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Exemplar Candidate Work

MATHEMATICS B (MEI)

H640
For first teaching in 2017

H640/03 Summer 2019 examination series

Version 1

Contents

Introduction

These exemplar answers have been chosen from the summer 2019 examination series.

OCR is open to a wide variety of approaches and all answers are considered on their merits. These exemplars, therefore, should not be seen as the only way to answer questions but do illustrate how the mark scheme has been applied.

Please always refer to the specification https://www.ocr.org.uk/qualifications/as-and-a-level/further-mathematics-b-mei-h635-h645-from-2017/ for full details of the assessment for this qualification. These exemplar answers should also be read in conjunction with the sample assessment materials and the June 2019 Examiners' report or Report to Centres available from Interchange https://interchange.ocr.org.uk/Home.mvc/Index

The question paper, mark scheme and any resource booklet(s) will be available on the OCR website from summer 2020. Until then, they are available on OCR Interchange (school exams officers will have a login for this and are able to set up teachers with specific logins – see the following link for further information http://www.ocr.org.uk/administration/support-and-tools/interchange/managing-user-accounts/).

It is important to note that approaches to question setting and marking will remain consistent. At the same time OCR reviews all its qualifications annually and may make small adjustments to improve the performance of its assessments. We will let you know of any substantive changes.

Question 2 (a)

2 **(a)** Find the transformation which maps the curve $y = x^2$ to the curve $y = x^2 + 8x - 7$. **[4]**

Exemplar 1 — 4 marks

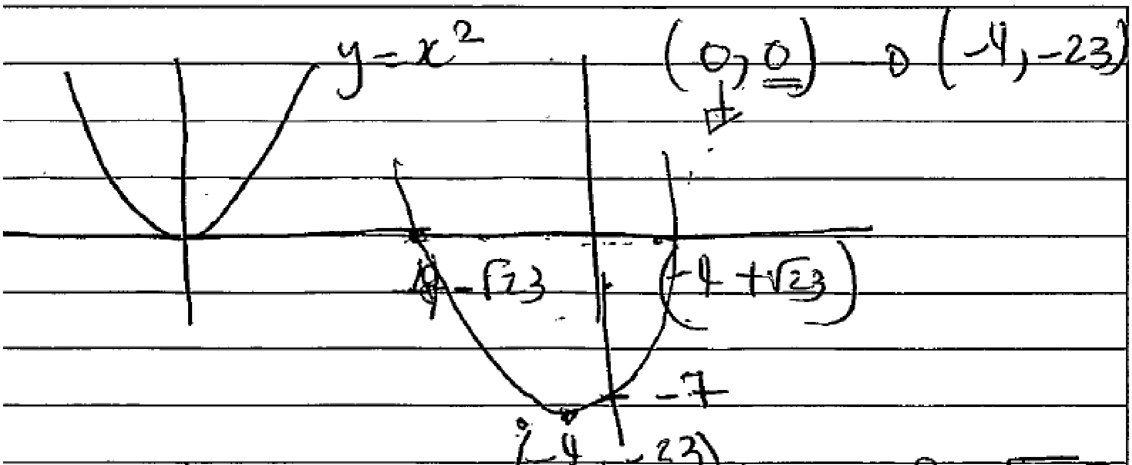
$x^2 + 8x - 7 = (x+4)^2 - 23$

$y = x^2$ is translated through the vector $\begin{pmatrix} -4 \\ -23 \end{pmatrix}$

Examiner commentary

This example shows a concise solution. The completed square form has been found either by inspection, or using the quadratic solve function of the calculator. The candidate has then clearly linked this form to the $y = x^2$ parabola to give the required vector form of the translation.

Exemplar 2 — 1 mark



Translation by the vector $\begin{bmatrix} 8 - \sqrt{23} \\ -23 \end{bmatrix}$

Examiner commentary

This candidate appears to have used their graphical calculator and the quadratic solve function to produce two nice sketches. However, they have not been able to capitalise on this initial work, only gaining the B1 for identifying that the transformation is a vector translation.

