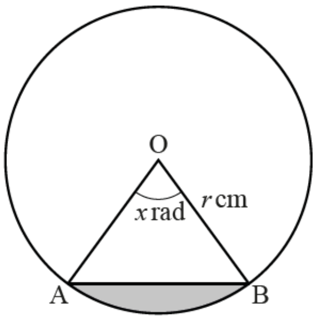


Fig. 14

(a) Show that $x - \sin x - \frac{1}{10}\pi = 0$. **[4]**

The Newton-Raphson method is to be used to find x.

(b) Write down the iterative formula to be used for the equation in part **(a)**. **[1]**

(c) Use three iterations of the Newton-Raphson method with $x_0 = 1.2$ to find the value of x to a suitable degree of accuracy. **[3]**

Exemplar 1 — 4, 0, 2 marks

14(a)

$\frac{1}{2} r^2(x) - [\frac{1}{2} r^2(\sin x)] = (\frac{1}{20}) \pi r^2$

$r^2(x) - r^2(\sin x) = \frac{1}{10} \pi r^2$

$\therefore x - \sin x - \frac{1}{10} \pi = 0$

Area of sector AOB is

$= \frac{1}{2} r^2 x$

Area of triangle AOB is

$= \frac{1}{2} (r)(r) \sin x$

$= \frac{1}{2} r^2 \sin x$

14(b)

$x - \sin x - \frac{1}{10}\pi = 0$

$x = \sin x + \frac{1}{10}\pi$

$\therefore x_{n+1} = \sin(x_n) + \frac{1}{10}\pi$

14(c)

$x_0 = 1.2$

$x_1 = \sin(1.2) + \frac{1}{10}\pi \approx 1.24679\ldots$

$x_2 = \sin(1.2468) + \frac{1}{10}\pi \approx 1.26193\ldots$

$x_3 = \sin(1.26193) + \frac{1}{10}\pi \approx 1.26684\ldots$

$x_4 \approx 1.268319\ldots$

$x_5 \approx 1.268760\ldots$

$x_6 \approx 1.268892\ldots$

$x_7 \approx 1.268931\ldots$

$x_8 \approx 1.268942\ldots$

$x_9 \approx 1.268946\ldots$

$x_{10} \approx 1.268947\ldots$

$\therefore x \approx 1.2689$

Examiner commentary

This exemplar shows a candidate looking back at their succinct proof for part (**a**) and adding further detail underneath – the arrow is a nice touch, indicating the order in which the pieces are to be read. It shows the value of explaining where the terms have come from and the lines of simplifying required to obtain the given answer in the correct form.

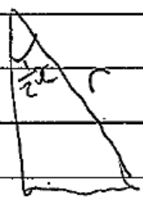
Part (**b**) and (**c**) are a good solution for a different question – had the question asked for any numerical method to find the value of *x*, this would have been a model solution. However, half marks only were awarded for the two parts taken together as the question explicitly asks for the Newton-Raphson method to be used. Maybe the candidate should have noticed that the limit of the sequence was not clear after only three iterations required in the question.

Exemplar 2 — 0, 1, 1 marks

14(a)

shaded area $= 0.05 \times \frac{1}{2}r^2$

$= 0.025r^2$



$\frac{1}{2}$ base $= r\sin\frac{1}{2}x$

height $= r\cos\frac{1}{2}x$

~~since~~

14(b)

$x_{n+1} = x_n - \dfrac{(x - \sin x - \frac{\pi}{10})}{1 - \cos x}$

14(c)

$x_0 = 1.2$

$\cancel{x_1 = 1.729986358}$
$\cancel{x_2 = 2.77198751}$
$\cancel{x_3 = 4.82598371}$

$\cancel{x_1 = -3942.7}$
$\cancel{x_2 = 83651.97975}$
$\cancel{x_3 = 135524.61829}$

$x_1 = -3942.907597$
$x_2 = 80365.63966$
$x_3 = -5611.055869$

Examiner commentary

This exemplar highlights that candidates can access partial credit on later parts even if they are unsure of how to approach the earlier parts. Here the candidate is not deterred by difficulties in part (**a**) from attempting parts (**b**) and (**c**). Part (**b**) was awarded full marks but it is not fully correct as some of the subscripts are missing from the formula – note that the mark would not have been awarded for the right hand side on its own. The only error in part (**c**) is not using radians as the calculator setting so the sequence does not converge. The context of the question and the use of calculus embedded in the Newton-Raphson method both require that *x* be in radians. It is even written in the question. Notice this paper requires the calculator be reset several times as the Mechanics questions need the angle to be in degrees and the pure questions are generally in radians.