Question 4

4 In this question you must show detailed reasoning.

Show that $\frac{1}{\sqrt{10}+\sqrt{11}}+\frac{1}{\sqrt{11}+\sqrt{12}}+\frac{1}{\sqrt{12}+\sqrt{13}}=\frac{3}{\sqrt{10}+\sqrt{13}}$. **[3]**

Exemplar 1 — 3 marks

$$\frac{1}{\sqrt{10}+\sqrt{11}}+\frac{1}{\sqrt{11}+\sqrt{12}}+\frac{1}{\sqrt{12}+\sqrt{13}}.$$

$$\frac{1}{\sqrt{10}+\sqrt{11}}\times\frac{\sqrt{10}-\sqrt{11}}{\sqrt{10}-\sqrt{11}}=-\sqrt{10}+\sqrt{11}$$

$$\frac{1}{\sqrt{11}+\sqrt{12}}\times\frac{\sqrt{11}-\sqrt{12}}{\sqrt{11}-\sqrt{12}}=-\sqrt{11}+\sqrt{12}\ldots$$

and

$$\frac{1}{\sqrt{12}+\sqrt{13}}=-\sqrt{12}+\sqrt{13}.$$

$$=-\sqrt{10}+\sqrt{13}$$

$$-\sqrt{10}+\sqrt{13}\times\frac{\sqrt{10}+\sqrt{13}}{\sqrt{10}+\sqrt{13}}=\frac{-10+13}{\sqrt{10}+\sqrt{13}}=\frac{3}{\sqrt{10}+\sqrt{13}}$$

As required.

Examiner commentary

This example shows a common approach, successfully actioned. The candidate has clearly shown that rationalising the denominator for each fraction will simplify the LHS to $-\sqrt{10}+\sqrt{13}$. It is worth noting that the question was presented so that the LHS of the identity was the sensible side to start with. This candidate has then gone on to 'unrationalise' their result to obtain the RHS.

Exemplar 2

3 marks

$$\frac{1}{\sqrt{10}+\sqrt{11}} \times \frac{\sqrt{10}-\sqrt{11}}{\sqrt{10}-\sqrt{11}} = \frac{-\sqrt{11}+\sqrt{10}}{10-11}$$

$$= \sqrt{11}-\sqrt{10}$$

$$\frac{1}{\sqrt{11}+\sqrt{12}} \times \frac{\sqrt{11}-\sqrt{12}}{\sqrt{11}-\sqrt{12}} = \frac{\sqrt{11}-\sqrt{12}}{11-12} = \sqrt{12}-\sqrt{11}$$

$$\frac{1}{\sqrt{12}+\sqrt{13}} \times \frac{\sqrt{12}-\sqrt{13}}{\sqrt{12}-\sqrt{13}} = \frac{\sqrt{12}-\sqrt{13}}{12-13} = \sqrt{13}-\sqrt{12}$$

$$\Rightarrow \cancel{\sqrt{11}}-\sqrt{10}+\cancel{\sqrt{12}}-\cancel{\sqrt{11}}+\sqrt{13}-\cancel{\sqrt{12}}$$

$$= \sqrt{13}-\sqrt{10}$$

TARGET:

$$\frac{3}{\sqrt{10}+\sqrt{13}} \times \frac{\sqrt{10}-\sqrt{13}}{\sqrt{10}-\sqrt{13}} = \frac{3(\sqrt{10}-\sqrt{13})}{10-13}$$

$$= \sqrt{13}-\sqrt{10}$$

$$\text{So} = \frac{3}{\sqrt{10}+\sqrt{13}}$$

Examiner commentary

This example shows the alternate, less traditional approach. Having found the simplified LHS of $-\sqrt{10}+\sqrt{13}$, they have then done a similar rationalisation of the RHS to show that both sides are the same.

Question 5

5 A student's attempt to prove by contradiction that there is no largest prime number is shown below.

> If there is a largest prime, list all the primes.
> Multiply all the primes and add 1.
> The new number is not divisible by any of the primes in the list and so it must be a new prime.

The proof is incorrect and incomplete.
Write a correct version of the proof. **[3]**

Exemplar 1 — 2 marks

If prime numbers are finite, there must be a large list :

$P_1, P_2, P_3, P_4 \ldots P_{n-1}$

Let Q be the product of the 'finite' prime numbers.

Q + 1

↳ Since this must be divisible by itself and 1 only, it must be added to the list. However, it is not, therefore there cannot be a finite number of primes, or a largest prime number. Since the list provides inaccurately.

so $P_1, P_2, P_3, P_4 + Q+1$

Examiner commentary

In this example, the first 5 words earn the M1 and they also earn the A1 for explaining the rest of the process and the conclusion. They would not have needed much more to get the final A mark as well, just adding 'this is a contradiction' after 'therefore'. It is vital to a proof by contradiction that this provides part of their conclusion.

It is worth noting that the June 2018 paper also contained a proof by contradiction question with a less familiar context. However the structure of the mark scheme was identical to this year.

Exemplar 2 — 1 mark

If there is a largest prime, list all the primes. Multiply all the primes and add 1. The new number is

Examiner commentary

The first 6 words are sufficient to earn the method mark. This is a good example for candidates to see that writing something, not necessarily a full solution may gain you marks.

Exemplar 3

0 marks

list all the primes for example $3 \times 5 \times 7 = 105$

Then you add an even constant to the number and then it is a new prime number. fore example $105 + 2 = 107$

This is because it will only divisible by itself and 1

Examiner commentary

Proof by contradiction can be problematic for lower ability candidates; examiners saw many scripts with no response or, like this one, with some specific calculations that may or may not lead to a prime number.

Questions 6 (a) and (b)

6 A circle has centre C (10, 4). The x-axis is a tangent to the circle, as shown in Fig. 6.

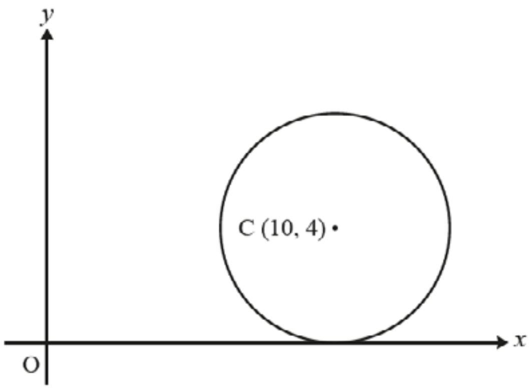


Fig. 6

(a) Find the equation of the circle. **[2]**

(b) Show that the line $y = x$ is not a tangent to the circle. **[4]**

Exemplar 1 — 0, 3 marks

6(a)

$(x-a)^2+(y-b)^2=r^2$

$(x-10)^2+(y-4)^2=r^2$

$(x-10)(x-10)+(y-4)(y-4)=r^2$

$x^2-20x+100+y^2-8y+16=r^2$

$x^2+y^2-20x-8y+116=r^2$

$x^2+y^2-20x-8y=-116$

6(b)

sub in $y=x \rightarrow$ $x^2+y^2-20x-8y=-116$

$(y)^2+y^2-20(y)-8y=-116$

$2y^2-20y-8y=-116$

$2y^2-28y=-116$

$2y^2-28y+116=0$ $\div 2$

$2\div$ $y^2-14y+58=0$

$y^2-14y+58=0$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{14\pm\sqrt{(-14)^2-4\times1\times58}}{2}$

$x=\frac{14\pm\sqrt{-36}}{2}$

* no real roots so $y=x$ does not touch the circle at any point so $y=x$ is not a tangent to the circle * Therefor

Can't square root negative number so no real roots *

Examiner commentary

This example was unusual in not scoring in part 6a but there was no indication that they thought the radius was 4.

This candidate's attempt at part (b) is notable though as, while no part of their working is accurate, they have used their wrong answer from part (a) and done the 3 steps of method demanded by the mark scheme. Unless stated otherwise, an E mark is dependent on everything before it being correct. It is also interesting that this candidate provided the conclusion that was missing from many higher ability candidates of 'so $y = x$ is not a tangent to the circle'.