Question 15 (a)

15 A model for the motion of a small object falling through a thick fluid can be expressed using the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 9.8 - kv,$$

where $v\,\mathrm{m\,s^{-1}}$ is the velocity after t s and k is a positive constant.

(a) Given that $v = 0$ when $t = 0$, solve the differential equation to find v in terms of t and k. **[7]**

Exemplar 1 — 1 mark

15(a)

$$\frac{dv}{dt} = 9.8 - kv$$

$$\int \frac{1}{9.8 - kv}\, dv = \int 1\, dt$$

$$t = \ln(9.8 - kv) + c$$

Examiner commentary

This question was attempted to relative degrees of success. In part (**a**) a range of errors were made in the rearranging of the differential equation and the subsequent attempts to integrate. In this exemplar the first M1A1 were awarded for the initial rearranging but then A0 for the integration.

Exemplar 2 — 1 mark

15(a)

$$\frac{dv}{dt} = 9.8 - kv$$

$$\cancel{\int (1/(9.8 - kv))}$$

$$\int \frac{1}{(9.8 - kv)}\, dv = \int 1\, dt$$

$$\int \left(\frac{1}{9.8} - \frac{1}{kv}\right) dv = t + c$$

Examiner commentary

In this exemplar, the initial M1 has been awarded for the correct separation of variables, but then the candidate has come unstuck dealing with the fraction.

Exemplar 3 1 mark

15(a)

$$\frac{dv}{dt} = 9.8 - kv$$

$$\int -\frac{1}{kv}\,dv = \int 9.8\,dt$$

Examiner commentary

Another common error, as shown in this exemplar, was careless manipulation of algebraic expressions when attempting to separate the variables. This candidate scored the M1 for their attempt.

Exemplar 4 1 mark

15(a)

$$\frac{dv}{dt} = 9.8 - kv$$

$$\int kv\,dv = \int 9.8\,dt$$

Examiner commentary

Again poor manipulation of the algebra means that this exemplar scored M1 for the attempt to separate variables but no further credit could be awarded.

Exemplar 5 0 marks

15(a)

$$\frac{dv}{at} = 9.8 - kv$$

$$V = \int (9.8 - kv)\,\cancel{dx}\,dt$$

Examiner commentary

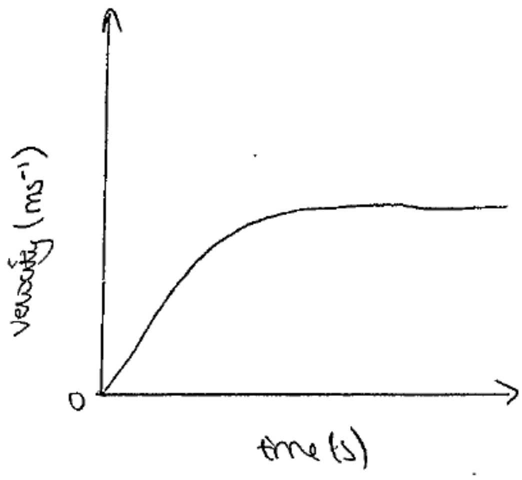
This final example shows a common response; no attempt has been made to separate the variables and no credit awarded.

Question 15 (b)

(b) Sketch the graph of v against t. **[2]**

Exemplar 1 — 1 mark

15(b)



Examiner commentary

This is an example of a recovery from a candidate who obtained no marks in part (**a**) but used the description in the question to draw a velocity-time graph with the correct shape. Had they realised the horizontal asymptote occurred when $\frac{dv}{dt} = 0$ they could have obtained full marks from the differential equation without a solution in part (**a**)

Question 16 (a)

16 A particle of mass 2 kg slides down a plane inclined at 20° to the horizontal. The particle has an initial velocity of 1.4 m s^{-1} down the plane. Two models for the particle's motion are proposed.

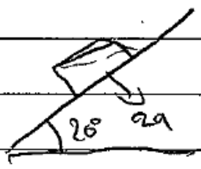
In model A the plane is taken to be smooth.

(a) Calculate the time that model A predicts for the particle to slide the first 0.7 m. **[5]**

Exemplar 1 — 2 marks

16(a)

$u = 1.4$



$s = 0.7$ $\qquad s = ut + \frac{1}{2}at^2$

$u = 1.4$ $\qquad 0.7 = 1.4t + \frac{1}{2} \times 2 \times 9.81\, t^2$

v $\qquad 0.7 = 1.4t + 9.81t^2$

$a = 2 \times 9.81$ $\qquad 9.81t^2 + 1.4t - 0.7.$

$t =$

$\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, $\dfrac{-1.4 + \sqrt{1.4^2 - 4 \times 9.81 \times -0.7}}{2 \times 9.81} = 0.205$ seconds.