Question 7 (a)

7 **In this question you must show detailed reasoning.**

(a) Express $\ln 3 \times \ln 9 \times \ln 27$ in terms of $\ln 3$. **[2]**

Exemplar 1 — 1 mark

$\ln 3 \times \ln 9 \times \ln 27$

$\ln 3 \times \ln (3)^2 \times \ln (3)^3$

$\ln 3 \times 2\ln 3 \times 3\ln 3 = 6\ln 3$

Examiner commentary

As this was a '**detailed reasoning**' question there was no marks for going straight to $21n \times 1n3$ without the working step showing $1n3^2$ and $1n3^3$. As this candidate did this they earned the first B1 however they lost the final B1 as they did not simplify correctly to $6(1n3)^3$. Those who did not earn the first mark could still earn the second.

Question 7 (b)

(b) Hence show that $\ln 3 \times \ln 9 \times \ln 27 > 6$. **[2]**

Exemplar 1

2 marks

$\cancel{6\ln(3)^3}$ $\ln(3) \times \ln(9) \times \ln(27) > 6$

$6(\ln(3))^3 > 6$

$(\ln(3))^3 > 1$

$\ln(3) > 1$

$3 > e^1$

$3 > e$

Examiner commentary

While it maybe feels more natural to start with $e < 3$, this solution shows the logically equivalent 'backwards' solution is just as acceptable.

Exemplar 2

0 marks

$6(\ln 3)^3 > 6$

$(\ln 3)^3 > 1$

$\ln 3 > 1$

$\ln 3 = 1.0986$ (4dp)

so yes $\ln 3 > 1$ as required.

Examiner commentary

This was the most common style of answer seen to this part with little reasoning and conclusions drawn mainly from calculators. It is worth restating that '**In this question you must show detailed reasoning**' generally means 'put your calculator down and show some maths'. Although they have made a good start, the jump from $\ln 3 > 1$ to a decimal approximation was not sufficient; the steps $e^{\ln 3} > e^1$ and $3 > e$ would have helped secure credit as seen in the previous example.

Question 9

9 In this question you must show detailed reasoning.

The curve $xy + y^2 = 8$ is shown in Fig. 9.

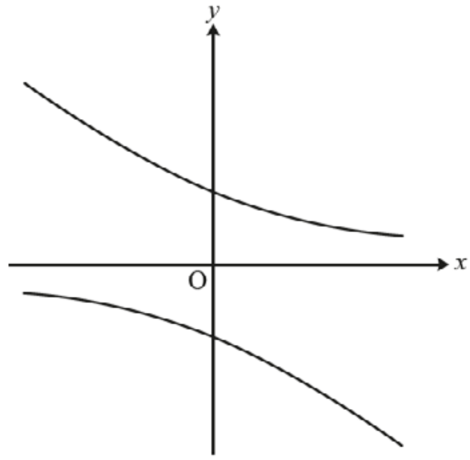


Fig. 9

Find the coordinates of the points on the curve at which the normal has gradient 2. **[6]**

Exemplar 1

6 marks

$xy + y^2 = 8$

Normal gradient $= 2$

$\therefore$

Graph gradient = neg reciprocal $= \underline{-\frac{1}{2}}$ $\left(\frac{dy}{dx}\right)$

$xy + y^2 = 8$

$y = uv$ → $\frac{dy}{dx} = vu' + uv'$

$u = x \quad v = y$

$u' = 1 \quad v' = \frac{dy}{dx}$

$\boxed{\frac{dy}{dx} = x\frac{dy}{dx} + y}$

$x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$

$x\frac{dy}{dx} + 2y\frac{dy}{dx} = -y$

$\frac{dy}{dx}(x + 2y) = -y \rightarrow \underline{\frac{dy}{dx} = -\frac{y}{x+2y}}$ $\qquad \frac{dy}{dx} = -\frac{1}{2}$

$-\frac{1}{2} = \frac{-y}{x+2y} \rightarrow x + 2y = 2y$

$x = 0$

$y^2 = 8$

$y = \pm\sqrt{8}$

~~Coordinates are $(0, \sqrt{8})$ and $(0, -\sqrt{8})$~~

Coordinates: $(0, \sqrt{8})$ and $(0, -\sqrt{8})$

Examiner commentary

This is a fully correct example for this question. The '**In this question you must show detailed reasoning**' requirement means each line of the solution needs to be justified. Many candidates missed marks for not making explicit how the gradient for the tangent was identified, here the link between the given normal gradient of 2 and the tangent gradient of $-\frac{1}{2}$ has been made. This candidate has also secured the final accuracy mark by clearly finding both points.

Exemplar 2 — 3 marks

$xy + y^2 = 8$

$u = x \quad v = y \qquad y + x\frac{dy}{dx}$

$du = 1 \quad dv = \frac{dy}{dx}$

$y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$\cancel{x\frac{dy}{dx} = -3y}$

$\cancel{\frac{dy}{dx} = -\frac{3y}{x}}$

$\cancel{-\frac{3y}{x} = -\frac{1}{2}}$

$x\frac{dy}{dx} + 2y\frac{dy}{dx} = -y$

$\frac{dy}{dx} = \frac{-y}{x+2y} = -\frac{1}{2}$

$y = 1$, $x = 0$

$(0, 1)$

Examiner commentary

This is a good example of a candidate who understands implicit differentiation but is let down by their GCSE algebra. They correctly differentiate, use the given gradient of normal correctly and rearrange correctly to get $\frac{-y}{x = 2y} = -\frac{1}{2}$. However there is no working to show where the 'correct' answer of $x = 0$ comes from and the preceding $y = 1$ suggests they have simply equated the numerators and then equated the denominators rather than cross multiplying and solving.

Question 12

12 Show that the equation of the line in Fig. C2 is $ry + hx = hr$, as given in line 24. **[2]**

Exemplar 1 2 marks

$y = mx + c$ grad $= \dfrac{h}{r}$

$y \cancel{h} = mx + h$

$y = -\dfrac{h}{r}x + h.$

$yr = -hx + hr$

$yr + hx = hr$

Examiner commentary

This is the first of a number of 'show that' questions in Section B. An appropriate way to think of the question could be 'Find the equation of the line, showing all your working. By the way the answer you should get is $ry + hx = hr$

Many candidates realised to start by finding the gradient and this candidate was given 'benefit of the doubt' for their incorrect $\frac{h}{r}$ as they recovered to $-\frac{h}{r}$ when they used it. Along with most other candidates this solution used $y = mx + c$ although other formats were just as acceptable.

Question 13 (a) (ii)

(ii) Hence show that the cross-sectional area is $\frac{\pi r^2}{h^2}(h^2 - y^2)$, as given in line 37. **[2]**

Exemplar 1 — 1 mark

$\pi x(2r - x)$

$ry + hx = hr$

$\therefore hr - ry = hx$

$r(h - y) = xh$

$x = \frac{r}{h}(h - y)$

$x = r - \frac{ry}{h}$

$= \pi\left(r - \frac{ry}{h}\right)\left(2r - \left(r - \frac{ry}{h}\right)\right)$

$= \pi r - \frac{\pi r y}{h}\left(r + \frac{ry}{h}\right)$

	r	$\frac{ry}{h}$
πr	πr^2	$\frac{\pi r^2 y}{h}$
$-\frac{\pi r y}{h}$	$-\frac{\pi r^2 y}{h}$	$-\frac{\pi r^2 y^2}{h^2}$

$= \pi r^2 - \frac{\pi r^2 y}{h} + \frac{\pi r^2 y}{h} - \frac{\pi r^2 y^2}{h}$

$= \pi r^2 - \frac{\pi r^2 y^2}{h}$

$= \pi r^2\left(1 - \frac{y^2}{h^2}\right)$

$= \frac{\pi r^2}{h^2}(h^2 - y^2)$

Examiner commentary

Many candidates scored 1 mark here like this example. They earned the method mark for rearranging the equation from 12 to find x and then substituting it in the Area equation from 13(a)(i). However, to get the second mark they had to use correct algebra to show the given formula and many candidates found it difficult to maintain accuracy for the whole process.

In this solution you will see h instead of h^2 in the denominator of their last term in the expression below their table. Therefore, at this point they are given A0.

At this level the examiners will be looking for clear and correct algebra, especially in 'Show that' questions.

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