$a = 9.81$

$b = 1.4$

$c = -0.7$

Examiner commentary

This candidate has got really muddled about weight – the diagram is not correct but was not required in this question. They have clearly stated what they think the acceleration of the particle is down the slope – M0A0 was given for finding acceleration. However, there is an otherwise correct quadratic equation using their acceleration to find the time for the subsequent M1A1A0. There is an expectation that candidates will make use of the available technology and solve quadratic equations with their calculator, even in questions that are looking for clear mathematical argument (see 'command words' in specification) the emphasis is on how the specific equation to be solved has been manipulated so that the standard algorithm can be applied and a clear rationale for why roots are discarded rather than on the manual application of the algorithm.

Question 16 (b)

(b) Explain why model A is likely to underestimate the time taken. **[1]**

Exemplar 1 — 1 mark

16(b) there would likely not be a constant acceleration and would accelerate slower, and so take longer.

Examiner commentary

This first exemplar was given the benefit of doubt; the incorrect statement was assumed to be lax wording and the subsequent correct reason gained the credit.

Exemplar 2 — 1 mark

16(b) friction is not taken into account so the speed is predicted to be higher

Examiner commentary

This second exemplar gains credit for identifying a correct reason, expressed the other way around.

Exemplar 3 — 0 marks

16(b) The plane is assumed to be smooth, assuming acceleration stays constant. This is unlikely to be true, the ~~acceleration~~ plane won't be smooth, causing friction, increasing time.

Examiner commentary

Most candidates realised that it was ignoring friction that causes a problem. However, it was necessary to say why an additional force would change the time, and the mark was only given where a reference was made to acceleration or velocity. In this third exemplar no reason is given for the increased time.

Exemplar 4

0 marks

16(b) Because there is likely to be a friction force opposing motion.

Examiner commentary

This fourth exemplar was quite borderline but it did not mention either velocity or acceleration and so did not gain credit.

Question 16 (c) and 16 (d)

In model B the plane is taken to be rough, with a constant coefficient of friction between the particle and the plane.

(c) Calculate the acceleration of the particle predicted by model B given that it takes 0.8 s to slide the first 0.7 m. **[2]**

(d) Find the coefficient of friction predicted by model B, giving your answer correct to 3 significant figures. **[6]**

Exemplar 1

1, 4 marks

16(c)

$s = ut + \frac{1}{2}at^2$

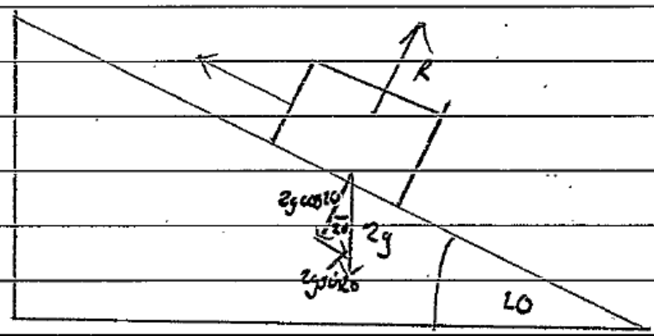
$0.7 = 1.4 \times 0.8 + \frac{1}{2} \times a \times 0.8^2$

$0.7 = 1.12 + 0.32a$

$-0.42 = 0.32a$

$a = 1.3 \text{ ms}^{-2}$

16(d)



$F = \mu R$

$\mu = \frac{F}{R}$

$F = m \times a$

$F = 2 \times 1.3$

$F = 2.6 \text{ N}$ Net force

$\therefore R(\searrow)\ 2g\sin 20 - F_r = 2.6$ $\qquad R(\nearrow)\ R - 2g\cos 20 = 0$

$-F_r = 2.6 - 2g\sin 20$ $\qquad R = 2g\cos 20$

$-F_r = -4.1 \text{ N}$

$F_r = 4.1 \text{ N}$

$\mu = \frac{4.1}{2g\cos 20}$

$\mu = 0.221$

Examiner commentary

This exemplar highlights a common sign issue that trips up mechanics candidates. They have not followed their correct working to state a negative value for acceleration – some candidates even got a correct negative value then explained it away and did not use it in part (**d**). There is a great example of a force diagram here, together with the components of the weight. This candidate has used their value of acceleration in part (**d**) so their otherwise correct equation for the frictional force was awarded the method mark. The equations for R and μ are correct but there is a rounding error in the value of μ .

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